Estimating Bayesian Model Averaging Weights and Variances of Ensemble Flood Modeling Using Multiple Markov Chains Monte Carlo

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Abstract

As all kinds of physics-based and data-driven models are emerging in the fields of hydrologic and hydraulic engineering, Bayesian model averaging (BMA) is one of the popular multi-model methods used to account for the various uncertainty sources in the flood modeling process and generate robust ensemble predictions based on multiple competitive candidate models. The reliability of BMA parameters (weights and variances) determines the accuracy of BMA predictions. However, the uncertainty in the BMA parameters with fixed values, which are usually obtained from the Expectation-Maximization (EM) algorithm, has not been adequately investigated in BMA-related applications over the past few decades. Given the limitations of the commonly used EM algorithm, the Metropolis-Hastings (M-H) algorithm, which is one of the most widely used algorithms in the Markov Chain Monte Carlo (MCMC) method, is proposed to estimate the BMA parameters and quantify their associated uncertainty. Both numerical experiments and the one-dimensional HEC-RAS models are employed to examine the applicability of the M-H algorithm with multiple independent Markov chains. The performances of the EM and M-H algorithms in the BMA analysis are compared based on the daily water stage predictions from 10 model configurations. The results show that the BMA weights estimated from both algorithms are comparable, while the BMA variances obtained from the M-H MCMC algorithm are closer to the given variances in the numerical experiment. Moreover, the normal proposal distribution used in the M-H algorithm can yield narrower distributions for the BMA weights than those from the uniform prior. Overall, the MCMC approach with multiple chains can provide more information associated with the uncertainty of BMA parameters and its prediction performance is better than the default EM algorithm in terms of multiple evaluation metrics as well as algorithm flexibility.

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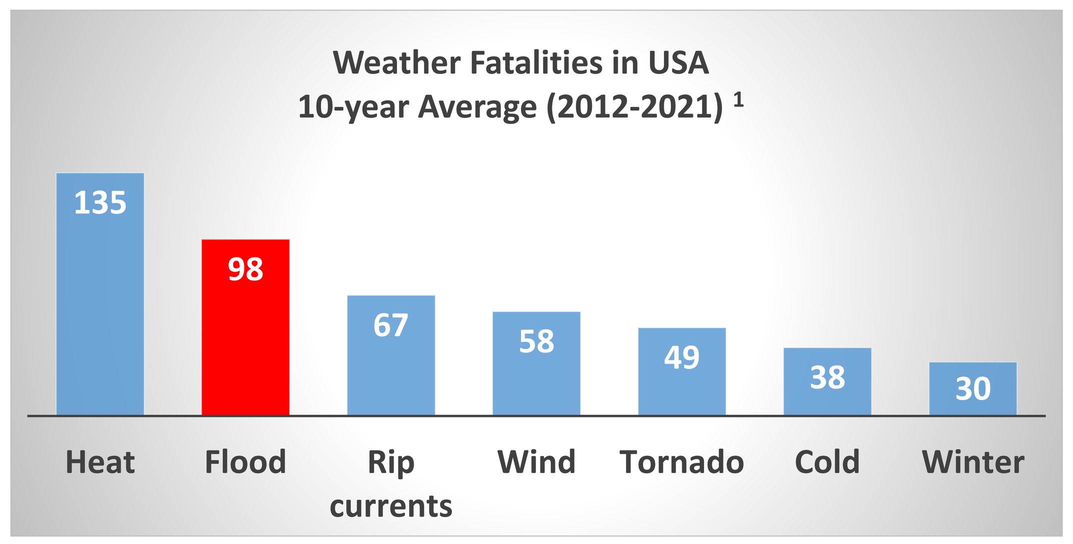
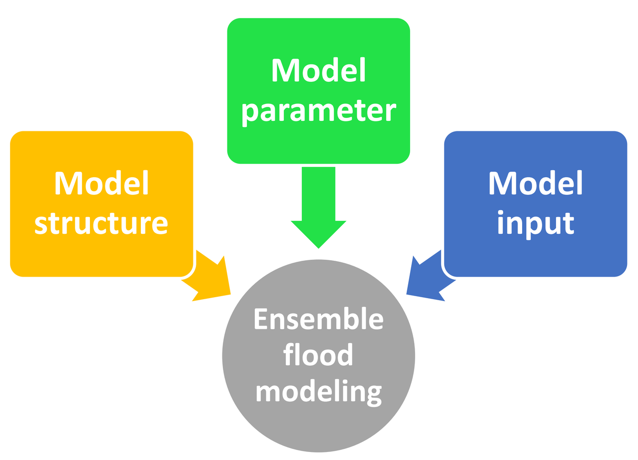
Introduction





