

# A Conceptual Model for Tropical Convective Organization

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## Abstract

Tropical convective organization and associated clusters of precipitation affect the global circulation and Earth’s energy budget, but many aspects such as the power-law distribution of precipitation clusters and the relation to convective self-aggregation remain poorly understood. Here, we present a physics-informed 2D conceptual model for tropical convective organization. The model is based on the budget equation of column moist static energy (CMSE) with terms that are parameterized based on diagnosing them in a high-resolution simulation with explicit convection. The conceptual model combines a reaction-diffusion equation, where the reaction term has memory, with a temporal red noise. We find that vertical advection has a strengthening effect on CMSE perturbations, whereas horizontal advection has a mitigating effect on CMSE perturbations. The CMSE tendencies from vertical and horizontal advection terms are larger in magnitude than those from radiation and surface fluxes. The strengthening effect of vertical advection corresponds to a negative gross moist stability (GMS) at short spatiotemporal scales, but the GMS becomes positive when the convection changes from shallow to deep, killing off the growth in CMSE. The conceptual model is faithful in simulating self-aggregation in that it shows a domain-size dependence of aggregation found in many prior works, and its CMSE power spectrum matches that of the high-resolution simulation except for a shallowing of the slope at high wavenumbers. Through analyzing the conceptual model equation, we show that 1) the domain-size dependence of self-aggregation is due to a competition between the strengthening effect of vertical advection and the smoothing effect of horizontal advection, and 2) the combination of temporal red noise and diffusive horizontal advection sets the shape of the CMSE power spectrum. Furthermore, the conceptual model also reproduces power-law distributions of precipitation clusters when precipitation clusters are viewed as thresholded islands on the CMSE topography. Thus, the simple conceptual model captures and helps to explain important aspects of convective self-aggregation and tropical convective organization.



