A method to identify the weakest link in urban water systems

Didrik Meijer[1], Hans Korving[1], Jeroen Langeveld[2], and François Clemens-Meyer[3]

[1]Deltares
[2]Delft University of Technology, Partners4urbanwater
[3]Norwegian University of Science & Technology, SkillsInMotion B.V.

December 7, 2022

Abstract

Urban water systems are composed of subsystems (gully pots, storm sewers and surface water), each with its own system dynamics. Engineers balance the functioning of the systems based on storage and discharge capacity of the subsystems. The load on, and capacity are influenced by e.g. ageing, urbanization, climate change. Consequently, the performance of and demand put on subsystems varies over time, potentially resulting in disturbances in the balance between the storage and discharge capacity of the subsystems. The Graph Based Weakest Link Method (GBWLM) is developed to analyse the behaviour of urban water systems to identify potential limitations due to deterioration, and/or changes in load. The proposed GBWLM is based on the structure of the networks. In addition, Graph theory is applied as alternative for series of hydraulic calculations. The GBWLM allows for an integrated performance assessments of urban water systems using multi-decades rainfall series. The results are sufficiently accurate to be able to determine the extent and frequency of urban flooding in order to compare the performance of the subsystems for various degrees of available discharge capacity. Keywords Criticality, flow paths analysis, Graph theory, linearised hydrodynamics, urban water systems, Weakest Link Method Highlights 1. Method for the analysis of urban water systems based on Graph theory 2. The use of linearised hydrodynamics in flow path analyses

A method to identify the weakest link in urban water systems

Short title: A method to identify the weakest link in urban water systems

Didrik Meijer[*^], Hans Korving[**], Jeroen Langeveld[***], and François Clemens-Meyer[****]

[*] Deltares, Boussinesqweg 1, 2629 HV Delft, *Postbus* 177 2600 MH *Delft,* the Netherlands and Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands
(Didrik.Meijer@Deltares.nl)

[**] Deltares, Boussinesqweg 1, 2629 HV Delft, the Netherlands
(Hans.Korving@deltares.nl)

[**] Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands and Partners4urbanwater, Graafseweg 274, 6532 ZV Nijmegen, The Netherlands
(J.G.Langeveld@tudelft.nl)

[****] Faculty of Engineering, Dept. Civil & Environmental Engineering, Norwegian University of Science & Technology, Høgskoleringen 6, 7491 Trondheim, Norway and SkillsInMotion B.V., Esdoornlaan 11, 3454 HH Utrecht, The Netherlands
(F.H.L.R.Clemens@tudelft.nl)

^Corresponding author: didrik.meijer@deltares.nl

Abstract
Urban water systems are composed of subsystems (gully pots, storm sewers and surface water), each with its own system dynamics. Engineers balance the functioning of the systems based on storage and discharge capacity of the subsystems. The load on, and capacity are influenced by e.g. ageing, urbanization, climate change. Consequently, the performance of and demand put on subsystems varies over time, potentially resulting in disturbances in the balance between the storage and discharge capacity of the subsystems. The Graph Based Weakest Link Method (GBWLM) is developed to analyse the behaviour of urban water systems to identify potential limitations due to deterioration, and/or changes in load. The proposed GBWLM is based on the structure of the networks. In addition, Graph theory is applied as alternative for series of hydraulic calculations. The GBWLM allows for an integrated performance assessments of urban water systems using multi-decades rainfall series. The results are sufficiently accurate to be able to determine the extent and frequency of urban flooding in order to compare the performance of the subsystems for various degrees of available discharge capacity.

Keywords
Criticality, flow paths analysis, Graph theory, linearised hydrodynamics, urban water systems, Weakest Link Method

Highlights
1. Method for the analysis of urban water systems based on Graph theory
2. The use of linearised hydrodynamics in flow path analyses
3. Analysis of the impact of the decline in system performance on flood frequency and extent

1 Introduction
Urban water systems are used to drain storm water to prevent pluvial flooding in urban areas. These systems consist of various subsystems (e.g. gully pots, storm sewers and surface water) and were built and have been adapted over extended periods of time and have often "grown organically" to their present state. During the construction period, there have been (substantial) changes in, for example, laws and regulations and the related design standards and design criteria, available techniques, costs of manpower and materials and external conditions like e.g., climate (Preston & van de Walle; 1978; Geels, 2006). As a result, urban water systems, and the

subsystems have their own specific characteristics in terms of storage and discharge capacity, that reflect the dominant dynamics at their specific temporal (minutes, hours, days) and spatial scales (building, street, neighbourhood, city) (Langeveld & Schilperoort, 2019).

The combination of storage and discharge capacity determines which storm event a subsystem can handle. The interaction between the subsystems determines the processing capacity of the system (Langeveld & Schilperoort, 2019). Backwater effects may change the internal boundary conditions between sub-systems, resulting in a reduction of the hydraulic capacity of these. A delayed discharge from a storm water system may result in increased water levels in the surface water system at an undesirable time.The balance between the subsystems and the performance of networks is threatened by the following (Reyes-Silva et al., 2020):
- Increasing load as a result of urbanisation, population growth and densification.
- Deterioration as a result of ageing (and lack of maintenance) resulting to a decrease in capacity.
- Terrorist attacks.

Understanding the contribution of each element to the system functioning is essential when (re)designing urban water systems, or when making decisions on asset management (van Riel 2016). In practise, integrated urban water systems are seldomly analysed, the main reasons for this are summarised as:
- In many countries individual parts of the systems are being managed by different organisations (e.g. in the Netherlands: surface water by Waterboards, drainage systems by municipalities, large rivers by the national government).
- Tools applied for subsystems are not always mutually aligned (Tscheikner-Gratl et al., 2019).
- Current methods for integrated analysis are complex and time consuming and demand simplifications. The current complex software tools for 1D-2D simulations, require a large processing capacity often with questionable results because input data is lacking (see e.g. Tscheikner-Gratl et al., 2019).

As a result, in practical cases, integrated analyses are rarely used due to the time-consuming calculation and the expertise needed to evaluate the model results. In order to reduce the calculational effort, either design storms are applied instead of precipitation series, or the network is simplified. Simplification of the network may reduce the useability of the flood simulation results (Fischer et al. 2009; Yang et al. 2018;) as it often implies a reduction in spatial resolution (leaving out manholes and/or conduits). The outcomes are not always accurate enough to determine whether the (sub)systems match well in terms of storage and discharge capacity. Replacing rainfall series by (a series of) design storm(s) prevents an accurate statistical analysis of the behaviour of the subsystems (Vaes, Feyaerts, & Swartenbroekx 2009).

This paper presents the results of a method that allows for applying rain series while maintaining a detailed spatial representation of the system under study. To this end a Graph based algorithm has been developed to assess hydraulic performance. With the Graph Based Weakest Link Method (GBWLM), the impact of capacity reduction of (parts of) urban water (sub)systems can be analysed with multi annual precipitation series. The starting point of the GBWLM is the system's topology.

This paper presents the proposed method along with an existing method to compare with, the results of the application of the GBWLM for a case study are presented. Finally, the results are discussed and conclusions are formulated for further application and developments.

2 Methods
The Achilles approach
A generally applicable method to determine the criticality of elements in networks is shown in Figure 1. This method has been adapted from the Achilles approach (Möderl et al., 2009; Mair et al., 2012;) and the percolation theory (Stockmayer, 1944; Broadbent & Hammersley, 1957; Stauffer & Aharony, 1991; Sahimi, 1994). This method aims to quantify the effect of a reduced capacity of an element to the system's performance level as a whole (Möderl et al., 2009; Mair et al., 2012). The Achilles approach can be used to determine vulnerable sites of (water)

infrastructure. For the determination of vulnerabilities, the outcomes of a hydrodynamic model are used. This method is referred to as the Achilles approach and is used as reference method.

The Achilles approach can be used to evaluate the contribution of subsystems or groups of elements to the functioning of the entire system, for example, an urban water system consisting of gully pots, storm sewers and surface water. In this case, the main steps taken in the Achilles approach are:

- Simulate the functioning of the system.
- Gradually reduce the capacity of (groups of) elements of the urban water system.
- Simulate the functioning of the adjusted subsystems of the urban water system and take into account the backwater effects.
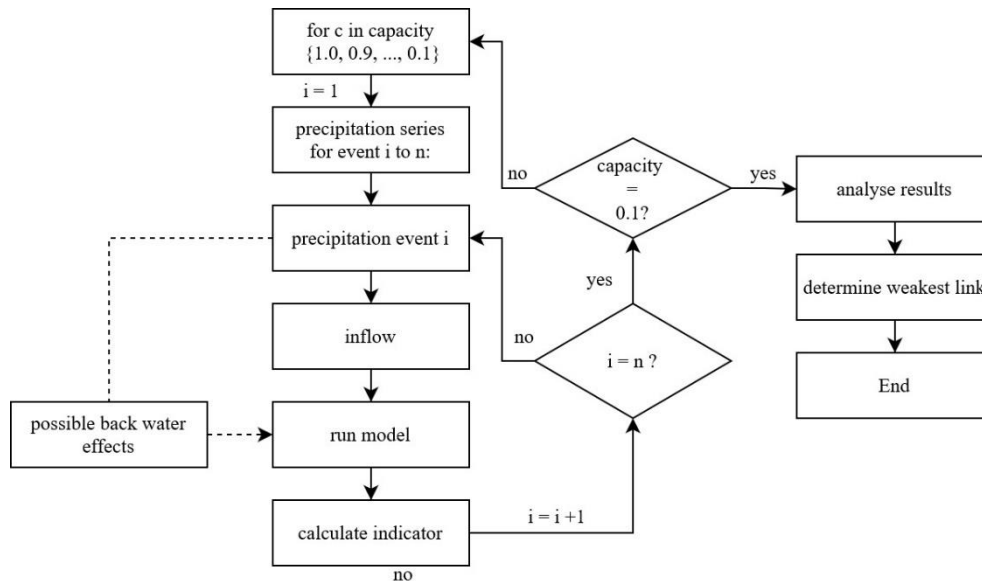- Evaluate the performance of the urban water system.