Multi-model method

- physics-based and data-driven models
- No "perfect" single model
- "Equifinality" in hydrologic & hydraulic modeling
- Fixed estimates cannot fit all kinds of scenarios

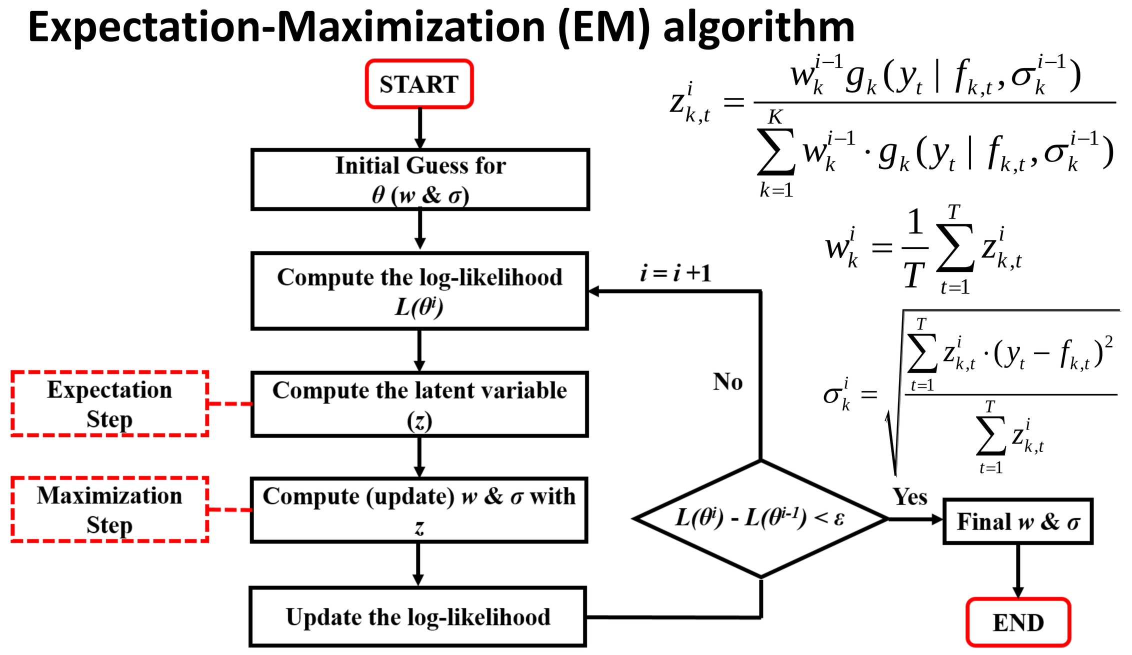
Bayesian Model Averaging (BMA)[2]

- **Law of total probability**

$$p(y\,|\,D)=\sum_{k=1}^{K}p(f_k\,|\,D)\cdot p_k(y\,|\,f_k,D)=\sum_{k=1}^{K}w_k\cdot p_k(y\,|\,f_k,D)\qquad \sum_{k=1}^{K}w_k=1$$

$$L(\theta)=\sum_{t=1}^{T}\log\left(\sum_{k=1}^{K}w_k\cdot p_k(y_t\,|\,f_{k,t},\sigma_k)\right)=\sum_{t=1}^{T}\log\left(\sum_{k=1}^{K}w_k\cdot\frac{1}{\sigma_k\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y_t-f_{k,t}}{\sigma_k}\right)^2}\right)$$

- **Expectation-Maximization (EM) algorithm**



$$z_{k,t}^{i}=\frac{w_k^{i-1}g_k(y_t\,|\,f_{k,t},\sigma_k^{i-1})}{\sum_{k=1}^{K}w_k^{i-1}\cdot g_k(y_t\,|\,f_{k,t},\sigma_k^{i-1})}$$

$$w_k^{i}=\frac{1}{T}\sum_{t=1}^{T}z_{k,t}^{i}$$

$$\sigma_k^{i}=\sqrt{\frac{\sum_{t=1}^{T}z_{k,t}^{i}\cdot(y_t-f_{k,t})^2}{\sum_{t=1}^{T}z_{k,t}^{i}}}$$