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Credit: Reid Wiseman

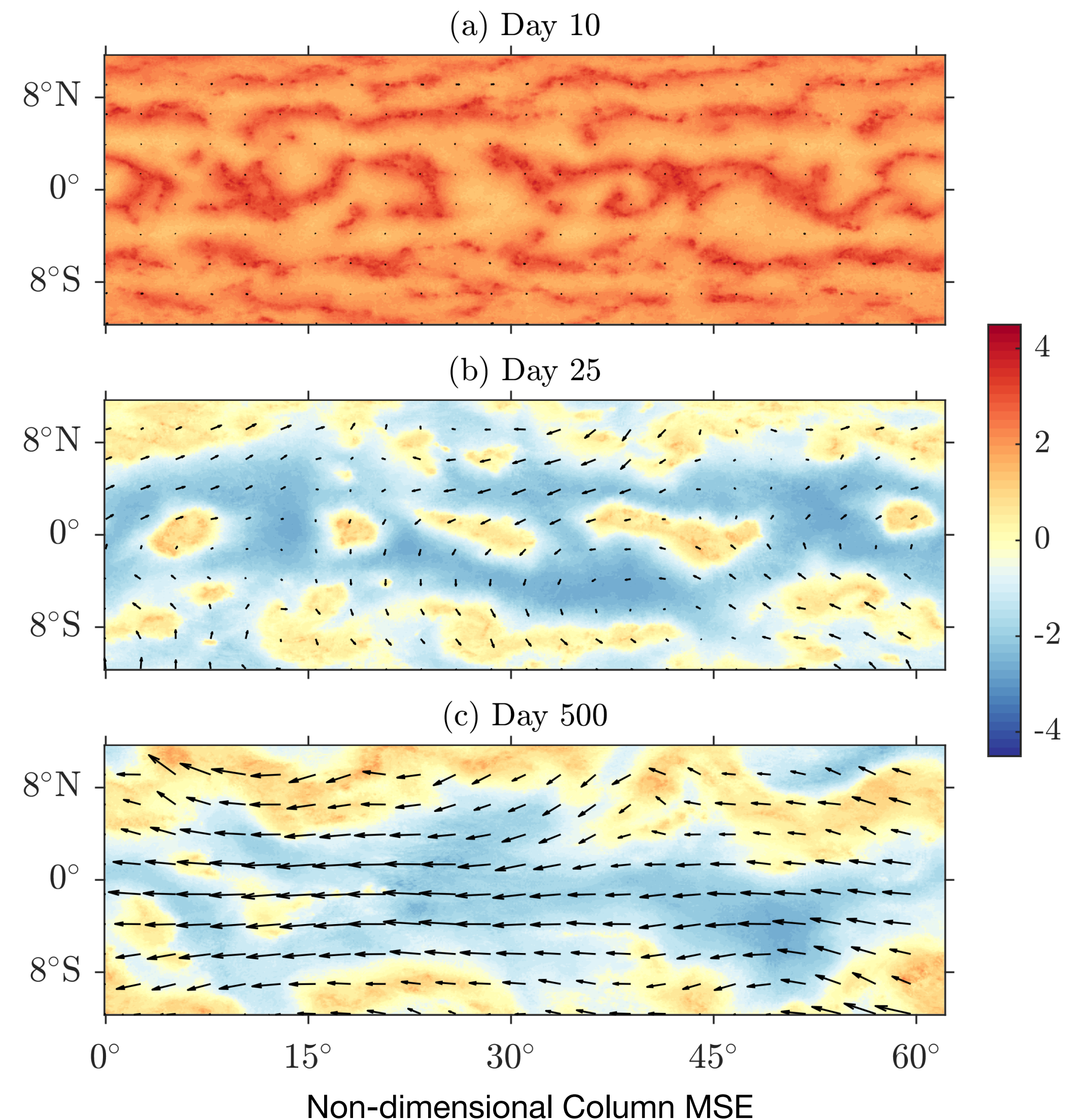


# Motivation: bridge self-aggregation to the real tropical atmosphere

- Convection self-aggregates due to radiative-convective instability (Emanuel et al., 2013)
- Diabatic heating, horizontal MSE convergence contribute to aggregation (Bretherton et al., 2005; Wing and Emanuel, 2013)
- Clusters “coarsen” in length-scale (Craig and Mack, 2013)

However, much is unknown about self-aggregation and its relation to statistical equilibrium

- Why is there a minimum domain size ( $\sim 300$  km) for aggregation? (Muller and Held, 2012, Jeevanjee and Romps, 2013)
- Connection to the observed power-law distribution of precipitation clusters? (Quinn and Neelin, 2017, Li et al., 2021)





# Method: design a conceptual model based on a high-resolution simulation with convection

- Diagnose a high-resolution simulation with SAM  
(as used in O’Gorman et al., 2020, Yuval et al., 2021)
- Design a 2D conceptual model based on the diagnosis

$$\frac{\partial}{\partial t} \text{MSE} = \text{Reaction} + \text{Diffusion} + \text{Stochasticity}$$

