Polinomial model revisited – a simple calculation of the kinematical parameters of a 100 m sprint

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Abstract

We summarize and extend here a simple procedure allowing to obtain all important parameters of the 100 m run based on the measured values for the distance S from the start line and expired time t. An example for testing the proposed model are the results for segment values for S and t for elite sprinters, male: C. Lewis, M. Green and U. Bolt, and female: F. Griffith-Joyner., F. Ashford and H. Drecksler. The distance is approximated by a third order polinomial function S = f(t), which is easily fitted from the split (segment) times. This function is a mathematical model enabling, by using a standard mathematical treatment, to obtain the equations for determining the point of the maximal sprinter velocity vdmax, corresponding to vanishing acceleration, its distance from the start line Sdmax and corresponding time moment tdmax. The function provides direct reading of the initial velocity vo as well as finding the expressions for the sprinter instantaneous velocity vt and acceleration at. It also provides the initial acceleration, enabling to determine the force acting at the begining of the run. Results obtained justified the proposed approach for a universal and practical preparation tool and also showed that there do not exist so large differences in the values of kinematical parameters between analyzed male and female sprinters. The didactical purpose of the paper is to demonstrate how the combination of (unrealistic) exactly solvable model with the knowledge of the realistic behaviour can lead to a good numerical fit.

KEYWORDS

100 m run, segment times, third order polynomial approximation, mathematical fit, initial, instantaneous and maximal velocity and acceleration

INTRODUCTION

Although unexpected, the athletic running, sprint in particular, is a typical chalenge for physicists, to try to test their concepts and methods. One very simple idea is to write down an equation of motion, based on Newton's second law and then solve it. Simplifying assumptions are always welcome. So, we assume that a) the force pushimg the athlete forwards is a constant one and b) that the air resistance is proportional to the velocity (proportionality coefficient k).

We shall consider only 100 m run, so it is the one dimensional motion. More details about kinematics of the one dimensional motion and the quantities defining it (distance, velocity, acceleration) can be found in general physics courses, for example.[1]

This problem was discussed already in seventies by Keller.[2, 3] Solving the differential equation based on above assumptions, he obtained the well-known solution

$_{\tau} = _{\mu\varsigma}\ (1 - \epsilon^{-\beta\tau})$ (1.1)

Here v_t [m/s] is the instantaneous velocity, t [s] is the time interval elapsed from the start, while v_{ms} [m/s] is the maximal value of the velocity. $\beta = \kappa/\mu$.

This behaviour, which occurs in many situations in physics is so called saturation[4], and this equation tells us that the velocity will asymptotically approach to v_{ms} after very long („infinite“) time. This situation might correspond to some long distance running, but definitely not to the sprint. So we must look for a different approach. We tested several ideas in our initial work[5], but further study[6-8]indicated that the best was an approach based on experimental data which are broadly available. Actually, we wish to demonstrate that a fit of the experimental data is much more efficient when combined with theoretical and euristical arguments.

It has become a common practice for athletic events, that in 100 m run, the path is divided into 10 equal segments of 10 m length, each. At the begining and end of each segment, one measures its distance $S$from the start line and the time t which took sprinter to reach that point. On the basis of the measured values of S and$t$, it is possible, following the procedure described in the papers[5-8] to calculate the values of the average segment velocities v_s and define the polinomial function $S = f(t\ )$. The function $S = f(t\ )$ is a mathematical model describing precisely the dependence of the distance covered on elapsed time and can be used to derive the expression for the maximal sprinter velocity during the race v_{dmax} and corresponding acceleration a_{dmax} , distanceS_{dmax} and elapsed time$t_{dmax.}$. This function enables direct reading of the initial velocity v_o and accelerationa_o as well as the expression for the instantaneous sprinter velocity v_t and acceleration a_t .

In order to determine the point in the path corresponding tov_{dmax}, t_{dmax} andS_{dmax} , certain researchers, like Henry and Trafton[9] tried to develop a mathematical model using the function of acceleration at 60 yards and presented a model based on separated exponential functions for acceleration and deceleration phase during 100 m run. Prendergast[10], following the idea of Lames[11], produced a unified model valid for all 100 m of the run, by combining the exponential functions describing both the increase and decrease of the sprinter velocity. The model includes a set of parameters with special meaning whose calculation demands very sophisticated computer program, making its application a complex one.

Looking for a more realistic, yet simple model, we tried to formulate the function $S=f(t)$ not as an exponential function, but as a third order polinomial of the form $S(t)=P_1\ t + P_2\ t^2 - P_3 t^3$. The reasons will be explained further on. Fitting this polinomial to the given values for S and t , we obtained the particular expression for the function $S=f(t)$ which enabled finding v_{dmax}, S_{dmax}, v_o, a_o, v_t anda_t for an individual sprinter. The values obtained justified the idea of the approach and its suitability for a universal and practical application.

