Derivation and analysis of partial integro-differential inequality on shout options with its underlying asset subject to jump-diffusion model

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Abstract

Up to present, research on shout options remains only on the assumption that the underlying asset follows either Brownian motion or geometric Brownian motion. But it can not be evaluated accurately by PDE on geometric Brownian motion. To solve this problem this paper derives a new partial integro-differential inequality (PIDI) for shout options pricing on the assumption that the price of the underlying asset follows the jump-diffusion model and constructs the mathematical model by combining specific features and terminal conditions. On the basis of this model we obtain some results about shout options pricing. For this mathematical model this paper proposes a new competitive algorithm to choose two aspects. One is employing high-order difference for integral and partial derivative terms, the other is using Howard algorithm (also called policy iteration) for the complementarity problem. Numerical examples show that this algorithm yields an accurate technique and is more efficient than the traditional approaches in the case of geometric Brownian motion and jump-diffusion model, respectively.

1 INTRODUCTION

Exotic options traded in the over-the-counter market has become increasingly important since the early 1980s and is larger than the exchange-traded market. An advantage of exotic options is that they can be tailored by a financial institution to meet the particular needs of a corporate treasurer or fund manager. One of them is called shout options [1] given the holder to shout to the writer once or more times according to specified rules during its life. At the end of the life of the option, the option holder receives either the usual payoff from a European option or the intrinsic value at the time of the shout, whichever is greater. The valuation and hedging of shout options is more complicated than that of standard options because there is an element of uncertainty in the investor's actions.

Up to present, academic research on shout options has not been extensive. Thomas [1] described the simple type of shout options in 1993. Cheuk and Vorst [2] considered shout options with multiple exercise opportunities and presented explicit type methods for pricing these types of derivatives under the assumption that the underlying asset followed geometric Brownian motion in 1997. Boyle et al [9] used Monte Carlo simulation to deal with Greek function integrals and dynamic programming to price 11*Corresponding author:Jun Liu E-mail address: junliu7903@126.com complicated shout options in 1999.In his paper Boyle presented when the number of factors exceeds four, the complication of this method far exceeds that of numerical partial differential method. Windcliff et al. [4] solved a system of interdependent linear complementarity problem to value the shout options and considered numerical issues related to interpolation and choice of time stepping method in detail in 2001.Dai and Kwok [6] developed a linear complementarity problem to analyze shout options and provided shouting boundaries using the binomial scheme and recursive integration approach in 2004. Goard [8] derived exact solutions for both the price of the shout call option and the strike reset put option where they each have a single shout right during the life of the contract in 2012. Ballestra and Cecere [21] presents a new numerical method for solving the linear complementarity problem controlled by partial integro-differential equations in 2016. Mallier and Goard [5] used an integral equation method to value shout options and found the behavior of the optimal exercise boundary for one and two shout options close to maturity in 2018.

More complicated shout options were embedded in other financial products, such as segregated funds sold by Canadian life insurance companies. These products provided a guarantee for the holders to permit to reset shout times, up to some limit during the life of the contract. It is also worth noting that some energy derivative contracts have included a feature called swing options [3], which is similar in many respects to complicated shout options. About these contracts embedded with shout options Windcliff et al. [7] explored the valuation using an approach based on the numerical solution of a set of linear complementarity problems in 2001.In his paper he indicated the shout option components of many of these contracts may be underpriced.

During last decade there have been some literature to propose and extend high-order difference approach for solving the partial integro-differential equation (PDE) arising from option pricing. Düring and Fournié [12] derived a high-order difference scheme for option pricing in Heston model in 2012 .They extended this method to non-uniform grids in 2014[13] and multiple space dimensions in 2015[14]. In 2019 Düring and Pitkin [15] applied this approach to extend to stochastic volatility jump modes for option pricing. The advantage of this approach is that it is very parsimonious in terms of memory requirements and computational effort and is more efficient than finite element approaches for option pricing [15].

About Howard algorithm (also called policy iteration [17]), Howard [16] proposed this technique for the solution of the Hamilton-Jacobi-Bellman(HJB)equations in finance in 1960.Thakoor et al. [18] developed a new procedure for the linear complementarity formulation and used Howard's algorithm to solve the discrete problem obtained through a higher-order Crandall-Douglas discretization in 2019.The advantage of this algorithm is to ensure convergence for solution of the discretized equations under sufficient conditions[17].

The purpose of this paper is to evaluate complicated shout options more accurately. So far research on shout options remains only on the assumption that the price of the underlying asset follows either Brownian motion or geometric Brownian motion. Now, Compared with geometric Brownian motion, jump-diffusion model can describe the underlying asset more accurately. Therefore, this paper derives a new partial integro-differential inequality (PIDI) for shout options pricing on the assumption that the price of the underlying asset follows the jump-diffusion model and constructs the mathematical model by combining specific features and terminal conditions. Another innovation is that this paper proposes a new competitive algorithm to choose two aspects for this mathematical model. One is employing high-order difference for integral and partial derivative terms, the other is using Howard algorithm (also called policy iteration) for the complementarity problem. The advantage of this action is to make full use of the advantages of Howard algorithm and the high-order difference algorithm, that is, to ensure the convergence and achieve valuation result more accurately than the traditional finite element method for the shout options.

The rest of the paper is organized as follows. In Section 2 we derive a new partial integro-differential inequality (PIDI) for shout options pricing on the assumption that the price of the underlying asset follows the jump-diffusion model and constructs the mathematical model by combining specific features and terminal conditions. In Section 3 we propose a new competitive algorithm by combining high-order difference and Howard algorithm. In Section 4 we present numerical examples to compare the convergence and efficiency of the scheme to traditional methods. Section 5 concludes.

2 MATHEMATICAL MOEL OF SHOUT OPTIONS

In this section, we derive a new partial integro-differential inequality (PIDI) for complicated shout options pricing on the assumption that the price of the underlying asset follows the jump-diffusion model and construct the mathematical model by combining specific features and terminal conditions.

The simple form of shout options is defined in John C. Hull [10]. Shout options is such an option that the holder can shout to the option seller during the life of the option. At the end of the life of the option, the option holder receives either the usual payoff from a European option or the intrinsic value at the time of the shout, whichever is greater. That is, the payoff function of simple shout options is

$$g\left(S,K\right)=\begin{cases} \&\max\left\{K-S_T,0\right\}, \quad \text{if no } \mathit{shout}, \\ \&\max\left\{S_\tau-S_T,0\right\}+K-S_\tau, \text{if shouting occur at the time } \tau,\tau\in\left(0,T\right), \end{cases}$$

where S denotes the price of underlying asset, T is the maturity time and τ is any time of the period.