horizontal advection

vertical advection,  
diabatic heating

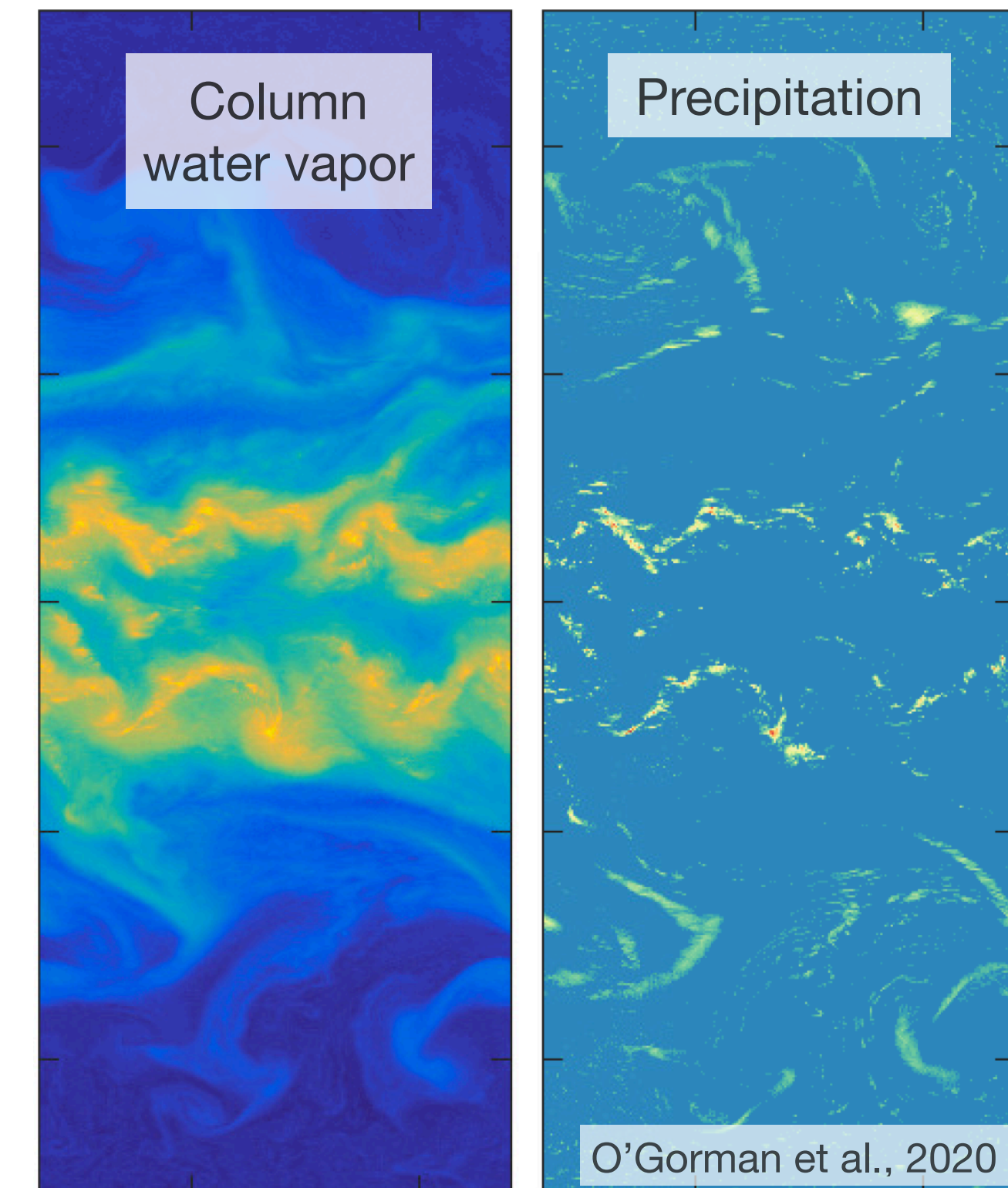
turbulence,  
gravity waves

- From conceptual model, we derive analytical results for:

Minimum domain-size for aggregation;

Column-MSE spectrum;