The example for testing how the polinomial function $S=f(t)$ can serve as a model for determining v_{dmax}, t_{dmax}, S_{dmax}, v_o, a_o, v_t and a_t ,will be the segment results for S and t in 100 m run of the elite sprinters. Tables with results for men appear in Ref.[5-8] and for women in[6,8] and we are not going to repeat them here. We intentionally use the same examples as in previous studies to demonstrate to what extent the application of the model can be extended.

The distribution of obtained results (Table 3) allowed to review to what extent there exist the differences in values of the kinematic parameters of male and female sprinters, clearly confirming expressed gender differences, which are not large in 100 m run and also to expose the dominant and impressive result of U.Bolt, who according to our model, achieved the maximal velocity of 12.66 m/s.

In order to give the paper a more pedagogical approach, we shall formulate our procedure „in three easy steps“. In this way, the athletic coaches can follow the procedure easily.

METHODS

The subject of our research were the results for elite sprinters, male: C. Lewis, M. Green and U. Bolt, and female: F. Griffith-Joyner, F. Ashford and H. Drecksler according to[12]. Nevertheless, the study design conformed to the ethical standards of the Helsinki Declaration and the ethical standards in sport and exercise science research described by Harriss and Atkinson[13].

Theory

2.1.1 Step one: simple mathematical expressions

If we wish to obtain the values for the parameters characterizing 100 m run, starting from the segment values of distance S and time$t$, we need an analytical expression for the dependence $S = f(t)$.

We have already mentioned that the solution (1.1) of the differential equation for given initial conditions includes an exponential term, providing so called saturation property, *i.e.* the maximal velocity is achieved only after very long (infinite) period of time, which is in discrepancy with the real situation.

On the other hand we know that during the sprint, velocity is rising achieving a maximum and then decaying after some time. So it is obvious that the mathematical form of the velocity *vs* time should be a curve having a maximum. One good candidate for such a behaviour is the inverted quadratic function (parabole). If we want to relate this to the above obtained solution, we can assume that the whole run happens in a short time period far away from the saturation. Then we can treat the obtained solution as the sum of Taylor's expansion in time variable. This also simplifies the mathematics. Taking into account the mathematical structure of the solution, we retain only certain terms of this series. Since we wish the velocity to be a quadratic function of time, the path must be one power higher, i.e. a cubic function which would then give a quadratic function for the derivative. In the same manner, the acceleration as a linear function of time. Denoting the positive coefficients attributed to t, t^2, t^3 as P_1, P_2, P_3 respectively, the expression for the distance$S(t)$ can be represented as a third order polinomial, without a constant term

$S(t)=P_1\ t + P_2 t^2 - P_3\ t^3$ (2.1)

where the coefficients can be obtained by fitting the data for $S$and t. Notice that the cubic term has the negative coefficient providing negative quadratic term in the expression for velocity, necessary for the function with the maximum. (No constant term is, of course, the consequence of the initial condition, $S=0$ for$t=0$.)

After the derivation, it follows from (2.1) that the velocity is

$v_t = dS/dt = P_1 + 2P_2\ t - 3P_3\ t^2$ (2.2)

and acceleration

$a_t = dv_t\ /dt = 2P_2 - 6P_3\ t$ (2.3)

Initial condition gives

$P_1 = v_0\ 2P_2 = a_o$ (2.4)

which allows a direct, yet simple determination of the initial velocity and acceleration. (For mathematical simplicity, we use the initial conditions for the moment $t=0$, although more accurate approach would be to measure time after start reaction time, but this will be the subject of some further studies.) Data fit can be performed by any convenient program, and in our previous work[5-8] we used the program Origin 6.1.

Simple mathematics offers some additional information, not discussed previously.

The sprinter achieves the maximal velocity $v_{dmax}$for vanishing acceleration, *i.e.*

$a_{dmax} = 0$ (2.5)

at the corresponding moment of time $t = t_{dmax}$, which, according to (4) and (6) equals to

$$t_{dmax} = \frac{P_2}{3P_3}$$

$t_{\mathrm{dmax}} = \frac{P_2}{3P_2}$ (2.6)

Substituting (7) into (3), one gets the maximal velocity

$v_{dmax} = P_1 + 2P_2 t_{dmax}$ (2.7)

corresponding to the distance

$$S_{dmax} = \frac{t_{dmax}}{3}(P_1 + 2v_{dmax})$$

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(2.8)

2.1.2 Step two: fitting the polinomial

Presented theoretical treatment allows us to determine the following parameters of the run: maximal sprinter velocity v_{dmax} in 100 m run, corresponding distance S_{dmax} and time moment t_{dmax} , as well as the initial velocity v_o , instantaneous velocity v_t and acceleration a_t .We need to know the function. S for any particular sprinter in a given run. The sample for our analysis will be measured segment data for S and t of elite sprinters; male: C. Lewis (1988, Seoul), M. Green (2001, Berlin) and U. Bolt (2009, Berlin) and female: F. Griffith-Joyner., F.Ashford and H. Drechsler (1988, Seoul). These data, presented in our previous works[5–8] were used to fit the polinomial coefficients, for each of the sprinters.