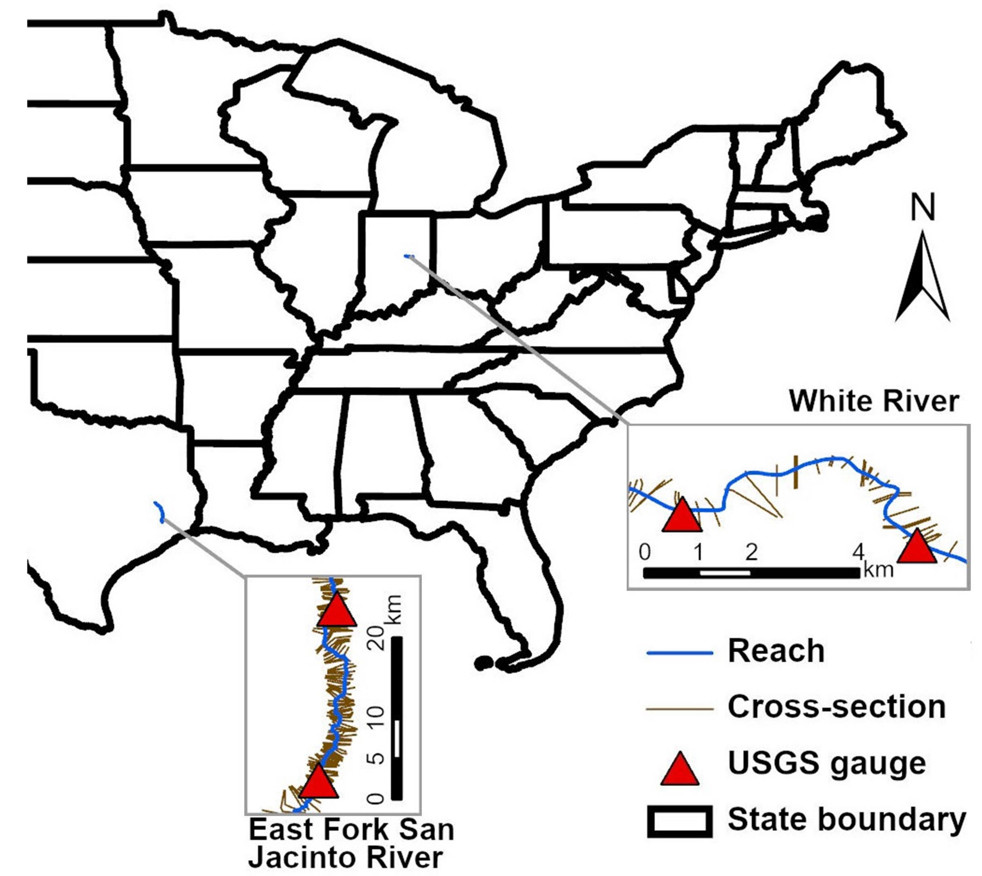
✓	✗
• combine estimations from multiple competing models • BMA weights are interpretable • provide a prediction distribution of variables of interest	• EM algorithm yielded fixed BMA weights and variances • Conditional PDF is limited by Gaussian assumption • Markov Chain Monte Carlo (MCMC) is rarely examined in BMA analysis

Research Objectives

Quantify the uncertainty in BMA parameters (weight & variance):

- estimate the BMA parameters using different numbers of samples in each Markov chain with the Metropolis-Hastings (M-H) algorithm
- compare the performance of EM and M-H MCMC algorithms for estimating the BMA parameters
- estimate the BMA weights using different proposal distributions in the M-H MCMC algorithm
- investigate the impact of different conditional PDFs of the predictor variable on the BMA parameters

Study Area and Data



Study stream	Channel length (km)	Average channel width (m)	Channel slope (%)
White	6.76	64	0.0631
East Fork San Jacinto	50.11	76	0.0438
Study stream	**Upstream USGS gauge**	**Downstream USGS gauge**	**Simulation Period (100 days)**
White	03348000	03348130	2021-3-15 to 2021-6-22
East Fork San Jacinto	08070000	08070200	2021-4-15 to 2021-7-23