Figure 1   General overview of the Achilles approach applied to urban water systems.

Graph Based Weakest Link Method

The performance of a(n urban water) system depends on the load and the system's response characteristics. As pointed out previously, the latter maybe effected by the interactions between the various subsystems and may change overtime or even during an event. Because subsystems are designed for different storage and drainage standards, they react differently to changes in load or capacity. The Graph Based Weakest Link Method (GBWLM) has been developed to analyse integrated urban water networks at three levels of subsystems: gully pots, storm sewers and surface water systems, with multi annual precipitation series. The metrics applied to evaluate the system's performance are the flood frequency and flooded area. The general outline of the GBWLM is shown in Figure 2. The various components are described in some detail in the following sections.
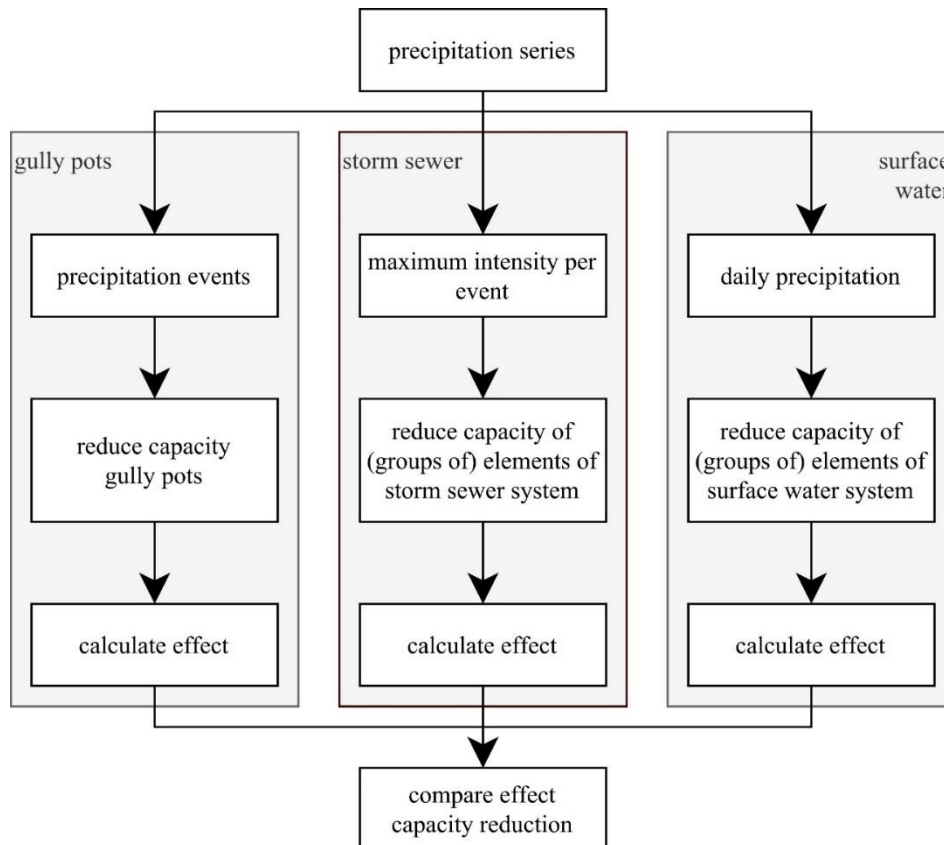
Figure 2   The graph-based weakest link method process for urban water systems focused on gully pots, storm sewers and surface water systems. For a detailed description see following subsections

Precipitation load on the subsystems

The test load applied is a multi-year precipitation series that is subdivided into mutually independent events (see the precipitation blocks at the top of the three columns in Figure 2). For the sake of simplicity, a rainfall runoff model is omitted, resulting in an inflow equal to the precipitation volume at all times. Events are considered to be mutually independent when the initial conditions of the system under study events are identical for each event (i.e.: the time for emptying the system after a storm event has to be considered as this determines the initial filling of the system for the next event, only when the time-gap between time windows in which precipitations occurs is larger than the time needed to empty the system these events are considered independent). The characteristic system response times are determined and used to divide the series into separate independent events.

The critical precipitation load of the subsystems depends on their primary function (storage or discharge). The leading principles (in flat areas) are the following:
- Gully pots: exceeding discharge capacity (rainfall intensity).
- Storm sewers: exceeding discharge capacity (rainfall intensity).
- Surface water systems: exceeding storage capacity (rainfall volume).

The inflow and outflow of subsystems are interdependent. The maximum drainage capacity of gully pots affects the maximum inflow of the storm sewers. In addition, the cumulative outflow of the storm sewer results in a cumulative inflow of the surface water system (see Figure 3 and, for more details, supplementary material).
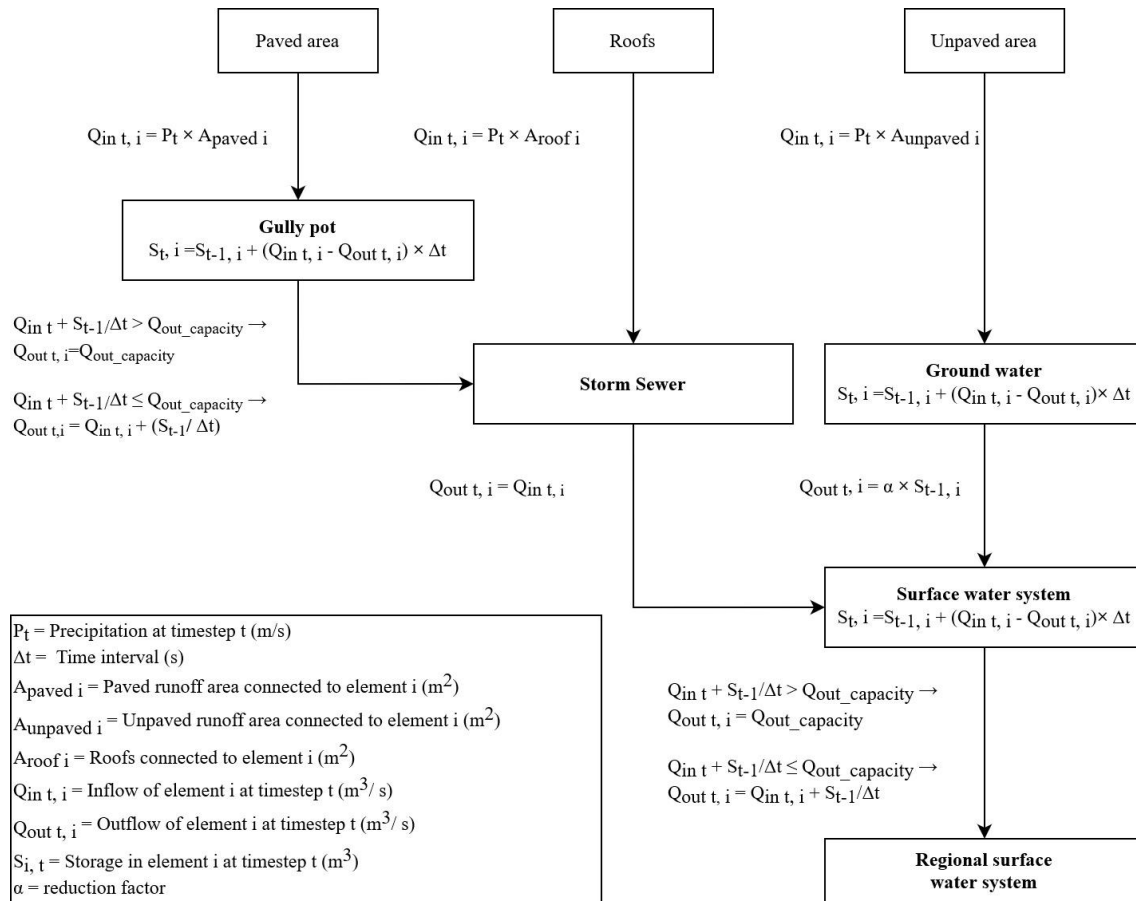
Figure 3   Overview of the water balance of graph-based weakest link method subsystems.

Analysis of network performance with flow paths analysis

The GBWLM is based on the Achilles approach. The hydrodynamic models have been replaced by water balances and flow path analysis. The functioning of urban water systems depends on the available discharge and storage capacity. The discharge capacity is used to evaluate the performance of the gully pots and storm sewers. The surface water system performance is evaluated on the combination of discharge and storage capacity.

The gully pots are represented as a reservoir with outflow. The storm sewers and surface water systems networks are described as digraphs. De performance of the networks is analysed with a flow path analysis. To perform flow path analysis, all connections in the network must be given a capacity and a cost. Capacity is the amount of water that can be drained through a connection. The cost is the amount of energy (head loss) required to drain the water (see Meijer et al. 2018). An analysis is carried out to determine how, at the lowest possible cost, the inflowing water can be discharged to the outfall point. In a storm sewer, the manholes are the inflow points (sources) and the Storm Sewer Outfalls (SSOs) are the outflow points (targets). In the surface water system, the SSOs are the sources and the final pumping station is the target.

For the analysis of the flows through the network, the minimum cost flow algorithm (Min_Cost_Flow) of the Networkx module in Python (Hagberg et al., 2008; NetworkX Developers, n.d.) is used to evaluate the consequences of a capacity reduction in networks. The algorithm can be applied to a digraph in which the edges have both a cost and a capacity. The nodes desire to send or receive some amount of flow. The algorithm calculates the minimum costs for a flow that satisfies the demands of all nodes. That means that for each node the net inflow or outflow is equal to the demand of that node.