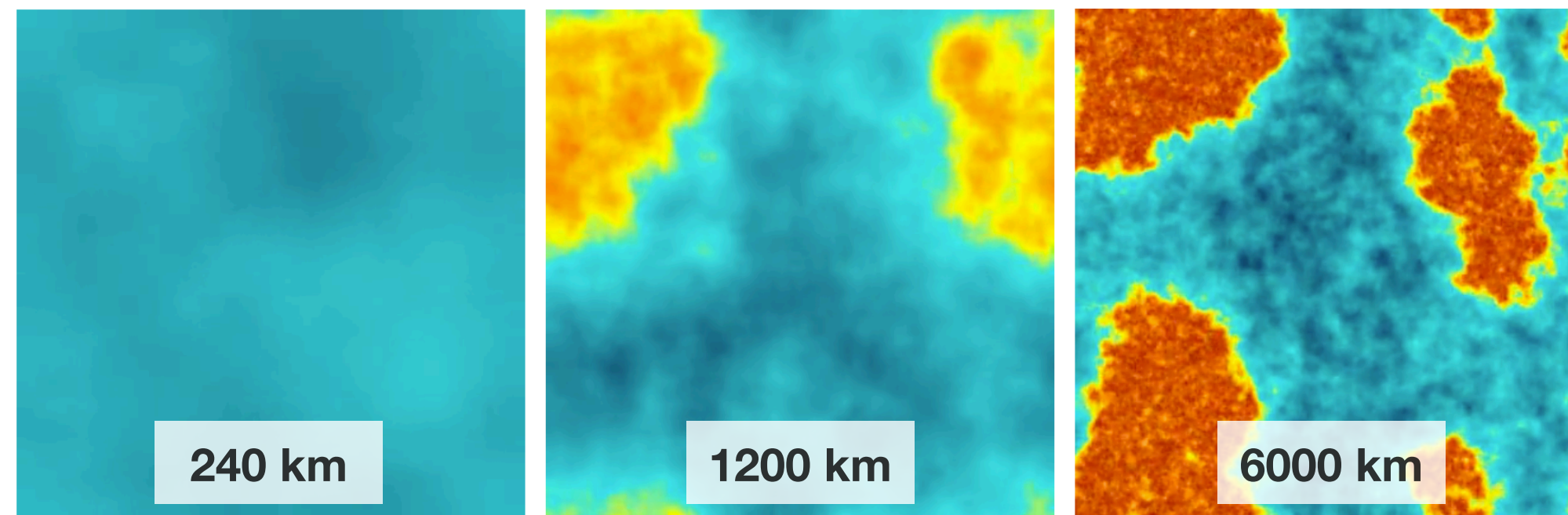
Growth exponent of column-MSE variance



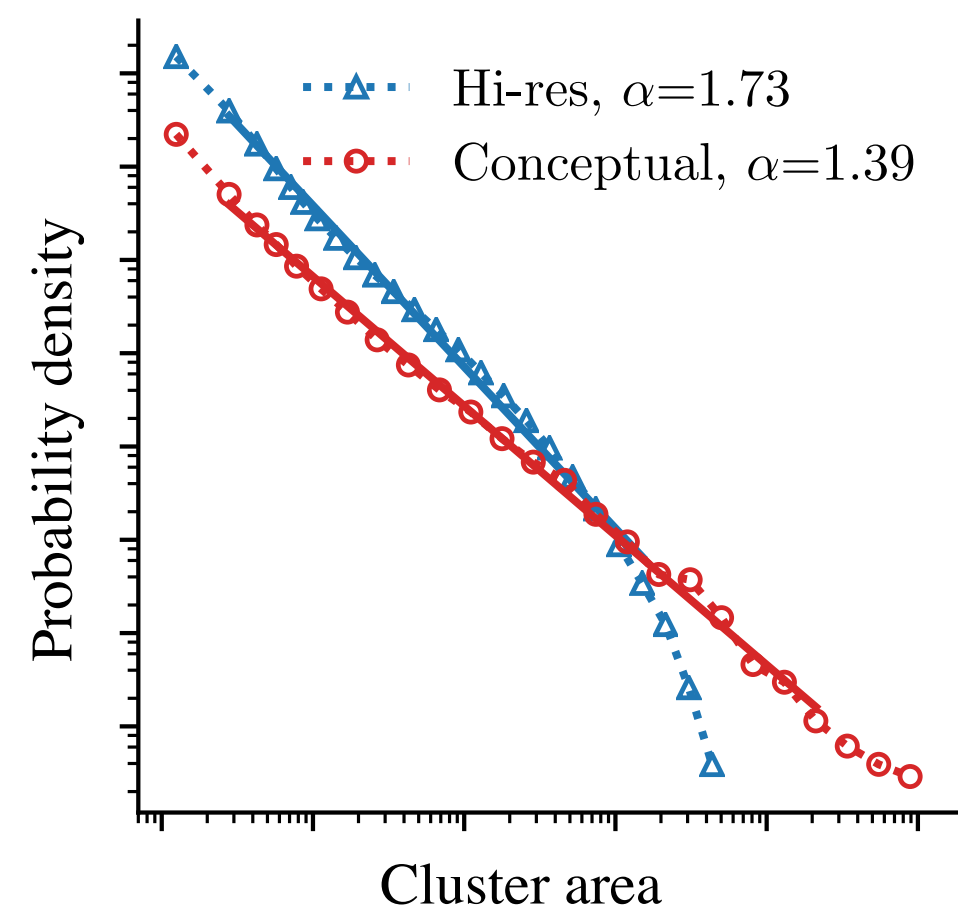


# **Result:** the conceptual model captures many characteristics of tropical convection and provides theoretical insights