The more complicated shout options is defined in Windcliff [4].The holder could have multiple rights according to the contract rules, in some cases with a limit placed on the number of rights which may be exercised within a given time period. That is, the holder has rights to convert the holding option contract into another one with the same form but less flexibility by exercising the options.

2.1. Mathematical model of shout options

So far research on shout options remains only on the assumption that the price of the underlying asset follows either Brownian motion or geometric Brownian motion. Now, Compared with geometric Brownian motion, jump-diffusion model can describe the underlying asset more accurately. Therefore, this paper assumes that the underlying asset follows jump-diffusion model, that is

$$\mathrm{dS}\left(t\right)=\left(r-\beta\lambda\right)\mathrm{Sdt}+\sigma\Sigma\left(t\right)\mathrm{dW}\left(t\right)+\left(y-1\right)S\left(t\right)\mathrm{dQ}\left(t\right), \tag{2.1}$$

where r is risk free rate, σ is volatility of underlying asset,$W\left(t\right)$ denotes Brownian motion. $S\left(t-\right)$ is the value of S immediately before the jump. $Q\left(t\right)$ is compound Poisson process with intensity λ and mean size β.

Suppose the jump sizes Y have a density $\mathrm{f}\left(y\right)$. In this case, the average jump size $\beta=\mathrm{EY}=\int_0^{+\infty}yf\left(y\right)\mathrm{dy}$. For example, the jump size Y of underlying asset price follows log-normal distribution, that is

$$f\left(y\right)=\frac{1}{\sqrt{2\pi}\psi\gamma}e^{-\frac{\left(logy-\alpha\right)^2}{2\gamma^2}},$$

then the average jump size $\beta=e^{\alpha+\gamma^2/2}$.

Under the assumption that the price of the underlying asset follows jump-diffusion model, we derive a new partial integro-differential inequality (PIDI) for complicated shout options pricing, that is, theorem 2.1 is a new result in this paper.

Theorem 2.1(The mathematical model for complicated shout put options) . *When the underlying asset price S follows the jump-diffusion model (2.1) and the jump size has a density function* $\mathrm{f}\left(y\right)$, *the shout put options value V satisfies the PIDI (2.2) and the inequality (2.3)*

$$\frac{\partial V}{\partial t}+\left(r-\beta\lambda\right)S\frac{\partial V}{\partial S}+\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}+\lambda\left[\int_0^{+\infty}V\left(t,\mathrm{yS}\right)f\left(y\right)dy-V\left(t,S\right)\right]-\mathrm{rV}\leq0, \tag{2.2}$$

$$\tilde{V}\leq V, \tag{2.3}$$

and the terminal condition

$V\left(S,K,U,T\right)=\{$

$\&0, \quad S\to+\infty,$
$\&K-S,S\to-\infty,$

where the value $\tilde{V}$ *of the contract the holder receives upon shouting for complicated shout options is defined as*

$\tilde{V}\left(S,K,U,t\right)=\{$

&$V\left(S,K,U+1,t\right)+D\left(S,K,U,t\right)$, if $U+1\leq U_{\max}$,
&$-\infty$, otherwise. (2.4)

Proof: Under the risk-neutral measure, the underlying asset price S follows the following stochastic process

$$\mathrm{dS}\left(t\right)=\left(r-\beta\lambda\right)\mathrm{Sdt}+\sigma\Sigma\left(t\right)\mathrm{dW}\left(t\right)+\left(y-1\right)S\left(t\right)\mathrm{dQ}\left(t\right).$$

The Itô formula implies

$$e^{-\mathrm{rt}}\left[V\left(t,S\left(t\right)\right)-V\left(0,S\left(0\right)\right)\right]$$

$$=\int_0^t e^{-\rho\varkappa}\left[-\rho^{\breve{}}\left(k,S\left(k\right)\right)+\frac{\partial V}{\partial t}\left(k,S\left(k\right)\right)+\left(r-\beta\lambda\right)S\left(k\right)\frac{\partial V}{\partial S}\left(k,S\left(k\right)\right)+\frac{1}{2}\sigma^2S^2\frac{\partial^2V\left(k,S\left(k\right)\right)}{\partial S^2}\right]\mathrm{dk}$$

$$+\int_0^t e^{-\mathrm{rk}}\sigma\Sigma\left(k\right)\frac{\partial V}{\partial S}\left(k,S\left(k\right)\right)\mathrm{dW}\left(k\right)$$

$$+\int_0^t e^{-\mathrm{rk}}\left[V\left(k,S\left(k\right)\right)-V\left(k,S\left(k-\right)\right)\right]\mathrm{dk}. \quad (2.5)$$

If S has a jump at time k, then the underlying asset price satisfies

$$S\left(k\right)=\mathrm{yS}\left(k-\right).$$

We examine the last term in (2.5)

$$\int_0^t e^{-\mathrm{rk}}\left[V\left(k,S\left(k\right)\right)-V\left(k,S\left(k-\right)\right)\right]\mathrm{dk}$$

$$=\int_0^\infty\int_0^t e^{-\mathrm{rk}}\left[V\left(k,\mathrm{yS}\left(k-\right)\right)-V\left(k,S\left(k-\right)\right)\right]\mathrm{dkdy}$$

$$=\int_0^\infty\int_0^t e^{-\rho\varkappa}\left[V\left(k,\psi\Sigma\left(k-\right)\right)-V\left(k,S\left(k-\right)\right)\right]d\left(N\left(k\right)-\lambda\varkappa\right)\mathrm{dy}$$

$$+\int_0^t e^{-\rho\varkappa}\left[V\left(k,\psi\Sigma\left(k-\right)\right)-V\left(k,S\left(k-\right)\right)\right]\lambda\delta\varkappa. \quad (2.6)$$

Substituting (2.6) into (2.5), we obtain

$$d\left(e^{-\mathrm{rt}}V\left(t,S\left(t\right)\right)\right)=e^{-\mathrm{rt}}\left\{-\mathrm{rV}\left(t,S\left(t\right)\right)+\frac{\partial V}{\partial t}\left(t,S\left(t\right)\right)+\left(r-\beta\lambda\right)S\left(t\right)\frac{\partial V}{\partial S}\left(t,S\left(t\right)\right)\ +\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{}\right.$$

$$+\int_{0}^{\infty} e^{-\rho\tau}\left[V\left(t,\psi\Sigma\left(t-\right)\right)-V\left(t,S\left(t-\right)\right)\right]d\left(N\left(t\right)-\lambda\tau\right). \tag{2.7}$$

