Nonsmooth, nonconvex regularizers applied to linear elctromagnetic inverse problems

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Abstract

Tikhonov's regularization method is the standard technique applied to obtain models of the subsurface conductivity dis- tribution from electric or electromagnetic measurements by solving UT (m) = ||F(m) - d||ˆ2 + P(m): The second term correspond to the stabilizing functional, with P(m) = ||m||ˆ2 the usual approach, and the regularization parameter. Due to the roughness penalizer inclusion, the model developed by Tikhonov's algorithm tends to smear discontinuities, a feature that may be undesirable. An important requirement for the regularizer is to allow the recovery of edges, and smooth the homogeneous parts. As is well known, Total Variation (TV) is now the standard approach to meet this requirement. Recently, Wang et.al. proved convergence for alternating direction method of multipliers in nonconvex, nonsmooth optimization. In this work we present a study of several algorithms for model recovering of Geosounding data based on Inmal Convolution, and also on hybrid, TV and second order TV and nonsmooth, nonconvex regularizers, observing their performance on synthetic and real data. The algorithms are based on Bregman iteration and Split Bregman method, and the geosounding method is the low-induction numbers magnetic dipoles. Non-smooth regularizers are considered using the Legendre-Fenchel transform.

GP43B-0991 Nonsmooth, nonconvex regularizers applied to linear electromagnetic inverse problems

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Abstract

A study of the application of nonconvex regularization operators to the electromagnetic sounding inverse problem is presented. A comparison is made among four regularization algorithms: Total Variation (TV), Infimal Convolution of TV with TV^2 and TV combined TV^2 and a non convex operator based recently proposed. The non convex regularization operator is approximated by the convex dual, and the minimization is then implemented considering the equivalence between the Bregman iteration and the augmented Lagrangian methods.

Introduction

Tikhonov's regularization method is the standard technique applied to obtain models of the subsurface conductivity distribution from electric or electromagnetic measurements.

$$U_T(m) = \|F(m) - d\|^2_{\mathcal{H}} + \lambda P(m). \tag{1}$$

$F : \mathcal{M} \supset \mathcal{D}(\mathcal{F}) \to \mathcal{H}$ represents the direct functional, applied to an element on the model space $\mathcal{M}$, a Banach space and returning a member of the Hilbert space $\mathcal{H}$ of data. $P(m) = \|\bigtriangledown m\|^2_{\mathcal{M}}$ is for Tikhonov's maximum smoothness functional the usual approach. We consider the discrete version of linear problem $F(m) = K(m) = \int \kappa m$, with κ the kernel of a Fredholm integral equation of the first kind. An important requirement for the regularizer is to allow the recovery of edges, and smooth the homogeneous parts. As is well known, Total Variation (TV) is now the standard approach to this requirement.

Nonconvex Regularization

A transformation that has been used to define functional approximations allowing to implement iterative algorithms is the Legendre-Fenchel transformation (LFT):

$$f^*(u) = \sup_{x\in\mathcal{H}} (\langle x, u\rangle - f(x)). \tag{2}$$

Consider the minimization of (1) with $P(m) = \int \phi(|\nabla m|)$, with ϕ a nonconvex, nonsmooth function such as

$$\phi_1(|t|) = \frac{|t|}{1+\rho|t|}, \tag{3}$$

with $\rho > 0$. Vese and Chan demonstrated, using LFT, that

$$\phi(t) = \inf_v (v^2 t + \psi(v)), \quad \forall t \geq 0, \tag{4}$$

obtaining $\psi_1(v) = \frac{(v-1)^2}{\rho}$. Recently, a methodology was proposed for the minimization of (1) with the nonconvex operator $\phi(|\nabla m|)$. The algorithm, named NSNC consider the dual functional

$$\begin{aligned} U_{NSNC}(m,b) =& \|Am - d\|^2 && (5)\\ &+\lambda\Big(\sum_k \phi^*[(D_x m)_k] + \sum_k \phi^*[(D_z m)_k]\Big) && (6)\\ &+\alpha\Big(\sum_k (D_x m)^2_k + \sum_k (D_z m)^2_k\Big), && (7)\end{aligned}$$

with ϕ^* the convex dual as defined in (4).