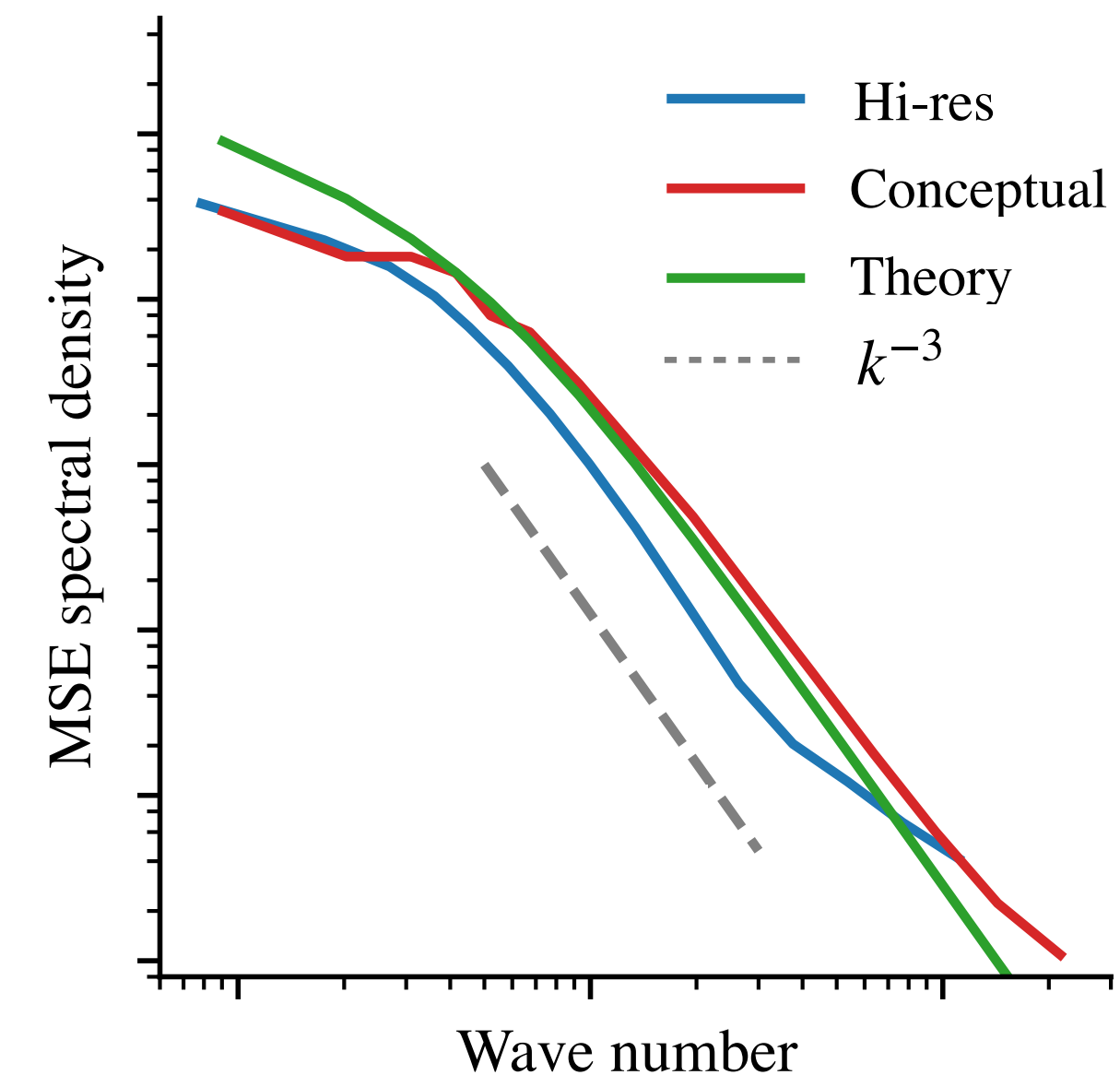
1. Competition between horizontal and vertical advection gives a minimum domain size for self-aggregation



2. Random noise gives power-law precipitation cluster distributions when viewed as islands on an MSE topography  
(Similar to Li et al., 2021)



3. Red noise and diffusive term give a -3 column-MSE power spectrum (consistent with Wing and Cronin, 2015)



Li, Z. et al.: A conceptual model for tropical convective organization, *in prep.*