Using Itô formula for $e^{-\mathrm{rt}}\pi\left(t\right)$, the following equation can be obtained

$d\left(e^{-\mathrm{rt}}\pi\left(t\right)\right)=e^{-\mathrm{rt}}\left[-\rho\pi\left(t\right)\mathrm{dt}+\delta\pi\left(t\right)\right]$

$=e^{-\mathrm{rt}}\left[\pi\left(t\right)\mathrm{dS}\left(t\right)-\mathrm{rS}\left(t\right)\pi\left(t\right)\mathrm{dt}\right]$

$=e^{-\mathrm{rt}}\left[-\sigma\Sigma\left(t\right)\pi\left(t\right)\mathrm{dW}\left(t\right)+\pi\left(t\right)S\left(t-\right)d\left(Q\left(t\right)-\beta\lambda\tau\right)\right]$

$$=e^{-\mathrm{rt}}\left[-\sigma\Sigma\left(t\right)\pi\left(t\right)\mathrm{dW}\left(t\right)+\pi\left(t\right)S\left(t-\right)\int_{0}^{\infty}d\left(N\left(t\right)-\lambda\delta\tau\right)\right]. \tag{2.8}$$

Let

$$\pi\left(t\right)=\frac{\partial V}{\partial S}.$$

Considering (2.7) and (2.8), we obtain

$d\left[e^{-\mathrm{rt}}V\left(t,S\left(t\right)\right)-e^{-\mathrm{rt}}\pi\left(t\right)\right]$

$=d\left[e^{-\mathrm{rt}}V\left(t,S\left(t\right)\right)\right]-d\left[e^{-\mathrm{rt}}\pi\left(t\right)\right]$

$$=e^{-\mathrm{rt}}\left\{-\mathrm{rV}\left(t,S\left(t\right)\right)+\frac{\partial V}{\partial S}\left(t,S\left(t\right)\right)+\left(r-\beta\lambda\right)S\left(t\right)\frac{\partial V}{\partial S}\left(t,S\left(t\right)\right)+\frac{1}{2}\sigma^{2}S^{2}\frac{\partial^{2}V\left(t,S\left(t\right)\right)}{\partial S^{2}}+\lambda\left[\int_{0}^{\infty}V\left(t,\mathrm{yS}\left(t\right)\right)f\left(y\right)\mathrm{dy}-V\left(\right.\right.\right.$$

Therefore, the following inequality can be established

$$-\mathrm{rV}+\frac{\partial V}{\partial t}+\left(r-\beta\lambda\right)S\frac{\partial V}{\partial S}+\frac{1}{2}\sigma^{2}S^{2}\frac{\partial^{2}V}{\partial S^{2}}+\lambda\left[\int_{-1}^{+\infty}V\left(t,\mathrm{yS}\right)f\left(y\right)\mathrm{dy}-V\left(t,S\right)\right]\leq 0.$$

Because the remaining times of contract held by the shout options holder after exercising the shout right is shorter than that of the original contract, and the number of shouts is less, there is

$$\tilde{V}\leq V.$$

For random variable U, when $U+1\leq U_{\max}$, the new contract value $\tilde{V}\left(S,K,U,t\right)$ with residual shout numbers U corresponds to the original contract value$V\left(S,K,U+1,t\right)$ with shout numbers $U+1$ and the dividend function $D\left(S,K,U,t\right)$ due to shout, that is

$$\tilde{V}\left(S,K,U,t\right)=\begin{cases}\&V\left(S,K,U+1,t\right)+D\left(S,K,U,t\right),\mathrm{if}U+1\leq U_{\max}, \\ \&-\infty, \qquad \text{otherwise.}\end{cases}$$

At the maturity time $t=T$ we can get the following equation

$V\left(S,K,U,T\right)=\ g\left(S,K\right).$

We will restrict attention in this work to shout put options, then the payoff function is given by

$g\left(S,K\right)=\max\left(K-S,0\right),$ (2.9) then for shout put option

$V\left(S,K,U,T\right)=\max\left(K-S,0\right),$

So the terminal condition of shout put options is

$V\left(S,K,U,T\right)=\{$

&$0,\qquad S\to+\infty,$
&$K-S, S\to-\infty.$

2.2 Some results from Theorem2.1

According to the Theorem2.1 we can obtain some results including the simplified form of the PIDI (2.2), discrete jump distribution and completeness and so on.

When the jump sizes follow specific continuous distributions, the PIDI (2.2) can be transformed into the partial differential inequality.

Corollary 2.1(Simplified form of the PIDI (2.2)) Assuming that the jump sizes Y_i of underlying asset price follow exponential distribution, that is, $f\left(y\right)=\theta e^{-\theta y}$, the PIDI (2.2) can be simplified as follows

$$-\mathrm{rV}+\frac{\partial V}{\partial t}+\mathrm{aS}\frac{\partial V}{\partial S}+\frac{b}{2}S^2\frac{\partial^2 V}{\partial S^2}\leq 0\ ,$$

where $a=r+\lambda\left(\frac{1}{\theta}-1-\beta\right), b=1+\sigma^2-\frac{2}{\theta}+\frac{2}{\theta^2}.$

Proof: For$\left[V\left(t,\left(y+1\right)S\right)-V\left(t,S\right)\right]$using Taylor expansion and substituting into the PIDI (2.2), we obtain

$$-\mathrm{rV}+\frac{\partial V}{\partial t}+\left[r-\beta\lambda+\lambda\int_0^\infty\left(y-1\right)f\left(y\right)\mathrm{dy}\right]S\frac{\partial V}{\partial S}+\left[\sigma^2+\int_0^\infty\left(y-1\right)^2f\left(y\right)\mathrm{dy}\right]\frac{S^2}{2}\frac{\partial^2 V}{\partial S^2}\leq 0. \tag{2.10}$$

If Y follows the exponential distribution with parameter θ, then

$$\int_0^\infty \mathrm{yf}\left(y\right)dy=EY=\frac{1}{\theta}\ , \tag{2.11}$$

$$\int_0^\infty y^2 f\left(y\right)dy=EY^2=\mathrm{DY}+E^2Y\ =\frac{1}{\theta^2}+\frac{1}{\theta^2}=\frac{2}{\theta^2}\ . \tag{2.12}$$

Let

$$a=r+\lambda\left(\frac{1}{\theta}-1-\beta\right),$$

$$b=1+\sigma^2-\frac{2}{\theta}+\frac{2}{\theta^2}\ .$$

Substituting (2.11) and (2.12) into (2.10), we obtain

$$-\mathrm{rV}+\frac{\partial V}{\partial t}+\mathrm{aS}\frac{\partial V}{\partial S}+\frac{b}{2}S^2\frac{\partial^2 V}{\partial S^2}\leq 0\ .$$

When the jump sizes are discrete random variables, the PIDI followed by the price of complicated shout options could be obtained easily.