We have at our disposal 11 pairs of data, sufficient to perform a fit by using any convinient program. The polinomial functions obtained in this way previously[5–8] are reviewed in the Tables 1 and 2. The quality of the fit was discussed in detail in our previous work[5].

Insert here **Table 1.**

Insert here **Table 2.**

Good agreement of measured points (pairs S and t) with the curve $S=f(t)$ as shown in the previous works[5–8], indicates the conclusion that for the description of the dependence of S on t , one can use the function in the form of third order polinomial with no constant term.

2.1.3 Step three: the values of the parameters

$$a_{dmax} = 0$$

$$S_{dmax} = \frac{t_{dmax}}{3}(P_1 + 2v_{dmax})$$

Using the numerical values of the coefficients P_1, P_2, P_3 in the equations (2.9) to (15) (Tables 1 and 2) to calculate the expressions (2.2) to (2.8), one obtains the values for the initial velocity v_o , instantaneous velocity v_t and acceleration a_t , as well as for the position S_{dmax} at which the sprinter achieves the maximal velocity v_{dmax} , (corresponding to $a_{max} = 0$ after the time t_{dmax} from the start, all presented in the Table 3. Part a) of the Table 3 presents results evaluated from the polinomial model, while part b) is based on crude estimates based on segemnt times and it is offered here for the sake of comparision.

Table 3 also includes the the initial acceleration a_o , i.e. the acceleration at the begining of the run. More important, multiplying it with the sprinter's mass, one can evaluate also the force acting at this moment, we call it ,,latent force" F_{lat} .

Insert here **Table 3** .

The distribution of the obtained results in Table 3 allows to observe the degree of variation of kinematic parameters and the gender differences related to 100 m, which exist, but are not substantial. Calculated values for the maximal velocity v_{dmax} clearly differ from the values of the maximal segment velocity v_{smax} listed in the papers by[5-8] and presented in the Table 3. This difference for studied sprinters ranges from 2.1 to 2.9 % for male, and from 3.2 to 3.9 % for female ones. More clear reasons for this difference follow from the very definitions for two different forms of the velocity, the instantaneous v_{dmax} and average one v_{smax} .

The value of the maximal velocity of 12.66m/s achieved by Bolt at the distance of 61,43m at the time 6.41 s in the segment between 60-70m, being the first sprinter to run it for 0,81 s and the world record of 9.58 s is more than impressive and a dominant one.

DISCUSSION

In order to demonstrate how many different information one can get from this simple fit, let us now analyze the results based on the polinomial fit $S=f(t)$, presented in Table 3. It also includes the data for the maximal average segment velocity v_{smax} taken from our previous papers[5-8] with the purpose to indicate the need of using the values of maximal instantaneous velocity v_{dmax} and confirm that v_{dmax} [?] v_{smax} . Evident differences (with respect ot time and distance) indicate to the error made in eventual declaring of v_{smax} for the maximal velocity of the run. As mentioned above, this error equals 2,1–2,9 % for male and 3,2–3,8 %

for female sprinters, in the examples discuassed above. This is incorrect also from the kinematic point of view since we are dealing with two different kinds of velocity: the instantaneous one v_{dmax} corresponding to a single point of a particular segment and the average onev_{smax} related to the whole segment in which the maximal instantaneous velocity was achieved. It is important for students to understand this difference.

The values for v_t and $a_t$listed in previous works, unambiguously indicate that sprinters during the short period of the 100 m run, pass through four different phases of the velocity which are:

1. initial velocity,

2. velocity corresponding to the phase of acceleration (a_t positive),

3. maximal velocity which is the highest possible velocity that an individual can realize running and which is considered to be the most important part of the run (a_{dmax}=0) [14-16] and

4. velocity endurance in the phase of deceleration from the maximal velocity till finish ($a_t$negative)[17, 18].

The results presented, confirm that these are top sprinters of excellent performances, so they can be used as the referent ones for the comparative analysis of the velocity of average sprinters[19].