The GBWLM can be applied to analyse the consequences of a reduced discharge and storage capacity in the subsystems. The consequences of a reduced discharge capacity have been analysed with both the Achilles approach and the GBWLM. If the surface water level rises, it

has been analysed whether this has a negative effect on the functioning of the storm sewer system and whether it leads to flooding from the storm sewer system (see Figure 4).
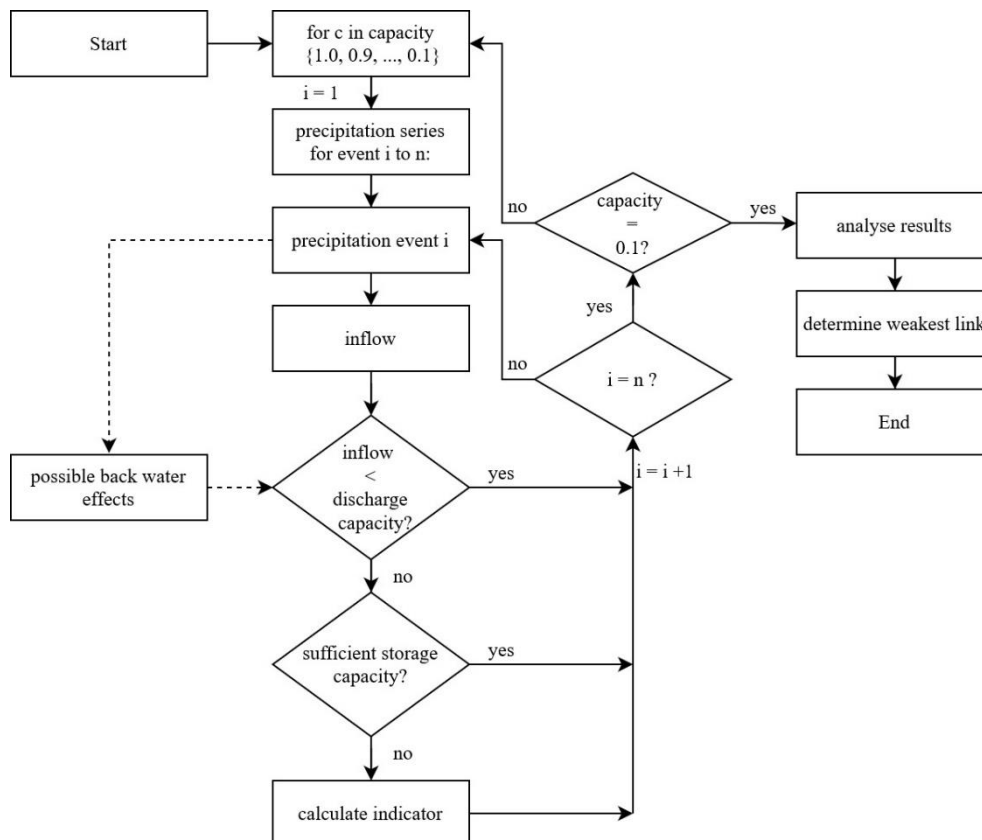

Figure 4   General overview of the hydrodynamic model-based weakest link method applied to urban water systems

Graph schematization of storm water and surface water networks
The design of the subsystems in the GBWLM can be tuned to the characteristics of the systems and the leading principle. A gully pot is represented as a reservoir with outflow. The storm sewers and surface water systems are described as digraphs (layer-1 in Figure 5 and Figure 6). In storm sewer digraphs, the nodes represent manholes and the edges represent conduits. In surface water digraphs, the nodes correspond to the watercourses between two structures, and the edges to the structures (weirs, orifices, pumps).

Surface water system
For a surface water system, a copy of digraph-1 is created (digraph-2). When combined, the two digraphs form an object referred to as a layered graph (layer-1 = digraph-1, layer-2 = digraph-2; see Figure 5). Layer-2 is used as a bypass to drain the water if the capacity of layer-1 is insufficient. The bypass is used to ensure that the algorithm used (minimum cost flow algorithm) always finds a solution. The layers are connected at the nodes (structures). Each node of layer-1 is connected with its copy in layer-2. On the outflow location(s) of the network, the direction of the connection between the layers is from layer-2 to layer-1. At the other nodes (structures), the direction between the layers is from layer-1 to layer-2.
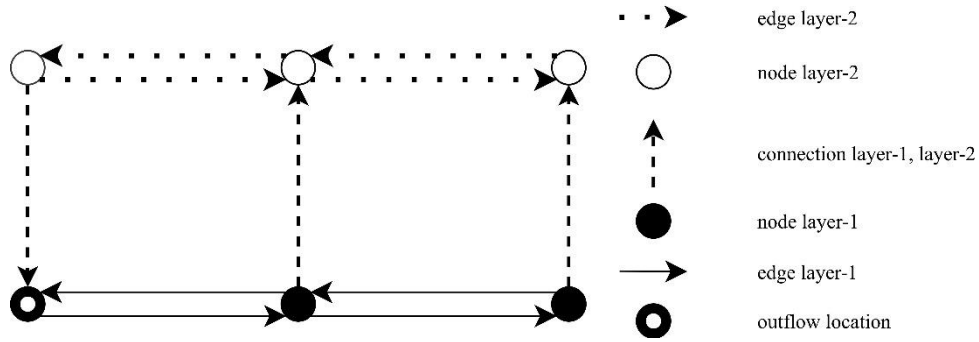
Figure 5   Overview of the edge types in the surface water component of the graph-based weakest link method. Layer-1 represents the structure of the network in which the nodes represent manholes watercourses. Edges correspond to the structures. Layer-2 is used when capacity of layer-1 is exceeded and indicates rising water levels in the surface water system.

For the surface water system, a water balance is used at each node to store the water that cannot be drained via the structures. The available storage is determined for the watercourses between two structures.

$$\left.\begin{array}{c} \Delta t \times Q_{in} + S_{t-1} > \Delta t \times Q_{structure} \rightarrow Q_{out} = Q_{structure}, \\ S_t = S_{t-1} + \Delta t \times Q_{in} - \Delta t \times Q_{out} \\ \\ \Delta t \times Q_{in} + S_{t-1} \leq \Delta t \times Q_{structure} \rightarrow Q_{out} = Q_{in} + \frac{S_{t-1}}{\Delta t}, \\ S_t = S_{t-1} + \Delta t \times Q_{in} - \Delta t \times Q_{out} = 0 \end{array}\right\} \qquad (1)$$

where Q_{out} is outflow of surface water node (m³/s), Q_{in} is inflow of surface water node (m³/s) $Q_{structure}$ is maximum discharge capacity structure (m³/s), Δt is time interval (s), and S_t is storage in surface water node at timestep t (m³).

Storm sewer system
For a combined or storm sewer system, two copies of digraph-1 are created (digraph-2 and digraph-3). In layer-1, the nodes represent the manholes and the edges the conduits of the storm or combined sewer system. Each node (manhole) of layer-1 is linked with the copy of this node in layer-2 (connection L1–L2) and layer-3 (connection L1–L3). At the outflow locations, the nodes of the layers are connected with an edge in the opposite direction (connection L2–L1 and connection L3–L1 instead of connection L1–L2 and connection L1–L3). Layer-2 is used to drain the water when the capacity of layer-1 is insufficient. Layer-3 is used as a bypass to drain the water when the capacity of Layer-1 and the connections L1–L2 are insufficient (see Figure 6). The use of three layers makes it possible to limit the capacity of the connection L1–L2 and still always arrive at a solution of the minimum cost flow algorithm via layer-3.
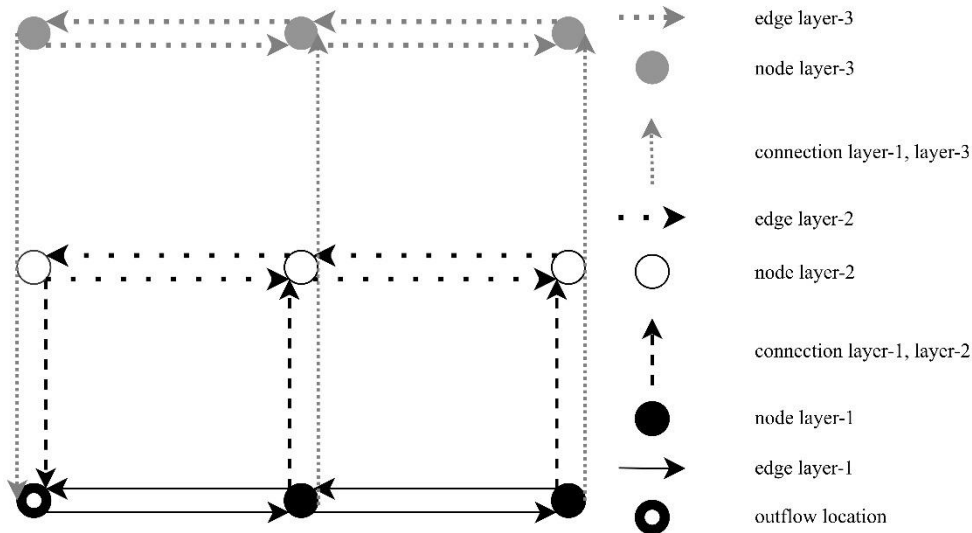
Figure 6　　　　Overview of the edge types in the storm or combined sewer component of the graph-based weakest link method. Layer-1 represents the structure of the networks in which the nodes represent manholes. Edges correspond to the pipes and structures. Layer-2 is used when capacity of Layer-1 is exceeded and indicates flooding. Layer-3 is a bypass in case the capacity of the links between Layer-1 and Layer-2 is exceeded.

Costs and capacity of the edges
The costs of most of the edges in the GBWLM are derived from the head loss (see Meijer et al. 2018), except for the connections between the layers. The capacity of the edges is based on linearised hydrodynamics. The general process to determine the capacity of the pipes is described in the next section. To determine the capacity or costs for the edges between the Graph layers, an additional operation is carried out or an exception can be made. These are in the sections after the linearised hydrodynamic section.