Remark 2.1(Discrete jump distribution). There are modifications of Theorem 2.1 for the case when the jump sizes Y_i have a probability mass function$p(y_1), \cdots, p(y_m)$ rather than a density $f(y)$ under risk-neutral measure. In (2.2), the term$\int_0^{+\infty} V(t, \mathrm{yS}) f(y) \mathrm{dy}$would be replaced by$\sum_{m=1}^{M} p(y_m) V(t, y_m S)$.

The mathematical model of complicated shout options (Theorem 2.1) always holds for an arbitrary shout options with different payoff functions.

Remark 2.2(Completeness). In this subsection, we have contracted the value for complicated shout put options on the underlying asset driven by jump-diffusion processes. It is clear from the analysis that the same argument would work for an arbitrary shout options with payoff$h(S(T))$ at time T written on a stock modelled this way. One could simply replace the put payoff by the function of h in equation (2.9).The partial integro-differential inequality (2.2) and the inequality (2.3) would still apply, although now with the terminal condition $\mathrm{V}(S, K, U, T) = h(S)$.

The following Remark2.3 shows that the previous research on shout option is only a specific case of this paper and Remark 2.4 gives the significance of inequality (2.3).

Remark 2.3 (Consistency). If there is not jump during the life of the contract, that is, the Eq. (2.1) does not contain the jump term, then the PIDI (2.2) reduce to PDE in [4].

Remark 2.4 (Sufficiency). Shout options holders have the right to exercise at any time or shout any times during the life of the contract. In view of this situation, the second inequality in (2.2) ensures that the seller of shout options has sufficient amount to meet the buyer's exercise.

3 AN NEW ALGORITHM FOR PRICING SHOUT OPTIONS

Another innovation is that this paper proposes a new competitive algorithm to choose two aspects for this mathematical model. One is employing high-order difference for integral and partial derivative terms, the other is using Howard algorithm (also called policy iteration) for the complementarity problem.

Let $x = \mathrm{logS}, \tau = T - t$, we obtain

$$-\frac{\partial V}{\partial \tau} + \left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)\frac{\partial V}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial x^2} + \lambda\left[\int_0^{+\infty} V(T-\tau, ye^x) f(y)\,\mathrm{dy} - V(T-\tau, e^x)\right] - \mathrm{rV} \le 0. \tag{3.1}$$

In order to calculate conveniently, it is necessary to limit x to the region $\Omega = [-\varphi, +\varphi]$. Considering the uniform grid with interval $h = 2\varphi/M$ that the grid points are given by $x_m = -\varphi + \mathrm{mh}, m \in [0, M]$, and $V_m(\tau) = V(x_m, \tau)$ is defined.

3.1 Handling integral term

Integral term is defined on$(-\infty, +\infty)$,then the region outside Ω_1 also needs to be calculated

We set

$z = logy, \tilde{H}(z, \tilde{y}, U, \tau) = H(e^z, \tilde{y}, U, \tau)$ and$\tilde{f}(z) = e^z f(e^z)$

so the integral term of (3.1)

$$\lambda\int_0^{+\infty} V(T-\tau, ye^x) f(y)\mathrm{dy}$$

$$= \lambda\int_{-\infty}^{+\infty} V\left(T-\tau, e^{x+z}\right) f(e^z) e^z \mathrm{dz}$$

$$= \lambda\int_{-\infty}^{-\varphi} V\left(T-\tau, e^{x+z}\right) f(e^z) e^z dz + \lambda\int_{-\varphi}^{+\varphi} V\left(T-\tau, e^{x+z}\right) f(e^z) e^z dz + \lambda\int_{+\varphi}^{+\infty} V\left(T-\tau, e^{x+z}\right) f(e^z) e^z \mathrm{dz}$$

By the composite trapezoidal rule in the region Ω, we have the following approximation

$$\lambda \int_{-\varphi}^{+\varphi} V\left(T-\tau, e^{x+z}\right) f\left(e^{z}\right) e^{z} \mathrm{dz}$$

$$\approx \frac{h}{3} \sum_{m=1}^{M}\left[V\left(T-\tau, e^{x+z_{m-1}}\right) f\left(e^{z_{m-1}}\right) e^{z_{m-1}}+4 V\left(T-\tau, y e^{x+z_{m}}\right) f\left(e^{z_{m}}\right) e^{z_{m}}+V\left(T-\tau, e^{x+z_{m+1}}\right) f\left(e^{z_{m+1}}\right) e^{z_{m+1}}\right]$$

Integral term is defined in the region$(-\infty,+\infty)$, then the region outsideΩ also needs to be calculated.

For shout put options, when φ is sufficiently large, we obtain

$$\lambda \int_{+\varphi}^{+\infty} V\left(T-\tau, e^{x+z}\right) f\left(e^{z}\right) e^{z} d z \approx \lambda \int_{+\varphi}^{+\infty} \max \left(1-e^{\xi}, 0\right) f(\xi) d \xi=0.$$

For the integral term of shout put options defined in the region$(-\infty,-\varphi)$, we estimate its value by use of the composite trapezoidal rule

$$\lambda \int_{-\infty}^{-\varphi} V\left(T-\tau, e^{x+z}\right) f\left(e^{z}\right) e^{z} d z \approx \lambda \int_{-\infty}^{-\varphi} \max \left(1-e^{\xi}, 0\right) f(\xi) \delta \xi.$$

3.2 Handling differential term

Let δ_{x} and δ_{x}^{2} denote the first and second order central difference approximations with respect to x, respectively, then

$$\delta_{x} V_{m}=\frac{V_{m+1}-V_{m-1}}{2 h}$$

and

$$\delta_{x}^{2} V_{m}=\frac{V_{m+1}-2 V_{m}+V_{m-1}}{h^{2}}.$$

At the internal node of region Ω we obtain

$$V_{\tau}=\left(r-\beta \lambda-\frac{1}{2} \sigma^{2}\right) \delta_{x} V_{m}+\frac{1}{2} \sigma^{2} \delta_{x}^{2} V_{m}-r V_{m}+\lambda \omega_{m}+\varepsilon_{m}, \tag{3.2}$$

where$\omega_m = \} (V_m) + \updownarrow (V_m)$.