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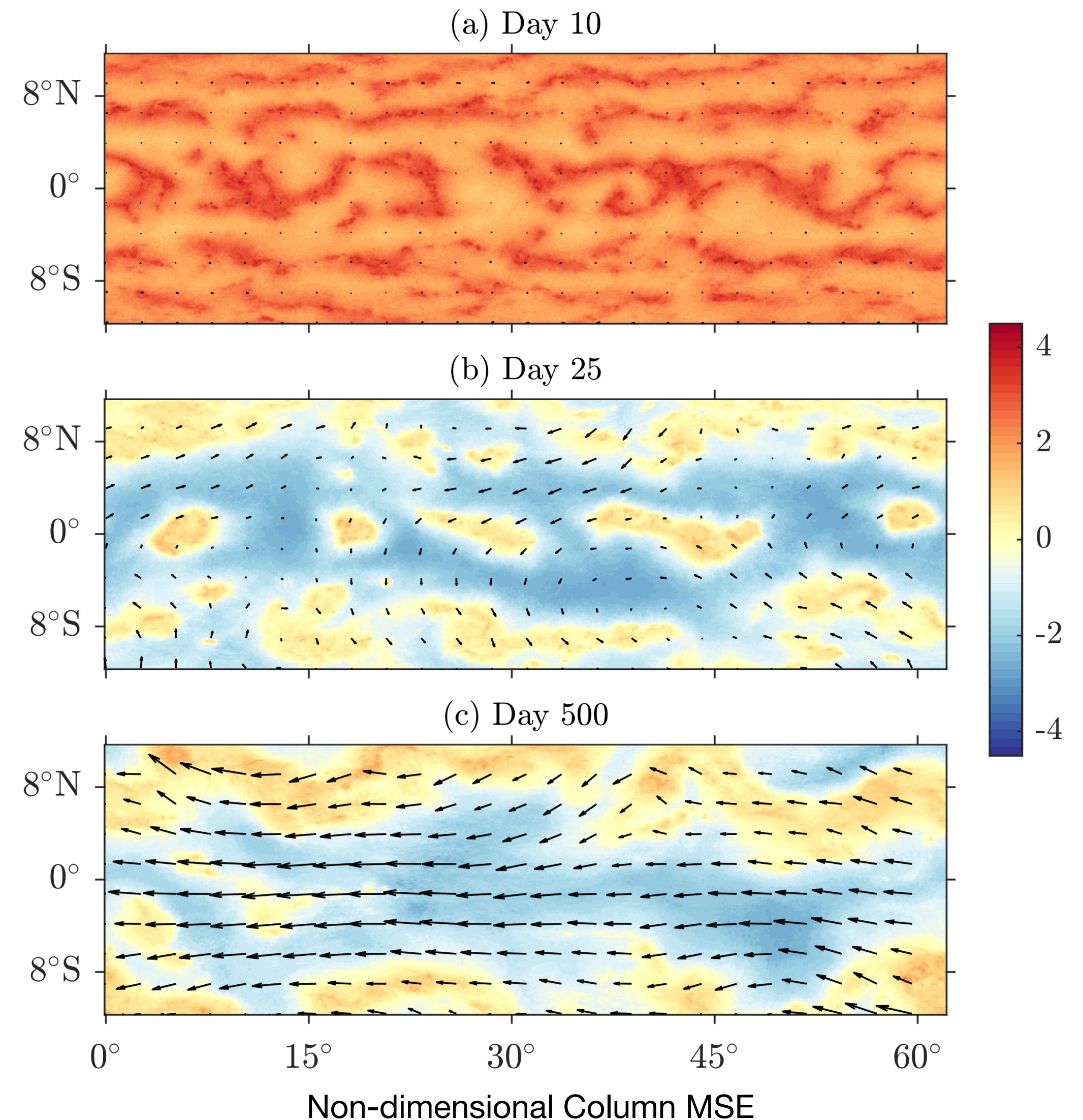


# First-order paradigm of tropical convective organization: self-aggregation

- Convection aggregates without large scale influence due to radiative-convective instability (Emanuel et al., 2013)
- “Coarsening” in length scale (Craig and Mack, 2013)
- Diabatic heating and horizontal convergence contribute to aggregation (Bretherton et al., 2005; Wing and Emanuel, 2013)