Alternating minimizations are applied over m and v, but the convexity of ψ allows to easily incorporate the procedure in a Bregman algorithm.

TV12

A combined first and second order variational operator for denoising is considered. The discrete version of the modeling is:

$$\min\left\{U_{TV12} = \frac{1}{2}\int_{\Omega} |Am - d|^s dx + \alpha_1\|\nabla m\|_1 + \alpha_2\|\nabla^2 m\|_1\right\}. \tag{8}$$

A split Bregman algorithm can be implemented for the minimization

$$\begin{aligned} U_{TV12}(m,v,w) =& \|Am - d\|^2 && (9)\\ &+\alpha_1(\|v\|_1 + \frac{\gamma_1}{2}\|\nabla m - v - d_1\|^2_2) && (10)\\ &+\alpha_2(\|w\|_1 + \frac{\gamma_2}{2}\|\nabla^2 m - w - d_2\|^2_2), && (11)\end{aligned}$$

that can be realized in three steps:

$$\begin{aligned} m^{k+1} &= \min_m U_{TV12}(m,v,w) && (12)\\ v^{k+1} &= \min_v \|v\|_1 + \alpha_1\frac{\gamma_1}{2}\|\nabla m - v - d_1^k\|^2_2, && (13)\\ w^{k+1} &= \min_w \|w\|_1 + \alpha_2\frac{\gamma_2}{2}\|\nabla^2 m - w - d_2^k\|^2_2. && (14)\end{aligned}$$

And the Bregman updates

$$\begin{aligned} d^{k+1} &= d^k + (d - Am^{k+1}) && (15)\\ d_1^{k+1} &= d_1^k + (v^{k+1} - \nabla m^{k+1}) && (16)\\ d_2^{k+1} &= d_2^k + (w^{k+1} - \nabla^2 m^{k+1}) && (17)\end{aligned}$$

Infimal Convolution

Infimal convolution was introduced by Chambolle and Lions in the context of total variation denoising. The infimal convolution of two functionals Φ and Ψ is defined as

$$(\Phi \square \Psi)(u) := \inf_{u=v+w} \Phi(v) + \Psi(w) \tag{18}$$

Benning et. al. proposed the infimal convolution model (ICTV) considering TV and TV^2

$$ICTV_{\beta}(u) := (TV \square TV^2)(u) := \inf_{u=v+w} (TV(v) + \beta TV^2(w)) \tag{19}$$

A Split Bregman procedure in each of the two terms is implemented here, considering $P(m) = \inf_{m=m_1+m_2}(\alpha_1\|\nabla(m - m_1)\|_1 + \alpha_2\|\Delta m_2\|_1)$. Defining $p = \nabla m$, using Split Bregman procedure on each term of IC, along with the Bregman update on the fitness term, the algorithm would aim to minimize

$$\begin{aligned} U_{ICTV}(m,p,v,w) =& \frac{1}{2}\|Am - d_1^k\|^2 && (20)\\ &+\alpha_1(\|v\|_1 + \frac{\gamma_1}{2}\|\nabla m - p - v - d_2^k\|^2_2) && (21)\\ &+\alpha_2(\|w\|_1 + \frac{\gamma_2}{2}\|\mathrm{div} p - w - d_3^k\|^2_2), && (22)\end{aligned}$$

and then update d_1^k, d_2^k and d_3^k.

Applications

Application to synthetic data for electromagnetic geosounding method at low induction frequencies is presented. Parameters used are observed on Table 1 for TV12 and IC algorithms. $\lambda = 0.01$ and 0.001 for NSNC and TV, respectively, and $\rho = 0.01$ in ϕ_1.