Methodology

- **Numerical experiment (ensemble of 10 members)**

D = 100 days of daily water stage data (in meters)	
$f_1 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.06^2)$	$f_2 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.06^2)$
$f_3 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.12^2)$	$f_4 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.12^2)$
$f_5 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.18^2)$	$f_6 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.18^2)$
$f_7 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.24^2)$	$f_8 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.24^2)$
$f_9 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.30^2)$	$f_9 = D+\varepsilon$, where $\varepsilon \sim N(0, 0.30^2)$

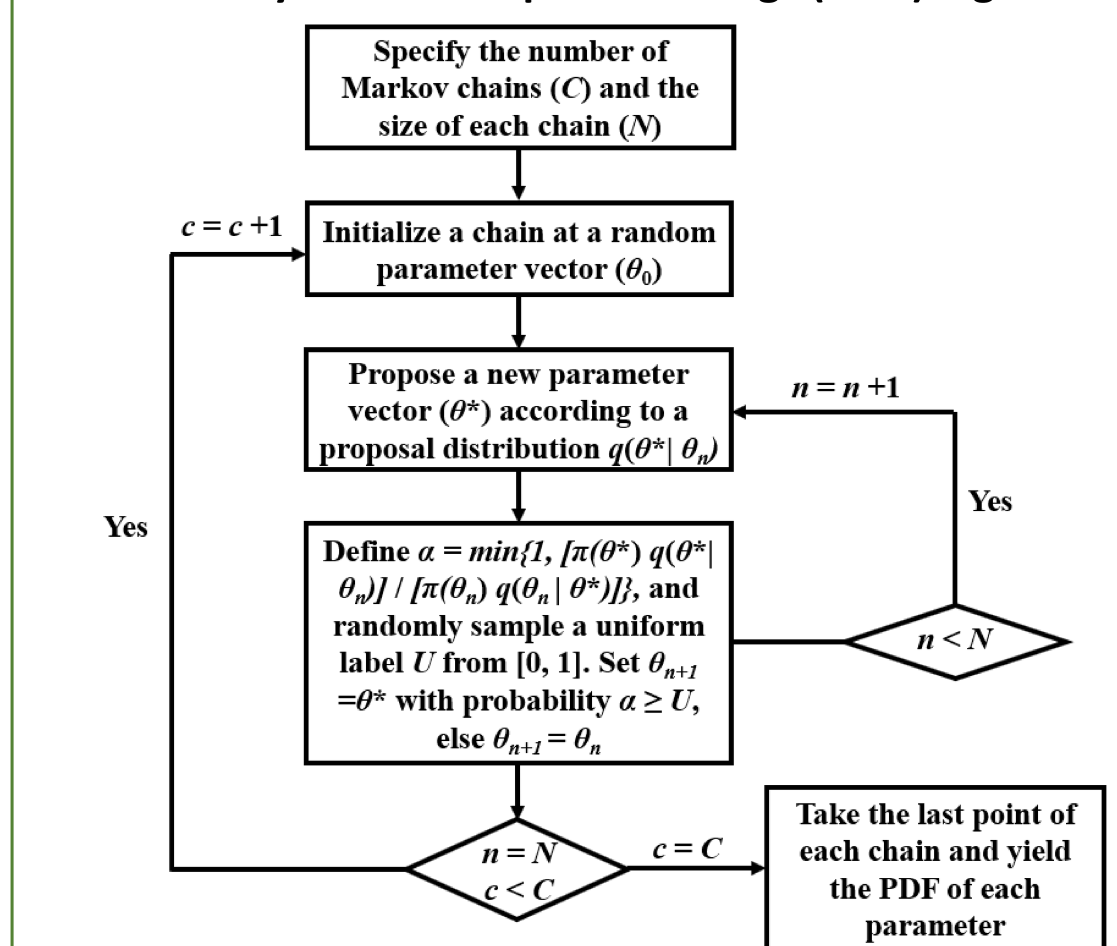
- **Ensemble flood modeling in 1D HEC-RAS**

No.	Channel Roughness	Upstream Flow Input	HEC-RAS Plan Files
1	0.8n	0.8Q	g01 & u01
2	0.8n	Q	g01 & u02
3	0.8n	1.2Q	g01 & u03
4	n	0.8Q	g02 & u01
5	n	Q	g02 & u02
6	n	1.2Q	g02 & u03
7	1.2n	0.8Q	g03 & u01
8	1.2n	Q	g03 & u02
9	1.2n	1.2Q	g03 & u03
10	Average of simulations from No.1-No.9		

Note: n is the Manning's n value for the main channel in the original HEC-RAS models, Q is the streamflow from USGS gauge stations, g** represents a geometry file of a HEC-RAS project, and u** represents a flow data file of a HEC-RAS project.