Linearised hydrodynamics in the GBWLM
To estimate the capacity of the pipes and the costs of the connections between layers of the graph, hydrodynamics are linearised. The quadratic hydraulic gradient equation (Equation 2) has been simplified to a linear relationship (Equation 3). Based on the maximum available hydraulic gradient, the maximum capacity of each pipe is computed (Equation 2). α_{linear} is determined for each pipe by linearising the discharge (between zero and the maximum pipe capacity) and the maximum available hydraulic gradient (Equation 3):

$$I = \alpha_{quadratic} Q^2 \qquad\qquad\qquad (2)$$

$$I = \alpha_{linear} |Q| \qquad\qquad\qquad (3)$$

where *I* is the hydraulic gradient (-), *Q* is the discharge (m³/s), $\alpha_{quadratic}$ is a quadratic hydraulic parameter (s²/m⁶) and α_{linear} is a linear hydraulic parameter (s/m³).

The process of linearisation in the GBWLM consists of the following steps:
1. Calculate the costs of the pipes based on the head loss.
2. Calculate the available hydraulic gradient. The available gradient for conduits depends on the crest level or surface water level of the outflow point, street level and path length and is determined as follows:
    i. For digraph-1, the shortest paths of all nodes to the closest outflow points are calculated with the Dijkstra algorithm (Dijkstra, 1959).
    ii. For each manhole (node), the minimum available hydraulic gradient is determined by dividing the differences between ground level and crest level by the path length without taking into account the available gradients of the other manholes.
    iii. For each manhole (from smallest to largest available hydraulic gradient from step b), the available hydraulic gradient is recalculated, taking into account the gradients of the other manholes. If a path crosses a path with a known hydraulic gradient, the

hydraulic gradient is computed between the level of the gradient line at junction, the street level and the path length.
3. Calculate the pipe capacity with Equation2.
4. Calculate α_{linear} with Equation 3.
5. Determine the runoff area that discharges via a pipe. The paths that include a pipe are selected. The runoff area of all starting nodes (sources) of these paths is summed.
6. Determine costs between connections for layer-1 and layer-2. The costs are equal to the freeboard (ground level minus water level) of all manholes which correspond with the static head loss. The water levels are calculated with an iterative process. The discharge and corresponding hydraulic gradient (Equation 3) in every pipe are calculated based on the runoff area that drains via a pipe (step 5) and a low rainfall intensity. Starting at the node with the smallest available hydraulic gradient, the water levels are calculated on the path from the closest outfall (see step 2 c). If the water level is below ground level at all manholes, the rainfall intensity is increased until the water level is above ground level at one or more manholes. The water levels calculated with the last intensity at which no floods occur are used to calculate the freeboard for every manhole.

Costs of the connections between the layers
The connections between the layers ensure that there is always a path through which all the water can be drained. The connections between the layers may only be used if there is no other outlet route available. Therefore, the cost should be equal to the total cost of pipes in layer-1 (see table 1).

Table 1, costs and capacity of the bypass edges

Subsystem	Edge type	Capacity	Costs
surface water	connection layer-1, layer-2	equal to the total inflow	total cost of the edges in layer 1
combined and storm water sewer	connection layer-1, layer-2	default value depending on the flood characteristics	the sum of costs of all edges of layer-1 and the freeboard at the location of the connection (manhole)
combined and storm water sewer	connection layer-1, layer-3	equal to the total inflow	total cost of the edges in layer 1 and 2

Structures (edges)
All the structures in both the combined or storm water sewers and surface water systems are made of multiple sub-connections in order to increase capacity with increasing head loss. For structures (and especially weirs), small changes in head loss can result in large changes in hydraulic capacity. The relationship between capacity and costs (head loss) of a structure is derived from its characteristics. The head loss is incrementally increased. For each head loss, a sub-connection is added to the graph. The cost of the sub-connection is equal to its head loss. For sub-connection 1, the capacity is equal to the capacity of the corresponding head loss. For the other sub links (i in 2 to n), the capacity is equal to the capacity of corresponding head loss of sub-connection i minus the capacity of the corresponding head loss of sub-connection $i-1$ (see Table 2).

Table 2   Costs and capacity of structures.

Sub-connection of structure	Head loss (m)	Costs	Total capacity structure (m³/s)	Capacity sub-connection structure (m³/s)
1	0.01	0.01	$Q = \alpha 0.01^\beta$	$Q = \alpha 0.01^\beta$
2	0.02	0.02	$Q = \alpha 0.02^\beta$	$Q = \alpha 0.02^\beta - \alpha 0.01^\beta$
3	0.03	0.03	$Q = \alpha 0.03^\beta$	$Q = \alpha 0.03^\beta - \alpha 0.02^\beta$
Etc.	…	…	…	…

The capacity and costs of all link types of the GBWLM are summarised respectively in Table 3 and Table 4.

Table 3   Overview of the capacity of the edges in the digraphs.

Edge type	Capacity storm sewer	Capacity surface water
layer-1	$Q = \alpha H^\beta$	$Q = \alpha H^\beta$
layer-2	$Q = \alpha H^\beta$	total inflow
layer-3	> total inflow	not applicable
connection L1–L2	default value for more details see Section 2	total inflow
connection L1–L3	total inflow	not applicable
connection L2–L1	total inflow	total inflow
connection L3–L1	total inflow	not applicable

Table 4   Overview of the costs of the edges in the digraphs of the graph-based weakest link method.

Edge type	Costs storm sewer	Costs surface water
layer-1	head loss	head loss
layer-2	head loss	head loss
layer-3	head loss	not applicable
connection L1–L2	sum costs of edges in layer-1 + freeboard	sum costs of edges in layer-1
connection L1–L3	sum costs of edges in layer-1 and layer- 2	not applicable
connection L2–L1	0	0
connection L3–L1	0	not applicable

Capacity reduction and effect
Gully pots
The results of a reduction of the discharge capacity of the gully pots can be evaluated (see left column of Figure 2 and Figure 4). Per gully pot, the number of events in which the discharge capacity is less than the rainfall intensity (which is identical to the runoff intensity, as no rainfall runoff model is used) is counted and multiplied with the runoff area per gully pot. This results in an estimation for the flooded area and a flood frequency.

Storm sewers
The consequences of a capacity reduction of the sewer system is evaluated using the function Min_Cost_Flow of the Networkx module in Python (Hagberg et al., 2008). This function determines the flow paths having minimum costs of all nodes to the central outflow point; this is performed for each event (see the centre column of Figure 2 and Figure 4). Water flows from the nodes of digraph-1 via the edges to the central outflow point. If the water flows through a layer-1–layer-2 edge, the manhole is labelled as flooded. The flooded area is set equal to the runoff area of the connected gully pots.

Surface water system
The effects of a reduction of the surface water system are determined in two steps: 1) the water levels are calculated (see Figure 4); 2) if these exceed a threshold, then the impact on the storm sewer is calculated (backwater effect, see Figure 7).
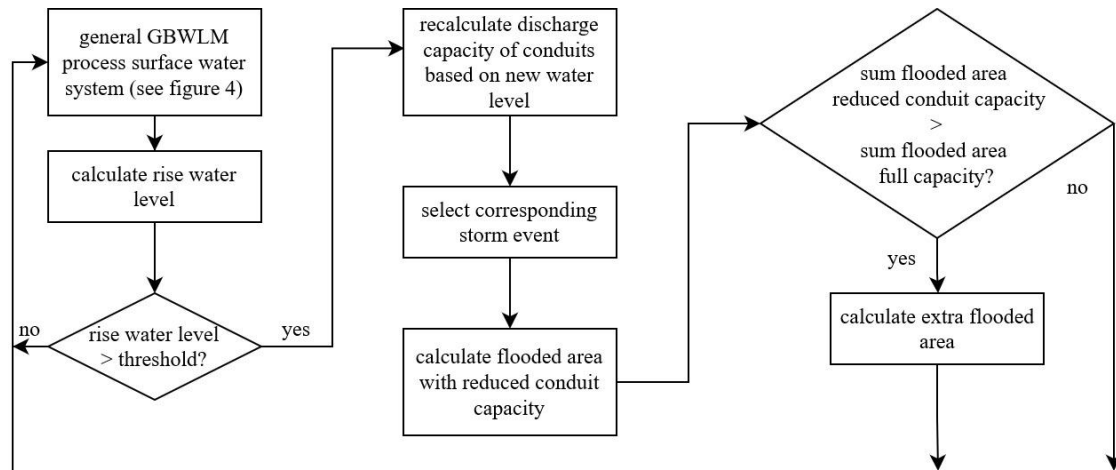
Figure 7   Process of the surface water component of the graph-based weakest link method.

Similarities and differences with the Graph Theory Method
The GBWLM has some similarities with the Graph Theory Method (GTM) for pressurised systems (Meijer et.al, 2020) and the Graph Theory Method for gravity-driven systems (Meijer et.al, 2018) but is substantially different. To clarify the differences among the three methods, their main characteristics of the three methods are summarised in Table 5 and Table 6. The tables show that the GBWLM costs of the edges are determined in a similar way but the outcomes, the network schematisation, and the application of the capacity of the pipes are different.

Table 5   Characteristics of the graph theory method for pressurised and gravity-driven systems.

	Pressurised systems	Gravity-driven systems
Outcome	Ranking of the elements	Ranking of the elements
Network schematisation	Graph	Digraph
Costs of edges	Dynamic head loss	Static and dynamic head loss
Capacity of edges	Not applicable	Not applicable

Table 6   Characteristics of the graph-based weakest link method for urban water systems consisting of gully pots, storm or combined sewer systems and surface water.

	Gully pots	Storm sewer	Surface water
Outcome	Flooded area and flood frequency	Flooded area and flood frequency	Flooded area and flood frequency
Network schematisation	Not applicable	Digraph 3 layers	Digraph 2 layers
Costs of edges	Not applicable	Static and dynamic head loss	Static and dynamic head loss
Capacity of edges	Not applicable	Linearised hydraulics	Linearised hydraulics

Metrics applied to compare the outcomes of the Achilles approach and the GBWLM
In comparing the Achilles approach and the GBWLM the following metrics are used:
- characteristics of the results:
  - Flooding volume of the storm sewer.
  - The number of flooded manholes at system level.
  - The locations of the flooded manholes.
  - The rise of the surface water level.