Operator	α_1	α_2	γ_1	γ_2	λ
TV12	0.0001	0.0001	100	100	-
IC	0.001	0.0001	20	10	-

Table 1: Algorithm Parameters

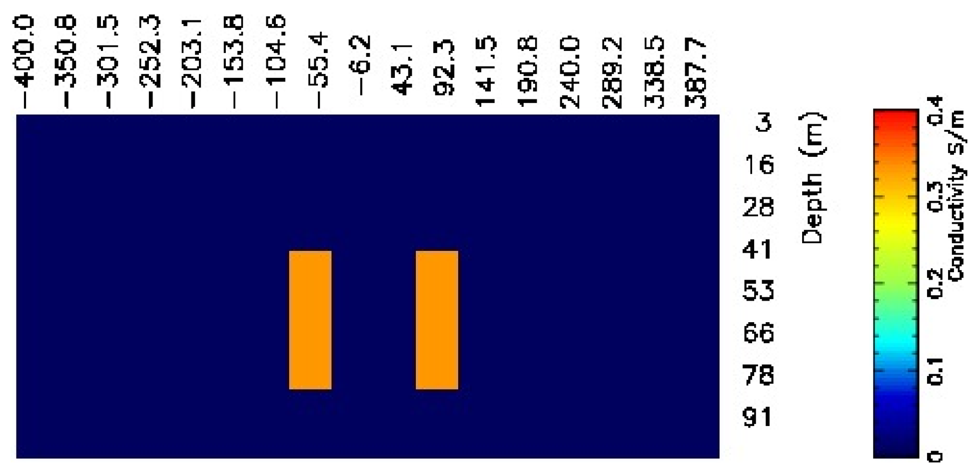

Figure 1: Model used to generate synthetic data

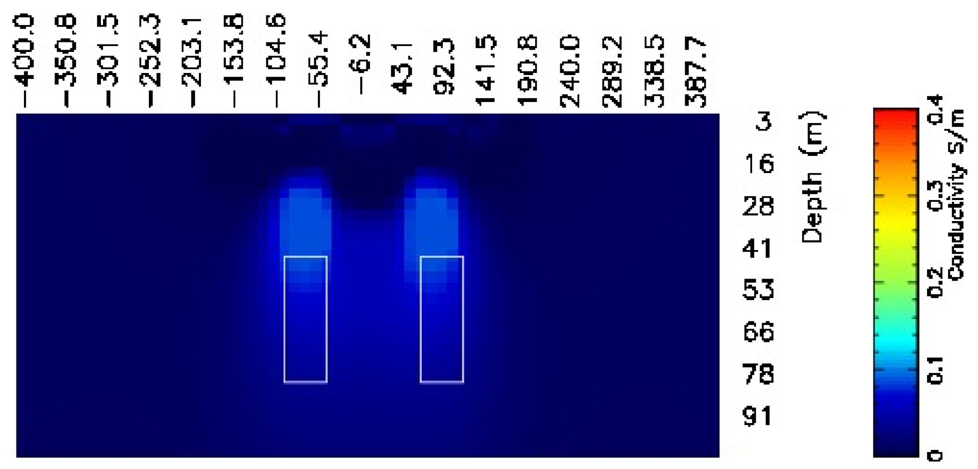

Figure 2: Resulting model for the TV regularizer

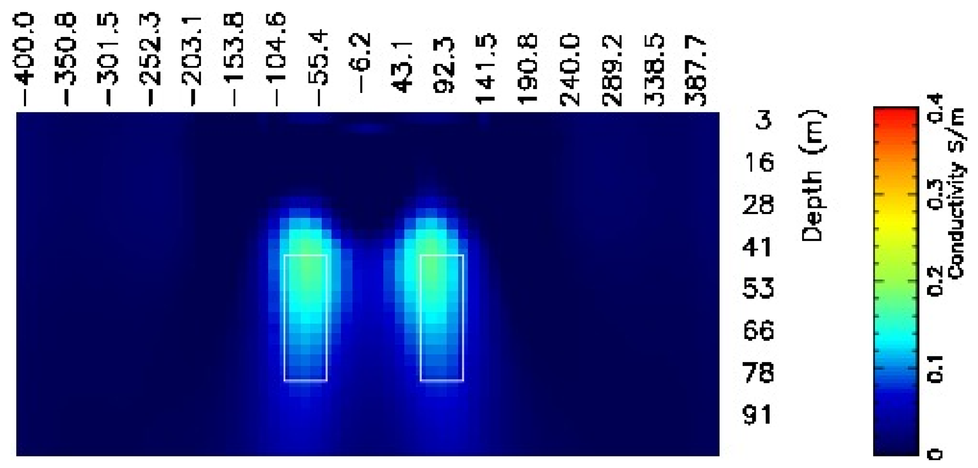

Figure 3: Model obtained with NSNC regularizer

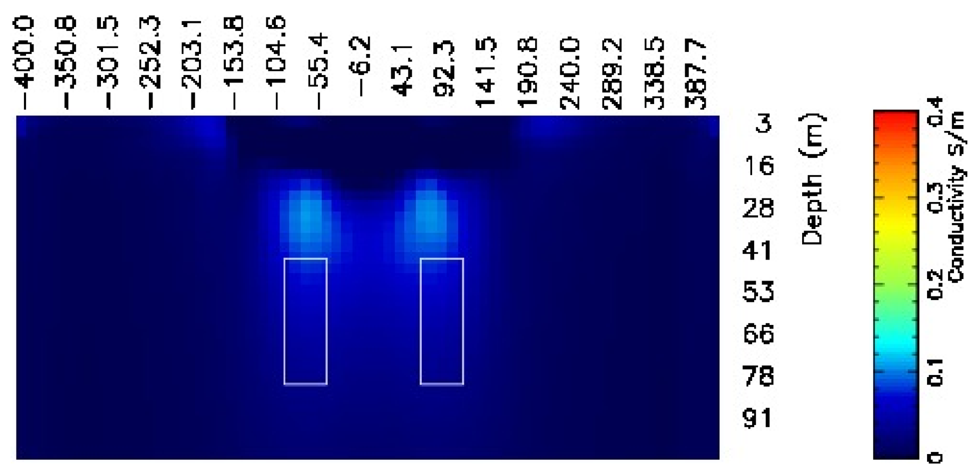