Local truncation error

$$\varepsilon_{m}=\frac{h^{2}}{12}\left(2\left(r-\beta \lambda-\frac{1}{2} \sigma^{2}\right)\left(V_{\mathrm{xxx}}\right)_{m}+\frac{1}{2} \sigma^{2}\left(V_{\mathrm{xxxx}}\right)_{m}\right)+\mathcal{O}\left(h^{4}\right).$$

Fourth-order discretization can be obtained by substituting higher derivatives in terms of V_x and V_{xx} and using central difference approximation. Referring to Thakoor et al [11], there is

$$\frac{1}{2} \sigma^{2} V_{\xi \xi \xi}=V_{\xi \tau}-\left(r-\beta \lambda-\frac{1}{2} \sigma^{2}\right) V_{\mathrm{xx}}+r V_{x}-\lambda \omega_{x},$$

and

$$\left(\frac{1}{2}\sigma^2\right)^2 V_{\mathrm{xxxx}} = \frac{1}{2}\sigma^2 V_{\xi\xi\tau} - \left(r - \beta\lambda - \frac{1}{2}\sigma^2\right) V_{\mathrm{x}} + \left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)^2 V_{\mathrm{xx}} - r\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right) V_x$$

$+\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)\omega_x + \frac{r}{2}\sigma^2 V_{\mathrm{xx}} - \frac{1}{2}\sigma^2\omega_{\mathrm{xx}}$.

Substituting V_{xxx} and V_{xxxx} into (3.2), we obtain the numerical discretization form

$$\left(\frac{1}{2}\sigma^2 + \frac{h^2}{12}\left(\frac{1}{2}\sigma^2\delta_x^2 + \left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)\delta_x\right)\right)(V_\tau)_m \qquad = \left(\left(\frac{1}{2}\sigma^2\right)^2 + \frac{h^2}{12}\left(\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)^2 - \frac{r}{2}\sigma^2\right)\right)\delta_x^2$$

It is noted that the finite difference operators δ_x andδ_x^2 are $\delta_x = \frac{1}{2h}$

$\left[-1 \quad amp;0 \quad amp;1\right]$,$\delta_x^2 = \frac{1}{h^2}$

$\left[1 \quad amp;-2 \quad amp;1\right]$, respectively, then for μ $[1, M-1]$,the operators satisfy the following equation

$\alpha\left(V_\tau\right)_{m-1} + \eta\left(V_\tau\right)_m + \gamma\left(V_\tau\right)_{m+1} = (\delta - \upsilon)V_{m-1} + \xi V_m + (\delta + \upsilon)V_{m+1} + \alpha\omega_{m-1} + \eta\omega_m + \gamma\omega_{m+1}$,

where

$$\omega_m = \}\left(x_m\right) + \updownarrow\left(x_m\right), \alpha = \frac{\sigma^2}{24} - \frac{\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)h}{}$$

english24,$\eta = \frac{5\sigma^2}{12}, \gamma = \frac{\sigma^2}{24} + \frac{\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)h}{}$

english24,

$$\delta = \frac{1}{12}\left(\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)^2 - \frac{r}{2}\sigma^2\right) + \frac{\left(\sigma^2\right)^2}{4h^2}, \upsilon = \frac{\sigma^2\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)}{}$$

english4h-ρ$\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)h$

ενγλιση24,

$\xi = -\frac{1}{6}\left(\left(r - \beta\lambda - \frac{1}{2}\sigma^2\right)^2 + \frac{5\sigma^2}{2}\right)$.

3.3 New algorithm

$$BV'(\tau) = \mathrm{AV}(\tau) + \mathrm{BW}(\tau) + (\tau),$$

where

$$V(\tau) = \left[V_1(\tau), V_2(\tau), \cdots, V_{M-1}(\tau)\right]^T, \omega(\tau) = \left[\omega_1(\tau), \omega_2(\tau), \cdots, \omega_{M-1}(\tau)\right]^T, (\tau) = \left[{}_1(\tau), 0, \cdots, 0, {}_{M-1}(\tau)\right]^T$$

$$_1(\tau) = -\alpha V_0^{'}(\tau) + \alpha\omega_0(\tau) + (\delta - v) V_0(\tau), _{M-1}(\tau) = -\gamma V_M^{'}(\tau) + \gamma\omega_M(\tau) + (\delta + v) V_M(\tau).$$

Let

$\delta_v^- = \delta - v, \delta_v^+ = \delta + v,$

define

$A =$

$$\begin{pmatrix} \xi & amp;\delta_v^+ & amp;0 & amp;0 & amp;0 \\ \cdots & amp;\cdots & amp;\cdots & amp;\cdots & amp;\cdots \\ 0 & amp;0 & amp;\delta_v^- & amp;\xi & amp;\delta_v^+ \\ 0 & amp;0 & amp;0 & amp;\delta_v^- & amp;\xi \end{pmatrix}, \mathrm{B} =$$

$$\begin{pmatrix} \beta & amp;\gamma & amp;0 & amp;0 & amp;0 \\ \alpha & amp;\beta & amp;\gamma & amp;0 & amp;0 \\ \cdots & amp;\cdots & amp;\cdots & amp;\cdots & amp;\cdots \\ 0 & amp;0 & amp;\alpha & amp;\beta & amp;\gamma \\ 0 & amp;0 & amp;0 & amp;\alpha & amp;\beta \end{pmatrix}.$$

Let

$$e(\tau) = [e_1, e_2, \cdots, e_{M-1}]^T,$$

where$e_m = 2\sum_{j=1}^{\frac{M}{2}-1} V_{2j}(\tau) f_{\mathrm{jm}} + 4\sum_{j=1}^{\frac{M}{2}} V_{2j-1}(\tau) f_{2j-1,m}$.