The results of the GBWLM are compared with the outcomes of a hydrodynamic storm sewer and surface water model serving as a reference.

The Root Mean Squared Error (RMSE) and correlation coefficient (R^2) are used to compare the outcomes of the hydrodynamic models and the GBWLM for the extent of flooding events (storms sewer component of the GBWLM) and increase of water level (surface water component GBWLM). The Kendall rank correlation coefficient (Kendall, 1945), commonly referred to as Kendall's tau-b coefficient (τ_b) is used to determine the relationship between the outcomes of the Achilles approach and the GBWLM. τ_b is a nonparametric measure of association based on the number of concordances and discordances in paired observations. τ_b can be interpreted as a measure of similarity between two data sets. Minus one (−1) implies a 100% negative association; one (1) is a 100% positive association. Equation 3 presents this calculation:

$$\tau_b = \frac{(P-Q)}{(\sqrt{(P+Q+X_0)*(P+Q+Y_0)})} \tag{3}$$

where τ_b is Kendall's tau b coefficient, P is the number of concordant pairs, Q is the number of discordant pairs, X_0 is the number of pairs tied only to the X variable and Y_0 is the number of pairs tied only to the Y variable.

The F1 score (or F1 measure) is a measure of the accuracy of a test. It combines the recall and precision in a single measure. The recall is a measure of the critical elements that were correctly identified as such, and the precision represents the proportion of correctly identified critical elements. If recall and precision are of equal weight, the following relations apply (Chinchor, 1992):

$$F_1 = \frac{2*P*R}{P+R} = \frac{2*TP}{2*TP+FP+FN} \tag{4}$$

$$R = \frac{TP}{TP+FN} \tag{5}$$

$$P = \frac{TP}{TP+FP} \tag{6}$$

where P is precision (-), R is recall (-), TP is true positive (-), FP is false positive (-), FN is false negative (-) and F_1 is F1 score (-).

For the GBWLM, precision is the number of correctly identified flooded manholes divided by the total number of flooded manholes identified by the GBWLM. The recall of the GBWLM is the correctly identified number of flooded manholes by the GBWLM divided by all the flooded manholes (in the hydrodynamic model).

Case study details.
A part of the urban water system in the municipality of Almere (the Netherlands) was used as a case study for testing the GBWLM. Almere is located in Flevoland, a province that was reclaimed from the former Zuiderzee, as such it is an entirely human-made and -controlled polder system. The case study focused on the surface water system in Almere Centrum (see Figure 8) and the storm sewer catchments Waterwijk Oost Noord (referred to as Waterwijk Noord) and Waterwijk Oost Zuid (referred to as Waterwijk Zuid), situated in the north-eastern part of the area (see Table 8). The pumping station that discharges water to a larger regional surface water system is situated just to the south of Waterwijk Zuid.

The "Weerwater" is a large pond (1.5 km²) located in the centre of the surface water system; because of its storage capacity, it has a dampening effect on variations in the surface water level. The surface water system is divided into a main system and a smaller subsystem. The water level in the main system, directly connected to the storm water systems, has a –5.5 m reference level. The reference water level in the smaller section in the southwestern part is –4.8 m and defines a boundary condition for the main system. During storm events, water is

discharged from the subsystem to the main system. The characteristics of the surface water system are summarised in Table 7.

Table 7 , The characteristics of the surface water system of Almere Centrum.

Characteristics	Surface water system Almere Centrum
Length of channels (km)	39.8
Area surface water (km^2)	2.6
Runoff area unpaved (km^2)	8.0
Runoff area paved (km^2)	12.4
Surface water level (m reference level)	-4.8 and -5.5
Number of internal weirs	3
Number of culverts	12
Pump capacity (m^3/s)	3.41

Table 8 The characteristics of the storm sewer systems Waterwijk Noord and Waterwijk Zuid

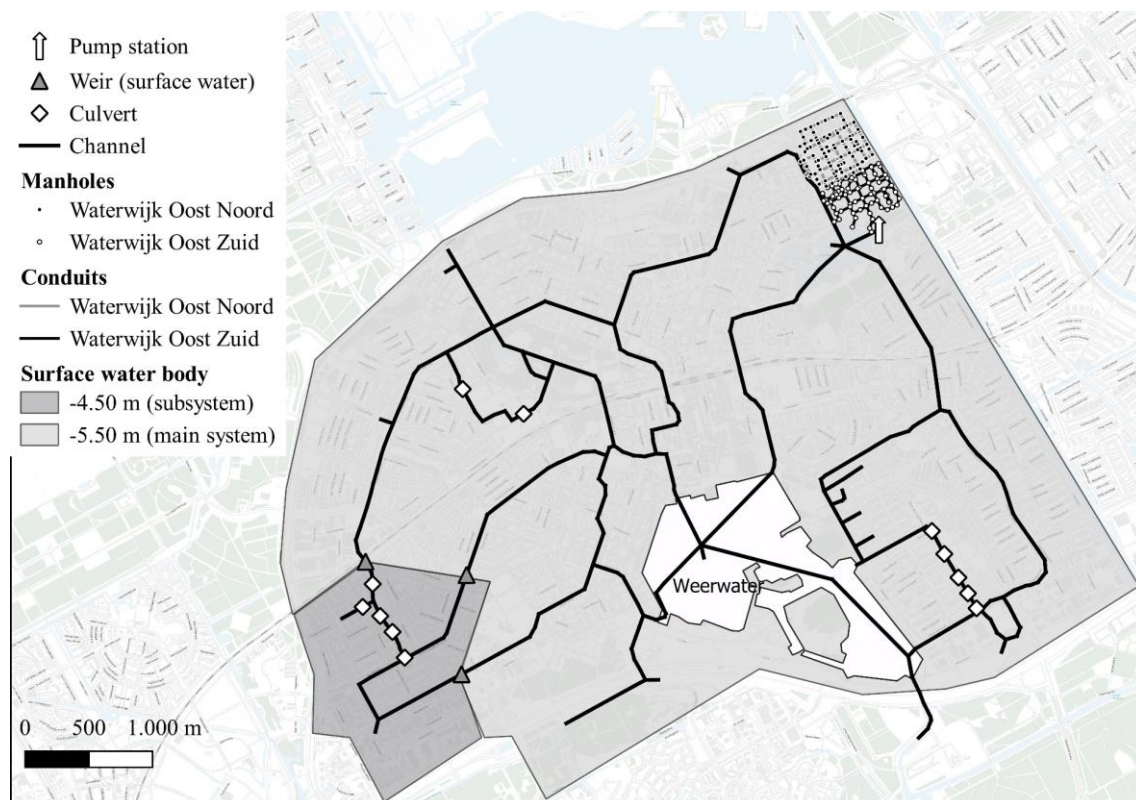| Characteristics	Waterwijk Noord	Waterwijk Zuid
System type	storm water	storm water
Catchment area	Flat	Flat
System structure	Branched	Branched
Surface level (m reference level)	-4.00 and -2.43	-4.34 and -3.06
Contributing Area (km^2)	0.15	0.09
Storage volume (mm / m^3)	1.73 / 253	0.83 / 76
Number of edges	118	92
Number of manholes	99	76
Outflow structure	3	4
Surface water level (m reference level)	-5.5	-5.5

Figure 8 The urban water system of Almere

Table 9 provides an overview of the runoff area per gully pot, determined using the Voronoi triangulation (Voronoï, 1908).

Table 9 The characteristics of the gully pots in Waterwijk Noord and Waterwijk Zuid

Characteristics	Waterwijk Noord	Waterwijk Zuid
Number gully pots	696	461
Average runoff surface per gully pot (m^2)	103	119
Maximum runoff surface per gully pot(m^2)	338	486

Applied Rainfall series

For the analysis of urban water system networks, perennial precipitation series were used to understand the consequences of interaction between the subsystems. The rainfall series, as registered by the Royal Dutch Meteorological Institute in De Bilt over the period 1955–1979 with a 15 minutes resolution, was used for the analysis of the functioning of the urban water system in Almere using the GBWLM.

The storm event in this rainfall series with the largest impact on surface water system in Almere is an event recorded between 11 October and 14 October 1960 (108.6 mm in 96 h) with a return period of ~50–100 years. This implies that, although the rainfall series was relatively short, it included a representative event with a return period close to the design return period ($T = 100$) of the surface water system.

The GBWLM was validated against the outcomes of a hydrodynamic surface water model and a hydrodynamic sewer model of Almere. For the validation of the surface water part of the GBWLM, the period 1955–1964 of the above-mentioned rainfall series was used. For the validation of the storm and combined sewer component, a storm event with increasing rainfall intensity from 1 mm/h to 30 mm/h in steps of 1 mm/h was used. The duration of each precipitation intensity was 12 hours to ensure (near) stationary conditions in the whole system. Between each of the two consecutive precipitation intensities, a dry period of 5 days was applied. This meant that the initial conditions at the different intensities were the same. For these 30 storm intensities, the results of the GBWLM were compared with the results from a hydrodynamic model.

3 Case studies results and their interpretation

Results of the case study of the GBWLM

Flood frequency and flood extent were used to analyse the impact of capacity reduction of the urban water system of Almere. The flood frequency and extent were determined for various capacity reductions of the three analysed subsystems: gully pots, storm sewer and surface water. The results are presented in bar charts (Figure 9 and Figure 10); the horizontal axis shows the applied capacity reduction (%), the vertical axis shows the resulting flood frequency (number per year), while the bar colour indicates the flood extent (flooded area as a percentage of total paved area).

The results for the surface water network in Figure 9 and Figure 10 present the increase of flooding compared to the reference situation in which all three subsystems have full capacity. The total flooding is therefore the sum of the flood in the storm sewer at 0% capacity reduction and the flood due to the surface water system.