Methodology (cont.)

- **BMA analysis & Metropolis-Hastings (M-H) algorithm**



$$L(\theta)=\sum_{t=1}^{T}\log\left(\sum_{k=1}^{K}w_k\cdot p_k(y_t\,|\,f_{k,t},\sigma_k)\right)=\sum_{t=1}^{T}\log\left(\sum_{k=1}^{K}w_k\cdot\frac{1}{\Gamma(\alpha)\beta^{\alpha}}y_t^{\alpha-1}e^{-\frac{y_t}{\beta}}\right)$$

$$\alpha=f_{k,t}^{\,2}/\sigma_k^{\,2}\qquad \beta=\sigma_k^{\,2}/f_{k,t}$$

- **Evaluation metrics for model performance[3]**

$$UC1=\frac{N_{obs\text{-}90\%}}{n}\cdot 100\%$$

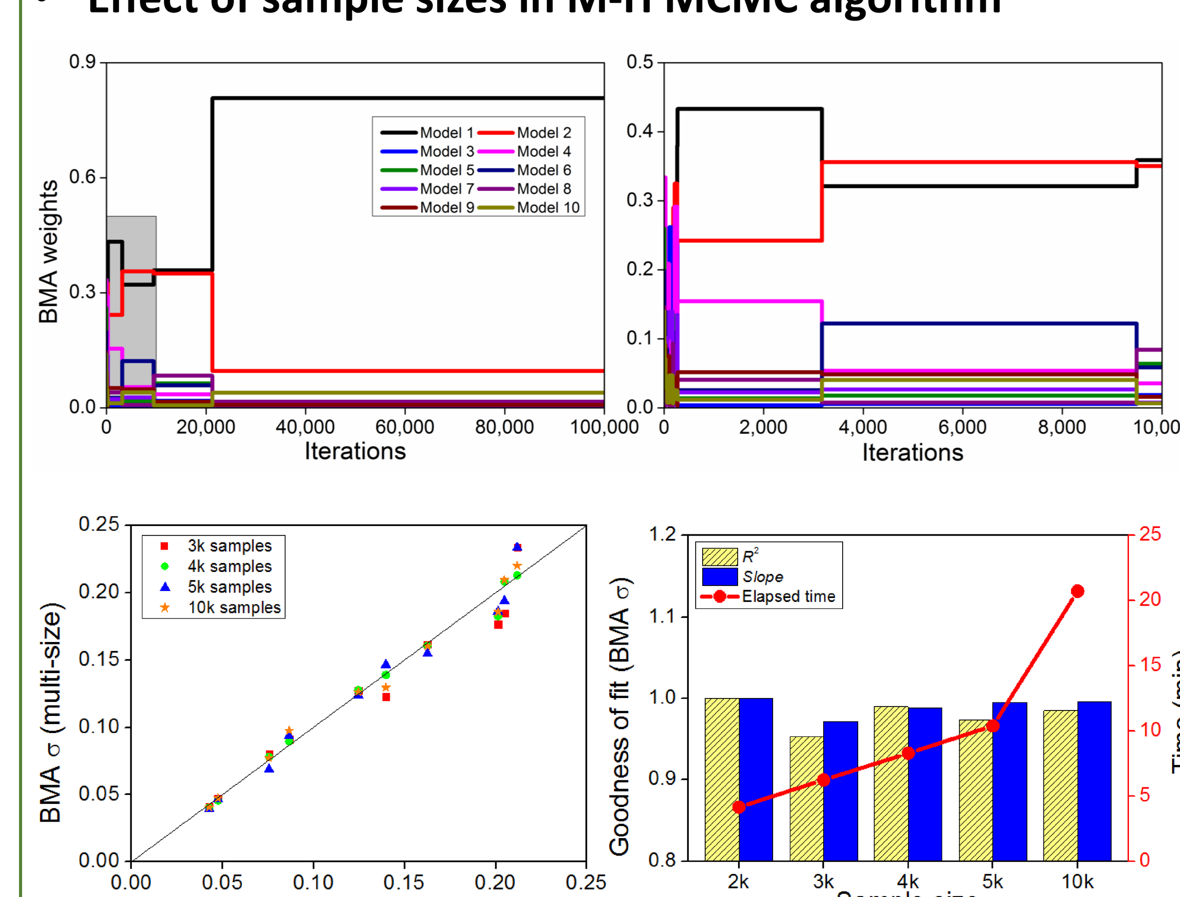
$$UC2=1-NSE=\frac{RMSE^2}{\sigma_{obs}^{\,2}}\cdot 100\%\qquad RMSE=\sqrt{\frac{\sum_{i=1}^{n}(y_{obs,i}-y_{BMA,i})^2}{n}}$$