Let

$$f_j = f(-\mathrm{jh})$$

then $e(\tau)$ can be written as

$e(\tau) = \mathrm{FGV}(\tau),$

where G is a diagonal matrix and can be written as $G = \mathrm{diag}[4, 2, 4, \cdots, 2, 4]$, $F$is Toeplitz matrix and can be written as

$F =$

$$\begin{pmatrix} f_0 & amp;f_1 & amp;f_2 & amp;\cdots & amp;f_{M-3} & amp;f_{M-2} \\ f_{-1} & amp;f_0 & amp;f_1 & amp;f_2 & amp;\cdots & amp;f_{M-3} \\ f_{-2} & amp;f_{-1} & amp;f_0 & amp;f_1 & amp;\cdots & amp;\vdots \\ \vdots & amp;\ddots & amp;\ddots & amp;\ddots & amp;\ddots & amp;f_2 \\ f_{-(M-3)} & amp;\cdots & amp;f_{-2} & amp;f_{-1} & amp;f_0 & amp;f_1 \\ f_{-(M-2)} & amp;f_{-(M-3)} & amp;\cdots & amp;f_{-2} & amp;f_{-1} & amp;f_0 \end{pmatrix}.$$

Let $(\tau) = [\eth_1(\tau), \eth_2(\tau), \cdots, \eth_{M-1}(\tau)]^T$, where $\eth_m(\tau) = \frac{h}{3}(g_{0m}(\tau) + g_{\mathrm{Mm}}(\tau) + \}(x_m))$, then $\omega(\tau)$ can be written as$\omega(\tau) = \lambda\left(\frac{h}{3}\mathrm{FGV}(\tau) + \eth(\tau)\right)$.

Coefficient B is a nonsingular matrix, then

$V^{'}(\tau) = \mathrm{CV}(\tau) + \theta(\tau), \tau\ [0, T]$,(3.3)

where $\mathrm{C} = B^{-1}A + \frac{\lambda\eta}{}$

english3FG,$\theta(\tau) = \lambda\eth(\tau) + B^{-1}d(\tau)$.

The payoff function of shout options is not smooth, so in order to achieve the fourth-order convergence rate, grid refinement technology is used around the strike price.

3.4 Cubic spline time integral

The time interval $[0,T]$ is divided into$N$ intervals with the length $k = T/N$ and the time node is $\tau_n = \mathrm{nk}, \nu\ [0,N]$. At each time interval $[\tau_n, \tau_{n+1}]$three spline approximations are found for the given values$V(\tau_n)$and the differential terms $V^{'}(\tau_n)\,V^{'}\left(\tau_n + \frac{k}{2}\right)V^{'}(\tau_n + k)$, that is

$$V(\tau) = V(\tau_n) + k\rho_0(q)\,V^{'}(\tau_n) + k\rho_1(q)\,V^{'}\left(\tau_n + \frac{k}{2}\right) + k\rho_2(q)\,V^{'}(\tau_n + k),$$

where polynomial ρ_i are

$$\rho_0(q) = q - \frac{3}{2}q^2 + \frac{2}{3}q^3, \rho_1(q) = 2q^2 - \frac{4}{3}q^3, \rho_2(q) = -\frac{1}{2}q^2 + \frac{2}{3}q^3.$$

Therefore, we obtain

$V(\tau_n + k) = V(\tau_n) + \frac{k}{6}\left(V^{'}(\tau_n) + 4V^{'}\left(\tau_n + \frac{k}{2}\right) + V^{'}(\tau_n + k)\right)$,(3.4)

$V\left(\tau_n + \frac{k}{2}\right) = V(\tau_n) + \frac{k}{24}\left(5V^{'}(\tau_n) + 8V^{'}\left(\tau_n + \frac{k}{2}\right) - V^{'}(\tau_n + k)\right)$.(3.5)

By use of (3.3) and (3.5), we have

$\left(I - \frac{k}{3}C\right)V\left(\tau_n + \frac{k}{2}\right) = \left(I + \frac{5k}{24}C\right)V(\tau_n) - \frac{k}{24}\mathrm{CV}(\tau_n + k) +_1(\tau_n)$,

where $_1(\tau_n) = \frac{k}{24}\left(5b(\tau_n) + 8b\left(\tau_n + \frac{k}{2}\right) - \theta(\tau_n + k)\right)$.

Removing $V\left(\tau_n + \frac{k}{2}\right)$, (3.4) can be written as the following form

$$V(\tau_n + k) = V(\tau_n) +_2(\tau_n) + \frac{k}{6}C\left(V(\tau_n) + 4V\left(\tau_n + \frac{k}{2}\right) + V(\tau_n + k)\right),$$

where$_2(\tau_n) = \frac{k}{6}\left(\theta(\tau_n) + 4\theta\left(\tau_n + \frac{k}{2}\right) + \theta(\tau_n + k)\right)$.

Time marching algorithm

$$\left(I - \frac{k}{2}C + \frac{k^2}{12}C^2\right)V(\tau_n + k) = \left(I + \frac{k}{2}C + \frac{k^2}{12}C^2\right)\ V(\tau_n) + c(\tau_n),$$

where $c(\tau_n) = \frac{2k}{3}C_1(\tau_n) + \left(I - \frac{k}{3}C\right)_2(\tau_n)$.

Although $_1(\tau_n)$ and$_2(\tau_n)$ depend on values at time $\tau_n + \frac{k}{2}$ and $\tau_n + k$, these values can be easily calculated because they only depend on terminal conditions.

3.5 Solutions obtained by the new algorithm

Linear complementarity problem (2.2) can be described as the form $\min(V_\tau - \mathrm{LV}, V - g) = 0$.

The discrete type of the above is

$\min\left(BV^{'}(\tau) -\ \mathrm{BV}(\tau) - \mathrm{Bw}(\tau) - (\tau),\ V(\tau)\ -\ g\right) = 0$,(3.6) where g $= [g_1, g_2, \cdots, g_{M-1}]^T$, $g_m = \text{Kmax}\left(1 - e^{x_m}, 0\right)$.

Vector $a = [a_1, a_2, \cdots, a_n]^T, b = [b_1, b_2, \cdots, b_n]^T$.

Let $\min(a_i, b_i)$ represents the component of vector $\min(a, b)$.

Let $V^n = V(\tau_n)$, then the calculation ofV^{n+1} can be obtained by solving the discrete problemmin $\left(BV^{n+1} - \zeta^n, V^{n+1} - g\right) = 0$, where$B = \left(I - \frac{k}{2}C + \frac{k^2}{12}C^2\right)$.