Figure 4: Model developed with TV12 regularizer

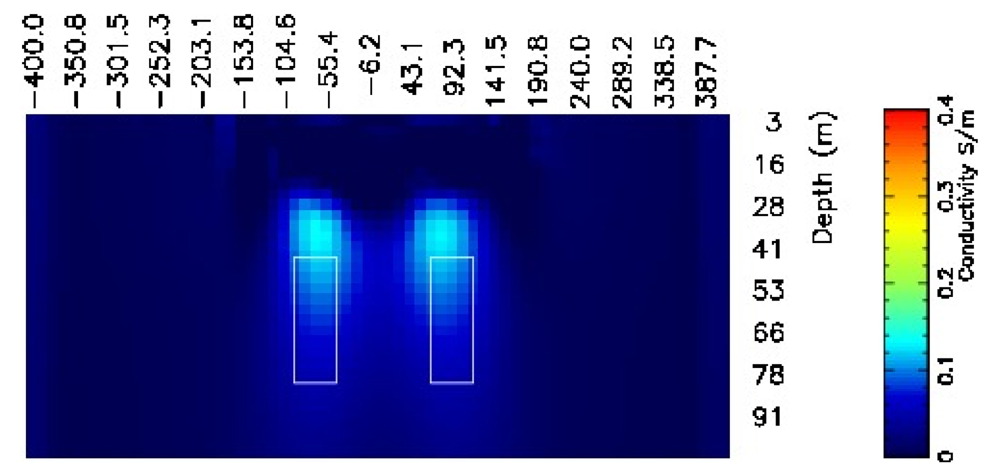

Figure 5: Model developed by the IC regularizer

Application to field data

The methods were applied to Las Auras field data. The purpose of the surveys was to identify sediment-filled faults affecting the construction of a dam. We are considering the vertical and horizontal dipole measurements taken at several points across strike for 10, 20 and 40 m separations.