$$UC3=1-KGE=\sqrt{(r-1)^2+\left(\frac{\sigma_{BMA}}{\sigma_{obs}}-1\right)^2+\left(\frac{\bar{y}_{BMA}}{\bar{y}_{obs}}-1\right)^2}\cdot 100\%$$

$$UC4=(1-R^2+|1-Slope|)\cdot 100\%$$

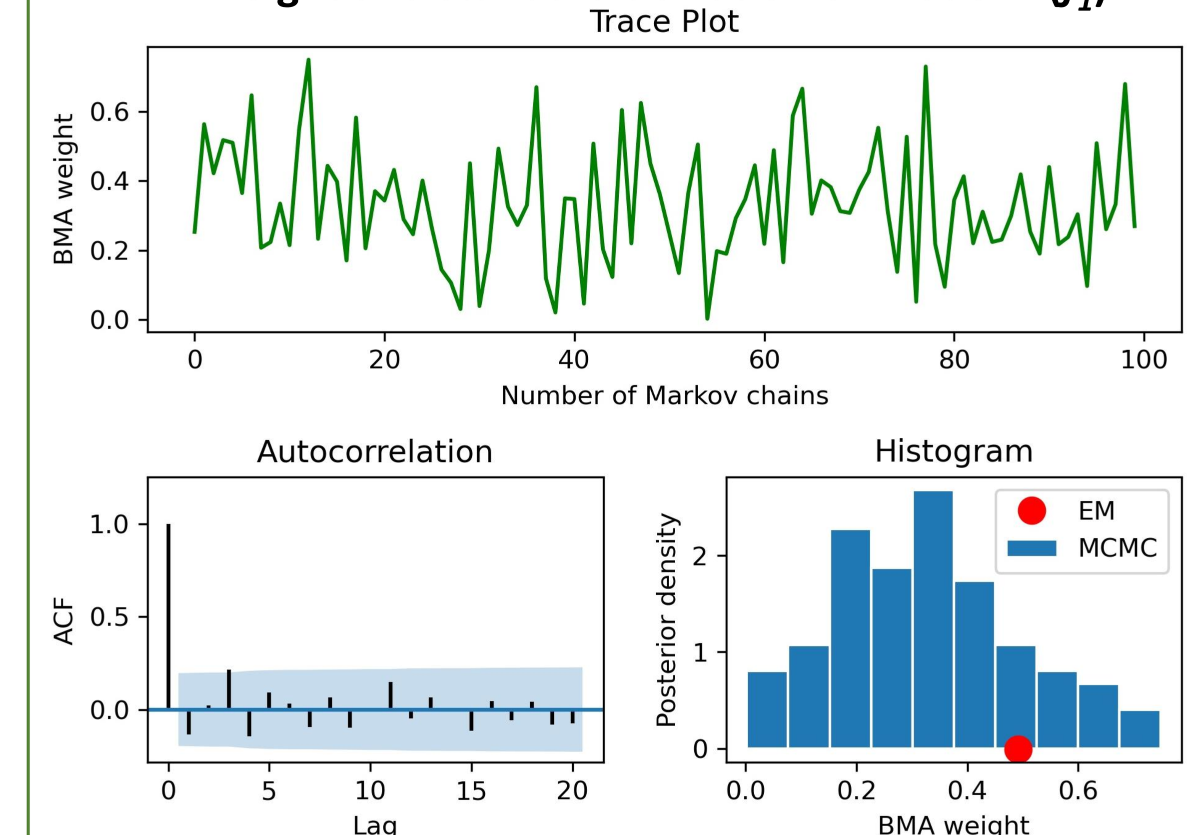
Results and Discussion

- **Effect of sample sizes in M-H MCMC algorithm**



Results and Discussion (cont.)