Let $c^n = c(\tau_n)$, then the vectorζ^n can be obtained by the following formula

$\zeta^n = \left(I + \frac{k}{2}C + \frac{k^2}{12}C^2\right)V^n + c^n$.

Let $\mathcal{Q} = \{0, 1\}, M' = M - 1$, considering the problem

$(B(\hat{\alpha})V - \varpi(\hat{\alpha}))$,(3.7)

where for $\hat{\alpha}\epsilon\mathcal{Q}^{M'}$,$B(\hat{\alpha})$ is a monotone matrix of dimension M', and $\varpi(\hat{\alpha})$ is a vector on M'.

For the case where$\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_{M'})\,\epsilon\mathcal{Q}^{M'}$and $B_{\text{ij}}(\hat{\alpha})$ depend only on $\hat{\alpha}_i$, for each $\hat{\alpha}\epsilon\mathcal{Q}$,$\hat{\alpha}^{\hat{a}} = (\hat{a}, \hat{a}, \cdots, \hat{a}) \in \mathcal{Q}^{M'}, B^{\hat{\alpha}} = B\left(\hat{\alpha}^{\hat{\alpha}}\right), \varpi^{\hat{\alpha}} = \varpi\left(\alpha^{\hat{\alpha}}\right)$, then (3.7) can be represented equivalently as following

$\left(B^{\hat{\alpha}}V - \varpi^{\hat{\alpha}}\right)$(3.8)

Therefore, Eq. (3.6) can be written as Eq. (3.8) in the case $B^0 = B, B^1 = I, \varpi^0 = \zeta^n, \varpi^1 = g$.

Then this algorithm is applied to find the solution of (3.8).

(1) Initialize$\hat{\alpha}_0 \in \mathcal{Q}^{M'} = \{0, 1\}^{M'}$

(2) For $k\,(\geq 0)$ iteration

(a) Find $V^{(k)} \in R^{M'}$, so that $B\left(\hat{\alpha}^k\right)V^{(k)} = \varpi\left(\hat{\alpha}^k\right)$. If $k \geq 1$and $V^{(k)} = V^{(k-1)}$, the iteration stops, otherwise do (b)

(b) For each $i = 1, 2, \cdots, M'$, let$\hat{\alpha}_i^{k+1} = \{$

&0, if $\left(B^0V^{(k)} - \varpi^0\right) \leq \left(B^1V^{(k)} - \varpi^1\right)$
&1, otherwise

(c) Set $k = k + 1$ and go back to (a)

This algorithm generates a series of approximate values V^k forV^{n+1} and if coefficient B is M-matrix, it has finite terminal. In all our numerical experiments, we have not encountered any violation of this nature.

4. NUMERICAL ANALYSIS

In this section, first we show the convergence of the algorithm in this paper is superior to traditional finite element method, then we compare the performance between the algorithm proposed in this paper and traditional method in two cases, one is without jump term, the other is in jump diffusion model.

4.1 Convergence of the algorithm

We use Figure1 to illustrate the time convergence of the numerical algorithm. The result indicates that the algorithm we propose in this paper has the fourth-order convergence rate in time because the logarithmic-logarithmic graph is parallel to the fourth-order convergence slope, while the Crank-Nicolson time-stepped finite element method has the second-order convergence rate. High-order discretization of space and time means that the algorithm has higher accuracy than second-order discretization under the same number of time steps and space nodes.

4.2 Performance Comparison of shout options with no jumps

The first example is carried out for the jump-diffusion model with the parameter $\lambda = 0$, that is, the underlying asset follows geometric Brownian motion only. For comparison, we also include results in [2] and [4] for the same shout options. The result in [4] were obtained using numerical PDE method and in [2] using a similarity reduction and an explicit difference method.

Parameters are used $\sigma = 0.20, r = 0.10, K = \$100, T = 5yr$ In Table 1.The Reference solution using asset grid nodes=n, time steps=n. Result in [4]: time steps=n, asset grid nodes=$2n-1$.The error given is for the course grid and was estimated using the converged answer at the finest level as an approximation for the exact solution.

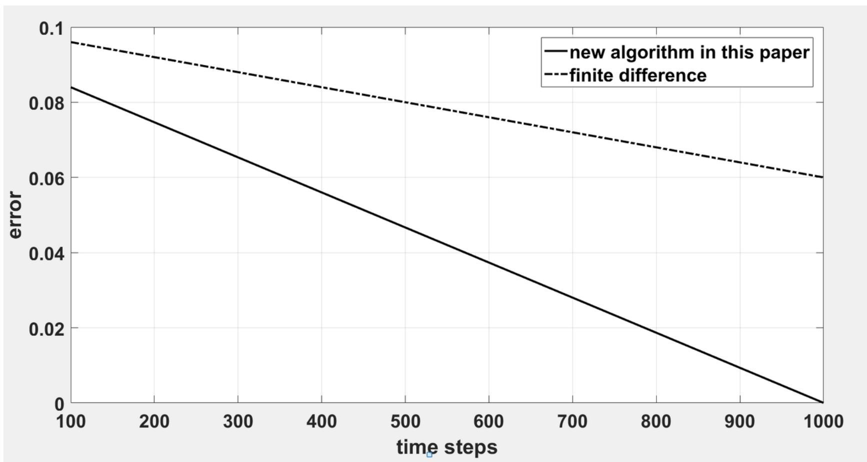

Figure1.Convergence of shout put options in jump-diffusion model

Table 1 Performance comparison between algorithm in this paper and that in [4] and [2] for pricing at-the-money shout put options with no jumps.

No shout counter resets	No shout counter resets	No shout counter resets	No shout counter resets	No shout counter resets	Shout counter reset yearly	Shout counter reset yearly	Shout counter reset yearly
Number of shouts	1	2	5	10	1/yr (5 total)	2/yr (10 total)	4/yr (20 total)
Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper
Asset nodes	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$
100	5.5701	7.7158	11.280	13.805	9.9502	12.411	14.474
200	5.5706	7.7226	11.285	13.818	9.9535	12.416	14.488
400	5.5713	7.7235	11.296	13.825	9.9549	12.419	14.492
800	5.5714	7.7242	11.299	13.829	9.9561	12.420	14.497
Course grid error	0.0052	0.0081	0.0095	0.014	0.00087	0.014	0.016
Result in [4]	Result in [4]	Result in [4]	Result in [4]	Result in [4]	Result in [4]	Result in [4]	Result in [4]