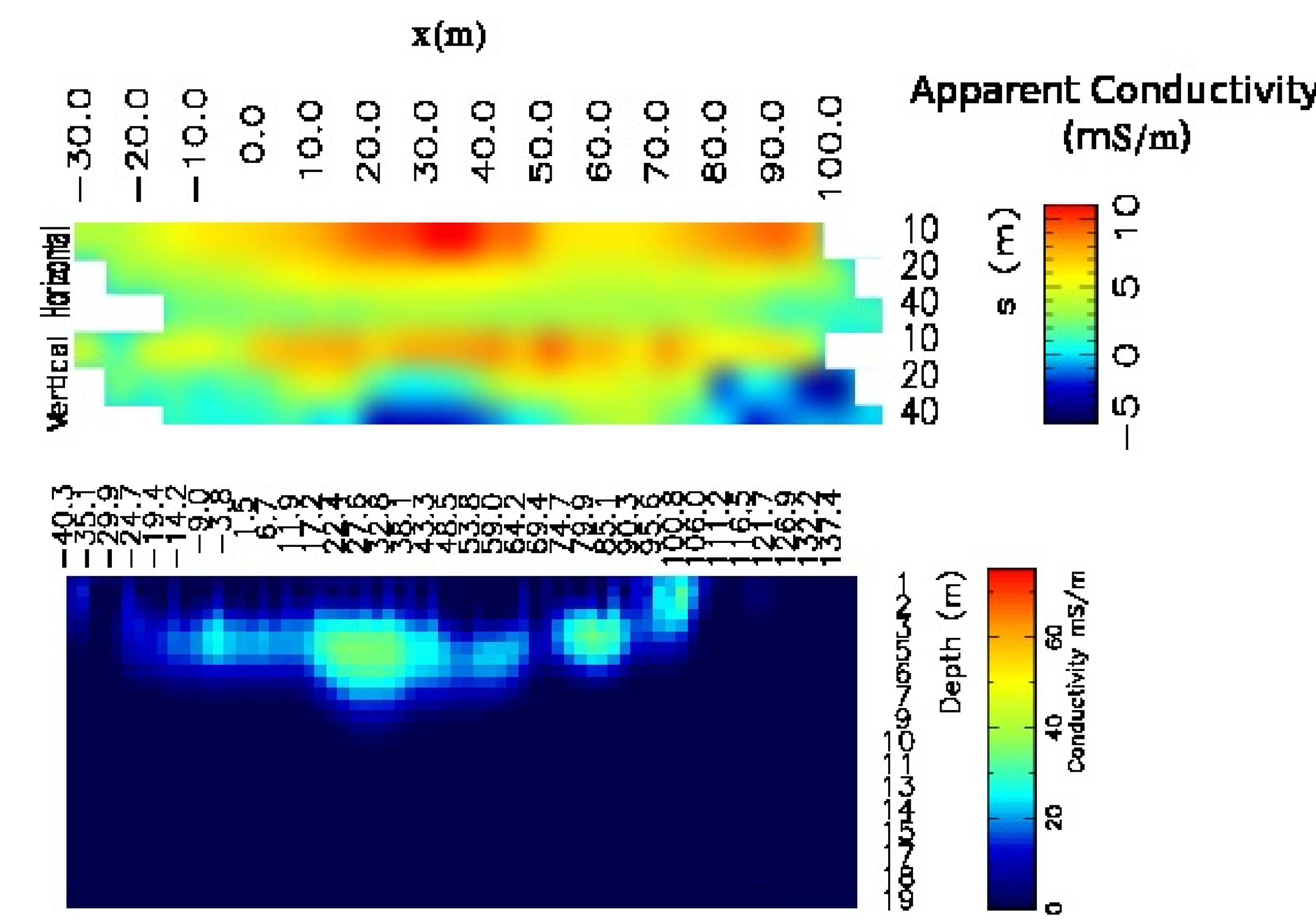

Figure 6: Results for the field data (a) field data, (b) NS2 result.

Conclusions

A comparison of the application of four regularization operators is presented. Total Variation, a combination of Total Variation with a second order TV, a Infimal Convolution of TV and second order TV and a nonsmooth, nonconvex optimization algorithm were implemented. TV presents the worst results, IC shows better model than TV12, but the best results are observed for the non convex operator, for the synthetic data.