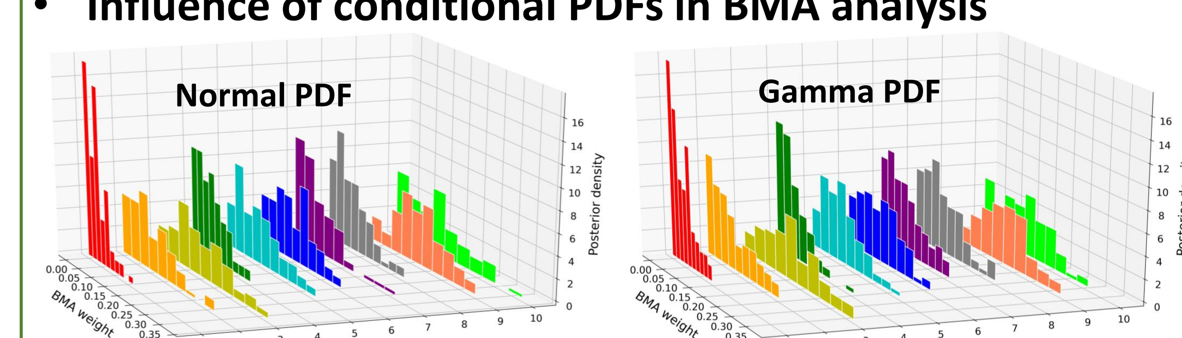
- **BMA weight and standard deviation of Model 1 (f_1)**



No.	EM weight	EM σ (m)	MCMC weight (Uniform & Normal)	MCMC σ (m)	Given σ (m)
1	0.492	**0.04**	0.332 & 0.354	**0.05**	**0.06**
2	0.497	**0.04**	0.320 & 0.35	**0.04**	**0.06**
3	0.0002	**0.02**	0.068 & 0.062	**0.08**	**0.12**
4	0	**0.05**	0.059 & 0.053	**0.09**	**0.12**
5	0	**0.08**	0.040 & 0.034	**0.14**	**0.18**
6	0	**0.02**	0.044 & 0.033	**0.12**	**0.18**
7	0	**0.11**	0.035 & 0.029	**0.16**	**0.24**
8	0	**0.05**	0.032 & 0.028	**0.20**	**0.24**
9	0.01	**0.001**	0.037 & 0.028	**0.20**	**0.30**
10	0	**0.09**	0.033 & 0.028	**0.21**	**0.30**

Algorithm	*RMSE* (m)	Average 90% prediction interval (m)	*UC1* (%)	*UC2* (%)	*UC3* (%)	*UC4* (%)
EM	0.041	0.19	5.00	1.61	1.06	2.00
MCMC (Uniform)	0.037	0.47	0.00	1.33	1.04	1.66
MCMC (Normal)	0.037	0.41	0.00	1.33	0.96	1.64

- **Influence of conditional PDFs in BMA analysis**



Algorithm	*RMSE* (m)	Average 90% prediction interval (m)	*UC1* (%)	*UC2* (%)	*UC3* (%)	*UC4* (%)
Normal PDF	0.421	1.32	13.00	7.54	5.69	7.76
Gamma PDF	0.419	1.34	11.00	7.48	5.67	7.71

Conclusions

- MCMC method with multiple independent chains is applicable in the BMA analysis and can demonstrate a full view of the uncertainty of BMA weights and variances.

References

1. Data source: https://www.weather.gov/hazstat/
2. Raftery, A. E., Gneiting, T., Balabdaoui, F., Polakowski, M. (2005). Using Bayesian model averaging to calibrate forecast ensembles. Monthly weather review, 133(5), 1155-1174.
3. Huang, T., & Merwade, V. (2023). Uncertainty Analysis and Quantification in Flood Insurance Rate Maps Using Bayesian Model Averaging and Hierarchical BMA. Journal of Hydrologic Engineering, 28(2), 04022038.