No shout counter resets	No shout counter resets	No shout counter resets	No shout counter resets	No shout counter resets	Shout counter reset yearly	Shout counter reset yearly	Shout counter reset yearly
Asset nodes	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$
100	5.5648	7.7144	11.278	13.793	9.9412	12.399	14.463
200	5.5697	7.7215	11.291	13.814	9.9524	12.415	14.485
400	5.5709	7.7233	11.294	13.820	9.9551	12.419	14.49
800	5.5712	7.7238	11.295	13.821	9.9557	12.419	14.491
Course grid error	0.0064	0.0094	0.017	0.028	0.0145	0.020	0.028
Result in [2]	Result in [2]	Result in [2]	Result in [2]	Result in [2]	Result in [2]	Result in [2]	Result in [2]
Time steps	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$	Option value at $S = 100$
100	5.59	7.74	11.22	13.53			
250	5.58	7.73	11.27	13.71			
500	5.58	7.73	11.28	13.77	N/A	N/A	N/A
1000	5.57	7.73	11.29	13.80			
5000	5.57	7.72	11.29	13.82			
Course grid error	0.02	0.02	0.07	0.29			

Results in table 1 indicates that the algorithm in this paper yields the more accurate value for the shout options according to the indicator course grid error.

4.3 Performance Comparison of shout options in jump-diffusion model

In this part, performance of shout options in jump-diffusion model will be compared between the algorithm in this paper and two traditional finite element methods. The first one used an exponential time integration (ETI) [20] and the other employed a second-order backward differentiation formula (BDF-2) [19]. Parameters are $S_0 = 80, K = 80, \sigma = 0.25, T = 1, r = 0.03$ and the jump term parameters $\lambda = 1.2, \alpha = 0.15, \gamma = 0.3$. The other parameters are $\varphi = 3$ and the number of time steps is twice that of spatial steps.

Table 2 shows that the accuracy of algorithm in this paper is much higher than that of finite element method in calculating shout options price. For example, when 480 spatial steps are used, the error of algorithm in this paper is only10-7, which is much smaller than that of finite element method10-3.

Table 2 Performance comparison on shout put options between algorithm in this paper and finite element algorithm in jump-diffusion model

M	ETI	ETI	BDF-2	BDF-2
	price	error	price	error
Linear element	Linear element	Linear element	Linear element	Linear element
30	12.3384	2.2E-1	14.6687	2.2 E-1
60	13.8231	7.1E-2	14.9471	7.1 E-2
120	15.0223	3.4E-2	15.0132	3.4 E-2
240	15.0346	4.8E-3	15.0296	4.8 E-3
480	15.0521	1.2E-3	15.0036	1.2 E-3

M	ETI	ETI	BDF-2	BDF-2
M	price	error	rate	rate
Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this paper	Algorithm in this
30	14.8266	0.8 E-1	-	-
60	15.0503	3.5 E-3	3.412	3.412
120	15.0512	1.4 E-4	3.487	3.487
240	15.0520	1.8 E-5	3.102	3.102
480	15.0520	6.5 E-7	3.034	3.034
Ref	15.0520			

It is obviously found from Table 3 that for shout put options, price generated by the algorithm in this paper using 272 spatial nodes can meet the accuracy requirement when two sets of parameters are selected. On a large number of grid nodes, the algorithm improves the speed because of processing integral terms.

Table 3 Shout put options in jump-diffusion model (parameters$S_0 = 80, K = 80, r = 0.03, T = 0.5, \varphi = 3$)

M	price	error	rate
$\sigma = 0.1, \gamma = 0.5, \alpha = -0.7, \lambda = 0.2$	$\sigma = 0.1, \gamma = 0.5, \alpha = -0.7, \lambda = 0.2$	$\sigma = 0.1, \gamma = 0.5, \alpha = -0.7, \lambda = 0.2$	$\sigma = 0.1, \gamma =$
34	2.2813	4.2 E-2	–
68	2.3014	4.2 E-3	4.112
136	2.3021	8.5 E-4	3.545
272	2.3100	2.1 E-4	3.826
Ref	2.3102		
$\sigma = 0.12, \gamma = 0.15, \alpha = 0.2, \lambda = 0.2$	$\sigma = 0.12, \gamma = 0.15, \alpha = 0.2, \lambda = 0.2$	$\sigma = 0.12, \gamma = 0.15, \alpha = 0.2, \lambda = 0.2$	$\sigma = 0.12, \gamma$
34	2.5033	2.2 E-2	
68	2.5108	1.8 E-3	4.232
136	2.5133	2.5 E-4	3.826
272	2.5133	3.6 E-5	3.131
Ref	2.5133		

5. CONCLUSIONS

In this paper, we derive a new partial integro-differential inequality (PIDI) for shout options pricing on the assumption that the price of the underlying asset follows the jump-diffusion model and construct the mathematical model by combining specific features and terminal conditions. Another innovation is that this paper proposes a new competitive algorithm. Numerical experiments confirm the convergence in pricing shout options and shows that the algorithm proposed in this paper is superior to the traditional algorithm in the case of geometric Brownian motion and jump diffusion model, respectively.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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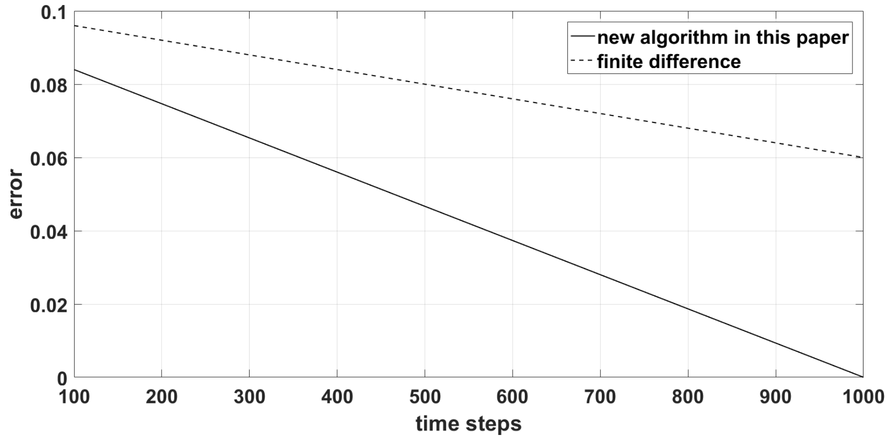
0.1
0.08
0.06
0.04
0.02
0
error
100
200
300
400
500
600
700
800
900
1000
time steps
new algorithm in this paper
finite difference