Gully pots

The function levels of the gully pots in Waterwijk Zuid and Noord at declining capacity were similar. However, both the flood frequency and flood extent were larger in Waterwijk Zuid. This corresponds with the (approximately 15%) larger average runoff area per gully pot in Waterwijk Zuid (see

Table 9). If full capacity was available, the flood frequencies of the gully pots were 0–2 times per year.

The flood frequency and flood extent increased with a non-linear relationship with decreasing capacity. When the capacity reduction was limited to maximum 40%, the flood frequency was

relatively limited (5 or 6 times per year) and the flood extent was limited to 5% of the area for most events. This implies that the overcapacity for most gullies was approximately 30%. This can be considered a safety factor in the design.

Surface water
A reduction of the discharge capacity of the surface water system had a slightly greater impact on the frequency and extent of flooding in Waterwijk Noord than in Waterwijk Zuid. If full capacity was available, the flood frequency of the surface water was less than 1 time per year. The impact of a capacity reduction of the surface water was relatively limited when the capacity reduction was limited to 70%. When the capacity reduction was larger, both the flood frequency and flooded area increased strongly.

It is noticeable that in case of a capacity reduction of 90%, the number of times that more than 75% of the surface was flooded was approximately 85 times per year in Waterwijk Noord and 70 times per year in Waterwijk Zuid. This is notable because when the capacity reduction in surface water was limited, there was less frequent flooding in Waterwijk Noord than in Waterwijk Zuid.
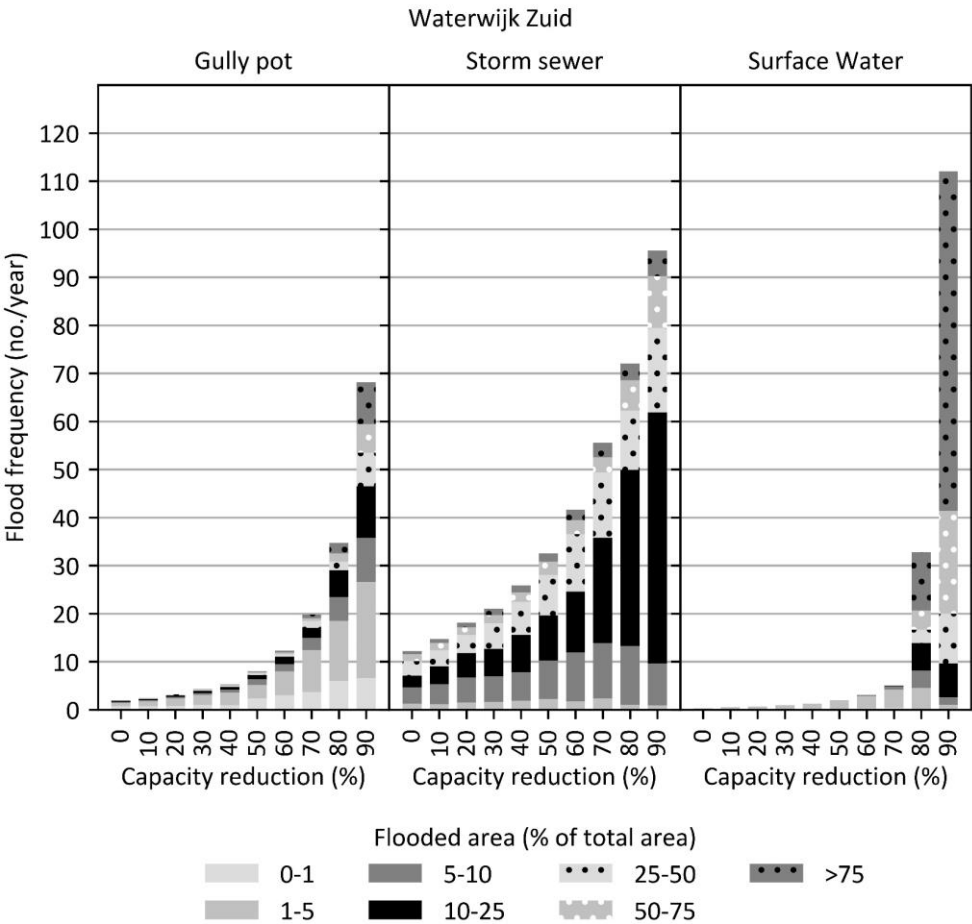

Figure 9   Overview of the yearly flood frequency for the various available capacities of gully pots, storm sewers and surface water system in Waterwijk Zuid. If full capacity is available, the flood frequency of the gully pots is 2 times per year the flood frequency of the storm sewer 12 times per year and the flood frequency of the surface water is less than 1 time/year.
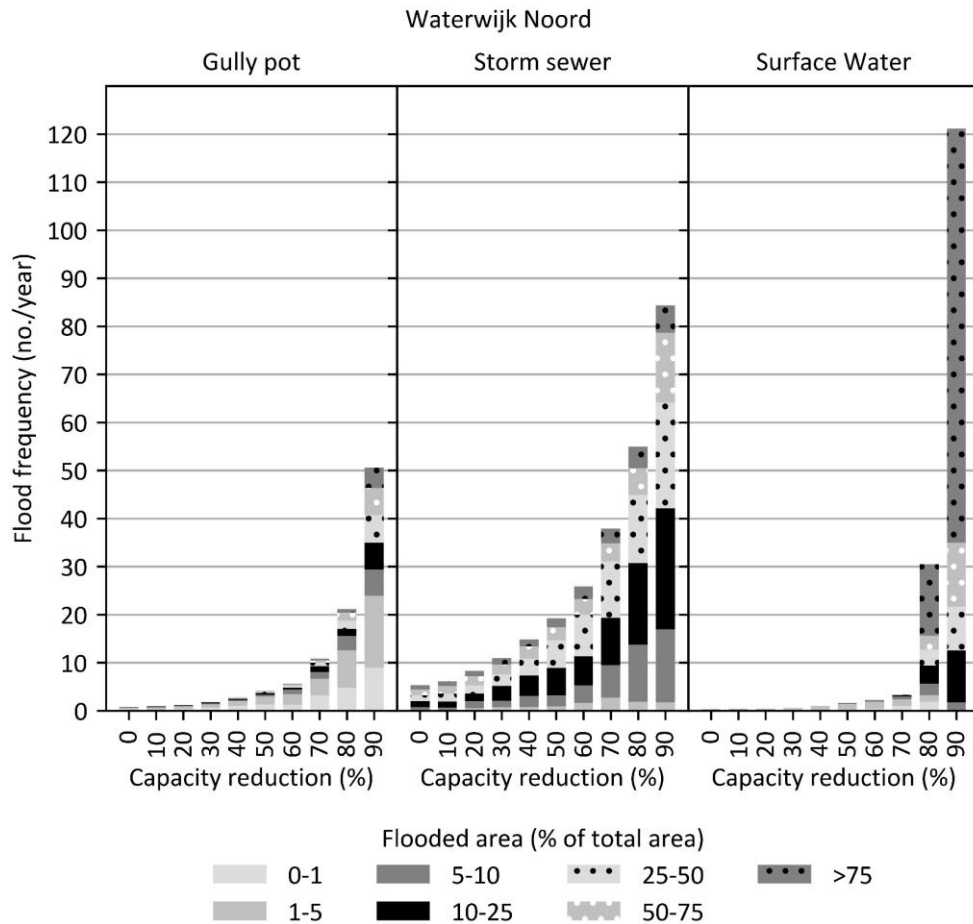
Figure 10 Overview of the flood frequency per year for the various available capacities of gully pots, storm sewers and surface water system in Waterwijk Noord. If full capacity is available, the flood frequency of the gully pots is less than 1 times per year, the flood frequency of the storm sewer 5 times per year and the flood frequency of the surface water is less than 1 time/year.

Storm water sewer
The differences in the performance of the two storm sewers were larger than the differences between the other subsystems in the two catchments. If the capacity reduction was set to the designed capacity, the flood frequency in Waterwijk Zuid was approximately 12 times per year and in Waterwijk Noord five times per year. The difference was mainly caused by a number of events with a small flood extent. The difference in flood frequency increased to approximately 10 times per year if the capacity declined.

The most striking difference was observed in the extent of flooding. If the capacity decreased, a clear increase could be seen in the frequency of events with a flood extent of 10–25% in Waterwijk Zuid. However, in Waterwijk Noord, the frequency of all flood extent categories increases. This means that a decrease in the capacity of the storm sewer in Waterwijk Zuid resulted in higher flood frequencies but with a limited extent. In Waterwijk Noord, both smaller and larger flood events occurred more frequently.

Validation of the GBWLM
The results of the GBWLM were validated against the Achilles approach. The validation was based on four criteria:

1. Flood volume at system level of the storm sewer.
2. Number of flooded manholes at system level.
3. Locations of flooded manholes.
4. Rise of the surface water level.

To validate the storm sewer system of the GBWLM, a storm event was used with a progressive rainfall intensity ranging from 1 mm/h to 30 mm/h. Between each of the two consecutive precipitation intensities, a dry period of 5 days was applied. Thus, the initial conditions at the different intensities were the same. The results are summarised in Table 10 (for more details see supplementary material). The table shows the minimum values of the 30 events.

The correlation between the flood volume of the Achilles approach and GBWLM for the storm sewer was significant ($R^2 = 0.97$–0.98 and $\tau_b = 0.95$–0.96, supplementary material Figure S.1). This implies a high degree of equality in the flood volume outcomes according to the Achilles approach and the GBWLM.