Presented values (Table 3) for achieved maximal velocityv_{dmax} and duration time of the acceleration and deceleration during the run, are dominated by the uprecendented results of U. Bolt. It seems that they demand reexamining of the existing concepts and models.

At this point we wish to make a particular comment on the relation between our values, and the values for the quantities concerning the maximal velocity, obtained by[20]. They performed measurements on sprinters (including 100 m run) during 2009 IAAF World Championship in Athletics in Berlin, combining cameras for segment times and lasers (using IR radiation with 50 Hz and 100 Hz frequency) obtaining in this way nearly 1000 pairs time – distance. Since laser beam was always directed at the same spot of sprinter's body, the results contained intracyclical oscillations which were eliminated by filtering. The curve was then differentiated to produce the velocity. The paper offers data for the first three runners, but we are interested in the results for U. Bolt. According to them, his maximal velocity was 12.34 m/s achieved at the distance of 67.90 m. It is important to notice that the velocity equal to 99 % of maximal velocity was achieved already at 51.27 m. Let us repeat that our results arev_{dmax} =12.66 m/s,S_{dmax} =61.43 m (t_{dmax} =6.41 s).

First of all, the difference in the value of the maximal velocity is 2.7 %, which in absolute sense is significant, but for a coach it does not mean much. There are two possible reasons for this discrepancy. First one, our fit is based on the points separated for 10 m apart, and it is actually possible that in a segment that large, the velocity can deviate from a typical parabolic peak to a more ,,flattened" curve and the fit can not ,,see" this. Also, within this segment, the position of the peak can be displaced with respect to the maximum of the parabole. So, it is the size of the segment which can cause this discrepancy. However, there exists another option, and that is that the filtering of data is the reason of ,,flattening" of the velocity curve. An argument in favour of this assumption is the fact that according to Graubner and Nixdorf, the maximal velocity is smaller than the maximal segment velocity which is rather difficult to accept. For this reason, we think that our polinomial fit is still a reasonably good measure for the coaches who need a simple and easily tractable procedure for determining the kinematical parameters of a 100 m sprint run.

Let us consider the force F_{lat} also given in the Table 3. In our previous work[6] we have estimated the horizontal starting force F_{st} as the ratio of the change of the linear momentum (mv_o) to the start reaction time which can be measured. This force is also quoted in the Table 3. It cn be seen that it is an order of magnited higher thanF_{lat} and at the moment we are not sure about their mutual relationship. However, it is our opinion that thisF_{lat} actually ,,produces" initial velocity.

CONCLUSION

$$a_{dmax} = 0$$

$$a_{dmax} = 0$$

$$a_{dmax} = 0$$

The general idea of the paper was to demonstrate how we can combine various approaches to look for the most suitable analytical form necessary for a fit. Developing our previous proposition to model the distance – time dependence ($S=f(t)$) by a third order polinimial without a constant term, we have shown in this paper how simple mathematical procedure can provide the values for the kinematic parameters related to the maximal velocity, more precisely: the positionS_{dmax} at which the sprinter achieves the maximal velocity v_{dmax} , (corresponding to

a_{dmax} =0) after the time t_{dmax} from the start. This are tthe information every coach would be glad to have. Model was tested for six top sprinters (both male and female) and in our opinion lead to reasonable results, especially with respect to maximal segment velocity, often used in practice.

The discrepancy between our fit and a more sophisticated fit[20] is also discussed.

The results produced in the present work have justified the idea of the polinomial treatment for a universal application leading to simple calculation of all essential kinematic parameters for 100 m sprint run.

5 WHAT NEXT ?

There are various options how to extend this approach. We must stress here that the expansion of the solution was only a guidance towards the form of the function for fitting. We treated polinomial coefficients just as the numbers to be fitted and nothing else.

However, one can also accept the idea that the expressions (2.1) is actually obtained from the expansion, and then the coefficientsP_2 and P_3 are expressed in terms of k/m. If this were taken into account, then there exist relations enabling us to calculate the coefficient k if the mass$m$ of the sprinter is known [5-7].

Another broad field is the idea that there may exist another form of the resistance force, the one proportional to v^2 . Probably the first research related to define an expression which would describe the effect of some aerodynamic factors influencing the result of 100 m run were the equations of Hill[21] which used measurement in the air (wind) tunnel to derive the expressions for the air resistance and wind velocity. Later, this kind of studies attracted more interest[22-26]. So, there is a lot of room for different interpretation of the physical meaning of the polynomial coefficients and it is a completely new field for research.

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CONFLICT OF INTERESTS

The authors declare that they have no competing interests.

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`Tables.doc` available at https://authorea.com/users/355397/articles/478652-polinomial-model-revisited-a-simple-calculation-of-the-kinematical-parameters-of-a-100-m-sprint