In the GBWLM, the flooding started at a slightly lower rainfall intensity (2 mm/h) when compared to the Achilles approach, while the GBWLM provide a satisfactory estimate for both the total number of flooded manholes and the locations where the flooding occurs (see Figure S.2 – Figure S.5, supplementary material). The scores for the number of flood locations and the locations of the flooded manholes were lowest at the transition between non-flooded and flooded regions. The indicator for the number of flooded manholes, the percentage correctly classified manholes (flooded, non-flooded) reached the lowest score of 72% correctly classified manholes. In this transition phase, the indicator for the locations of flooded manholes, the F1 score, may drop below 0.3. The F1 score is so low because only a few manholes were flooded in the transitional phase. If some of these manholes are not correctly classified, this results in a low F1 Score. If the rain intensity after the transition between non-flooded and flooded increased by 2 mm/h, the F1 score also increased rapidly.

The surface water system was validated with a 10-year rainfall event and the water level variation of the GBWLM were compared with the results of the Achilles approach. The results of the validation of the surface water part of the GBWLM showed a strong correlation with the results of the Achilles approach ($R^2 = 0.93$; see supplementary material, Figure S.6). This means that the GBWLM followed the pattern of surface water level rise in the Achilles approach.

Table 10   Overview of the validation results for the graph-based weakest link method (GBWLM) for a sewer system, based on a storm event with an increasing intensity of 1–30 mm/hr. The values of the flooded manholes and the F1 score are the minimum values of the 30 events.

		Waterwijk	Waterwijk Zuid	Waterwijk Noord
Flood volume	R^2	0.98	0.98	0.97
	τ_b	0.95	0.95	0.96
Flooded manholes	True positive/negative (minimum%)	84	69	82
	False positive (maximum%)	12	21	6
	False negative (maximum%)	6	14	18
F1 (overall)	F1	0.84	0.87	0.8
	Precision	0.78	0.84	0.7
	Recall	0.90	0.90	0.92
F1 (minimum value)	F1	0.21	0.24	0.11
	Precision	0.16	0.19	0.06
	Recall	0.33	0.33	0.84

Applicability of the GBWLM at increased rainfall intensities
As stated in Section 1, the ratio between the discharge and storage capacity of the subsystems can be affected due to either a change in load or capacity of the networks. In this subsection, the focus is on network capacity.

For a storm sewer system, a comparison was made between the freeboard (freeboard is the distance between ground level and water level) at a doubling of the precipitation intensity and at a halving of the discharge capacity. Three sets of calculations were performed:
1. The storm sewer system was tested with 2 stationary events with 4 mm/h and 8 mm/h intensities.
2. The intensity of the events was doubled and the storm sewer system was tested with 2 stationary events of 8 mm/h and 16 mm/h.
3. The discharge capacity of the storm sewer system was halved and tested with 2 stationary events of 4 mm/h and 8 mm/h.

Figure 11 shows that the storm sewer network reacted similarly to a capacity reduction of the pipe diameter and an increased storm intensity. Figure 11 shows that when the storm intensity increased by a factor of two (Figure 11, bottom-left) or the available capacity decreased by a factor two (Figure 11, bottom-right), the changes in freeboard were as expected and of the same order of magnitude (Figure 11, top-right).
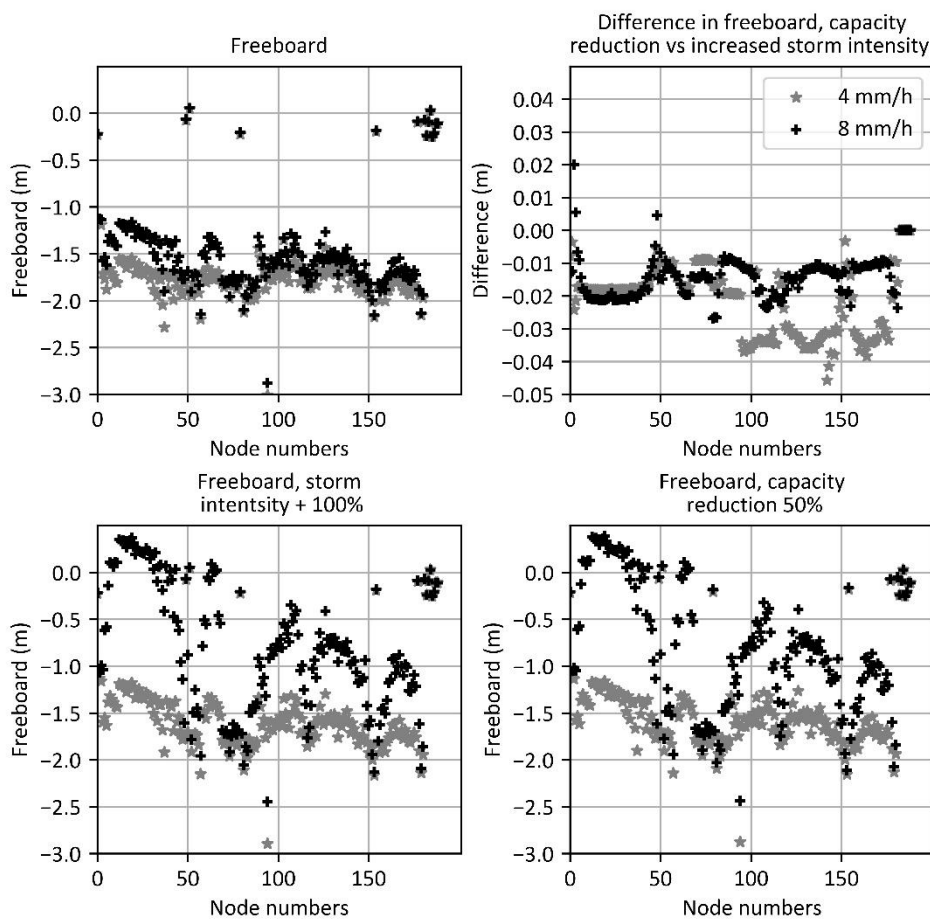

Figure 11   Comparison of the freeboard in the event of an increase of the storm intensity and a decrease of the available capacity for a storm event of 4 mm/h and 8 mm/h. Note: A different Y-scale has been used for the graph top right.

Due to climate change, the rainfall intensities of storm events with return periods of 0.5–1000 years and a duration of 10 min–12 hours are expected to increase by 17.1–21.3% (Beersma et al., 2019). Based on Figure 11, this is expected to correspond to a capacity reduction of approximately 20%. Figure 9 and Figure 10 show that such a capacity reduction had little impact on the flood frequency and extent caused by gully pots and surface water. However, it could lead to a 50% increase in the storm sewer flood frequency and an increase in the extent of flooding.

4 Discussion
The objective of the GBWLM is to assess the potential functional limitations due to degradation-induced capacity losses and changes in network load of different subsystems of an urban water system. The GBWLM can be used to balance between required data, effort and accuracy. In this section, these aspects of the GBWLM are discussed.

Required input data and effort
The GBWLM is more accessible than the Achilles approach because fewer data and less computational effort are needed. The computational effort required for the Achilles approach is approximately 30 times greater than that for the GBWLM.
Correct representation of the network structure and geometry is important in both approaches; however, in the GBWLM, hydrodynamic models are replaced by path analysis of digraphs. As a result, the GBWLM requires less information about manholes and structures when compared to the Achilles approach.
Table 11 shows the data not included in the GBWLM.

Table 11   Overview of the required data per sub system included in hydrodynamic models and excluded in the GBWLM.

Rainfall runoff models	Gully pots	Storm sewers	Surface water system
Runoff parameters	Inlet parameters	Street level manhole	Thalweg (centre line of the water system)
Differentiation of roof types	z-coordinate	Dimensions manholes	Cross sections
Differentiation of paved area types		Weir characteristics (except height)	Weir characteristics (except capacity)
Differentiation of unpaved area types		Pump characteristics (except capacity)	Pump characteristics (except height)

Collecting data regarding structures normally may require a significant effort, particularly when the sewer must be accessed to obtain these data. Information about manholes can be collected relatively easily and is often combined with sourcing characteristics that are necessary for both the Achilles approach and the GBWLM.

Spatial time resolution of the GBWLM
The purpose of the GBWLM is to identify potential functional limitations due to degradation-induced capacity losses and changes in network load. The GBWLM should be able to assess the functioning of the system under different conditions. For these conditions, the results should show mutually significant differences.

The τ_b and F1 score (see Table 10) have indicated a relationship between the outcomes of the Achilles approach and the GBWLM. However, these tests have also hinted at some differences caused by the simplification of hydrodynamics in the GBWLM.

The application of linearised hydrodynamics in the GBWLM results in over- and underestimations of the discharge capacity of elements (see Figure S.2.). In the GBWLM, the capacity of a conduit is estimated based on the geometry of the conduit and the hydraulic gradient. Simplifications are made by assuming that hydraulic gradients are the same for all conduits in one path (see Section 2).

With the GBWLM, the flood locations could be determined reasonably well. For most precipitation intensities, the F1 was greater than 0.6 (see **Error! Reference source not found.**). However, at the transitions from non-flooded to flooded regions, the number of flooded locations was overestimated. The size of the flooding extent was sometimes overestimated and sometimes underestimated, but for Almere the locations matched up well (see **Error! Reference source not found.** and **Error! Reference source not found.**).
In case of sewer systems with a large storage capacity, the storage could be included in the storm or combined sewer graph, in a similar manner to that applied to the surface water component of the GBWLM. For the tested storm sewers of Almere, storage was not included in

schematisation because the systems are directly connected (in open access) to the surface water and the majority of the pipes are always surcharged.

The degree of similarity between Achilles approach and GBWLM
The results of the GBWLM can be considered to be representative for the Achilles approach. The comparison (see Section 3 and Supplementary material) demonstrated a relationship in the development of the flood volume at different precipitation intensities between the Achilles approach and the GBWLM (storm sewer R^2 > 0.9 and τ_b > 0.9; see Table 10). A relationship between the Achilles approach and the GBWLM was also observed in the calculated rise in surface water level due to storms (surface water R^2 > 0.9; see Appendix **Error! Reference source not found.**). The minimum overall F1 score was 0.69. This means that although the GBWLM results did not fully match those of the Achilles approach, the overall impression was similar in terms of both the number of flooding locations and the locations themselves (see Table 10 and supplementary material).

The largest differences occurred at the transitions from non-flooded to flooded regions. They occurred at the beginning of a flood event and at the edges of the flooded area. Especially at the beginning of a flood event, when the number of flooded manholes in both the Achilles approach and the GBWLM was low, relatively small deviations could result in a low F1 score.

An integrated analysis with the GBWLM
The interaction between subsystems is included in the GBWLM. The backwater effects of the surface water system on a storm sewer are included in the GBWLM by reducing the capacity of the storm sewers based on the occurring surface water levels. The GBWLM allows for an analysis of the overall functioning of an urban water system using a multiyear rainfall series. A multiyear analysis is necessary because event selection in advance is difficult due to the differences in storage and drainage characteristics of subsystems. The combination of storage and discharge capacity determines the time it takes for the system to return to its initial status after a storm event.

The drainage capacity of a gully pot connection becames critical at approximately the same time as flooding occurs in the sewer system. In the GBWLM, this was labelled as storm sewer flooding. Gully pots and storm sewers are connected through pipes that are usually relatively short (i.e. length < 10 m, diameter 125 mm). A difference in water level of 1 cm results in a gradient of 0.1% and a discharge capacity of the connection pipe between gully pot and sewer of approximately 2.5 l/s (neglecting inflow and outflow losses). The average runoff surface per gully pot in Almere is < 120 m^2. Therefore, even when the water level in the storm sewer is close to ground level, the discharge capacity of a connection is still 215 l/(s.ha) and is therefore not a limiting factor.

Application options
Section 3 shows that the results of the GBWLM and Achilles approach were consistent in this study. The added value of the GBWLM compared to the Achilles approach is that:
- The GBWLM allows the analysis of urban water systems with multiyear rainfall series.
- The GBWLM allows for a combined analysis of subsystems of urban water subsystems.
- The GBWLM facilitates sensitivity analysis of urban water systems due to ageing or climate change.
- The GBWLM permits comparing flood frequency and flood extent caused by capacity reduction of urban water subsystems.

5 Conclusions
To compare the overall performance of urban water systems in terms of flood frequency and flood extent with relatively limited resources, the GBWLM was used to determine the extent and frequency of urban flooding. The GBWLM is an integrated method to analyse the robustness of water networks consisting of gully pots, storm sewers and surface water networks with full (multiple decade) rainfall series. This method was used to analyse urban water systems at different levels of detail, ranging from gully pots to complete surface water systems. The subsystems could be analysed separately, but also in conjunction, including the corresponding interactions between the subsystems using multi-decade precipitation series.

The GBWLM differs from the Achilles approach in that the former relies on network structure and flow path analysis, while the latter requires the application of full hydrodynamics. In comparison, the advantages of the GBWLM approach can be summarised as follows:

- A full and integrated analysis of urban water systems (gully pots, storm sewers and surface water).
- An analysis with full (multiyear) rainfall series consisting of multiple events.
- Less required computational effort, resulting in a significant reduction of simulation run times. At the price of a simplification of the representation of the hydrodynamics (in GBWLM this is lineairised).
- Determination of the effects of both reduced (sub)system capacity and increased rainfall intensities.

The parameters "flood extent" and "flood frequency" were used to assess the system's performance. Both are indicators for the severity of floods and can be used as a basis to determine the consequences in terms of impact such as damage or health risk.
The outcomes of the analysis can be used to prioritise maintenance activities and system rehabilitation. For the system of Almere, it was clear that the storm sewer system is the weakest link and most prone to capacity reduction. Due to climate change, the flood frequency may increase by approximately 50% in 2050.

The case study also shows that the GBWLM is applicable for systems where the functioning is determined by both discharge and storage capacity. This means that many different types of systems could be analysed with the GBWLM. Further research is needed to determine whether the GBWLM is also applicable on larger networks with other parameters, such as concentration times. The latter allows an analysis of large river systems with the GBWLM.

References

Beersma, J., Hakvoort, H., Jilderda, R., Overeem, A., & Versteeg, R. (2019). Neerslagstatistiek en -reeksen voor het waterbeheer 2019 [Rainfall statistics and series for water management 2019] (STOWA Report No. 2019-19). Stichting Toegepast Onderzoek Waterbeheer. https://www.stowa.nl/sites/default/files/assets/PUBLICATIES/Publicaties%202019/STOWA%202019-19%20neerslagstatistieken.pdf

Broadbent, S. R., & Hammersley, J. M. (1957). Percolation processes: I. Crystals and mazes. Mathematical Proceedings of the Cambridge Philosophical Society, 53(3), 629–641. https://doi.org/10.1017/S0305004100032680

Chinchor, N. (1992). MUC-4 evaluation metrics. In MUC4 '92: Proceedings of the 4th Conference on Message Understanding (pp. 22–29). Association for Computational Linguistics. https://doi.org/10.3115/1072064.1072067

Dijkstra, E. W. (1959). A note on two problems in connexion with graphs. Numerische Mathematik, 1(1), 269–271. https://doi.org/10.1007/BF01386390

Fischer, A., Rouault, P., Kroll, S., Van Assel, J., & Pawlowsky-Reusing, E. (2009, December). Possibilities of sewer model simplifications. Urban Water Journal, 6(6), 457–470. https://doi.org/10.1080/15730620903038453

Geels, F. W. (2006). The hygienic transition from cesspools to sewer systems (1840–1930): The dynamics of regime transformation. Research Policy, 35(7), 1069–1082. https://doi.org/10.1016/j.respol.2006.06.001

Hagberg, A. A., Schult, D. A., & Swart, P. J. (2008). Exploring network structure, dynamics, and function using NetworkX. In G. Varoquaux, T. Vaught, & J. Millman (Eds.), Proceedings of the 7th Python in Science Conference (SciPy2008) (pp. 11–15).

Kendall, M. G. (1945). The treatment of ties in ranking problems. Biometrika, 33(3), 239–251. https://doi.org/10.2307/2332303

Langeveld, J., & Schilperoort, R. (2019). Stedelijk Water 2040: hemelwater verbindt! [Urban water 2040: Rainwater connects!] [White paper]. Partners4UrbanWater. https://www.urbanwater.nl/Files/190604_Whitepaper_hemelwater_verbindt.pdf

Mair, M., Sitzenfrei, R., Kleidorfer, M., Möderl, M., & Rauch, W. (2012). GIS-based applications of sensitivity analysis for sewer models. Water Science & Technology, 65(7), 1215–1222. https://doi.org/10.2166/wst.2012.954

Meijer, D., Post, J., Van der Hoek, J. P., Korving, H., Langeveld, J., & Clemens, F. (2020). Identifying critical elements in drinking water distribution networks using graph theory. Structure and Infrastructure Engineering, 17(3), 347–360. https://doi.org/10.1080/15732479.2020.1751664

Meijer, D., Van Bijnen, M., Langeveld, J., Korving, H., Post, J., & Clemens, F. (2018). Identifying critical elements in sewer networks using Graph-Theory. Water, 10(2), 136. https://doi.org/10.3390/w10020136

Möderl, M., Kleidorfer, M., Sitzenfrei, R., & Rauch, W. (2009). Identifying weak points of urban drainage systems by means of VulNetUD. Water Science & Technology, 60(10), 2507–2513. https://doi.org/10.2166/wst.2009.664

NetworkX Developers. (n.d.). min_cost_flow. NetworkX. https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.flow.min_cost_flow.html

Preston, S. H., & van de Walle, E. (1978). Urban French mortality in the nineteenth century. Population Studies, 32(2), 275–297. https://doi.org/10.2307/2173562

Reyes-Silva, J. D., Helm, B., & Krebs, P. (2020). Meshness of sewer networks and its implications for flooding occurrence. Water Science & Technology, 81(1), 40–51. https://doi.org/10.2166/wst.2020.070

Sahimi, M. (1994). Applications of percolation theory. Taylor & Francis.

Stauffer, D., & Aharony, A. (1991). Introduction to percolation theory (Rev. 2nd ed.). Taylor & Francis. https://doi.org/10.1201/9781315274386

Stockmayer, W. H. (1944). Theory of molecular size distribution and gel formation in branched polymers: II. General cross linking. The Journal of Chemical Physics, 12(4), 125–131. https://doi.org/10.1063/1.1723922

Tscheikner-Gratl, F., Bellos, V., Schellart, A., Moreno-Rodenas, A., Muthusamy, M., Langeveld, J., Clemens, F., Benedetti, L., Rico-Ramirez, M. A., Fernandes de Carvalho, R., Breuer, L., Shucksmith, J., Heuvelink, G. B. M., & Tait, S. (2019). Recent insights on uncertainties present in integrated catchment water quality modelling. Water Research, 150, 368–379. https://doi.org/10.1016/j.watres.2018.11.079

Vaes, G., Feyaerts, T., & Swartenbroekx, P. (2009). Influence and modelling of urban runoff on the peak flows in rivers. Water Science & Technology, 60(7), 1919–1927. https://doi.org/10.2166/wst.2009.638

van Riel, W. (2016). On decision-making for sewer replacement [Doctoral thesis, Delft University of Technology]. TUDelft. https://doi.org/10.4233/uuid:92b10448-795d-43ac-8071-d779af9d374d

Voronoï, G. (1908). Nouvelles applications des paramètres continus à la théorie des formes quadratiques : Deuxième mémoire : Recherches sur les parallélloèdres primitifs [New applications of continuous parameters to the theory of quadratic forms: Second dissertation: Research on primitive paralleloids]. Journal für die reine und angewandte Mathematik, 1908(134), 198–287. https://doi.org/10.1515/crll.1908.134.198

Yang, Q., Dai, Q., Han, D., Zhu, X., & Zhang, S. (2018, May 16). Impact of the Storm Sewer Network Complexity on Flood Simulations According to the Stroke Scaling Method. Water, 10(5), 645. https://doi.org/10.3390/w10050645