However, much is unknown about self-aggregation

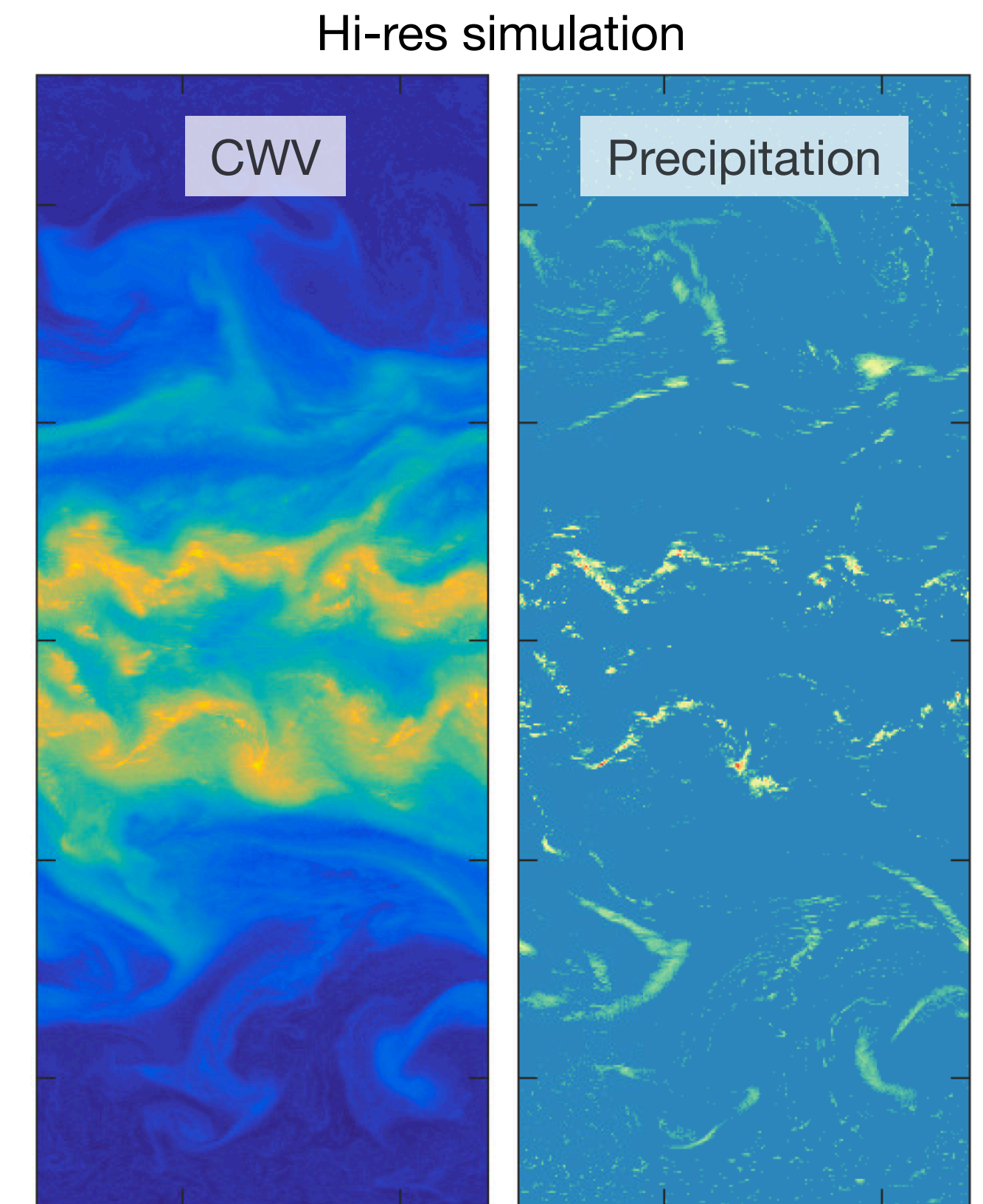
- Minimum domain size ( $\sim 300$  km) for aggregation (Muller and Held, 2012, Jeevanjee and Romps, 2013)
- Connection to the power-law distributions in precipitation clusters in real atmosphere (Quinn and Neelin, 2017, Li et al., 2021)





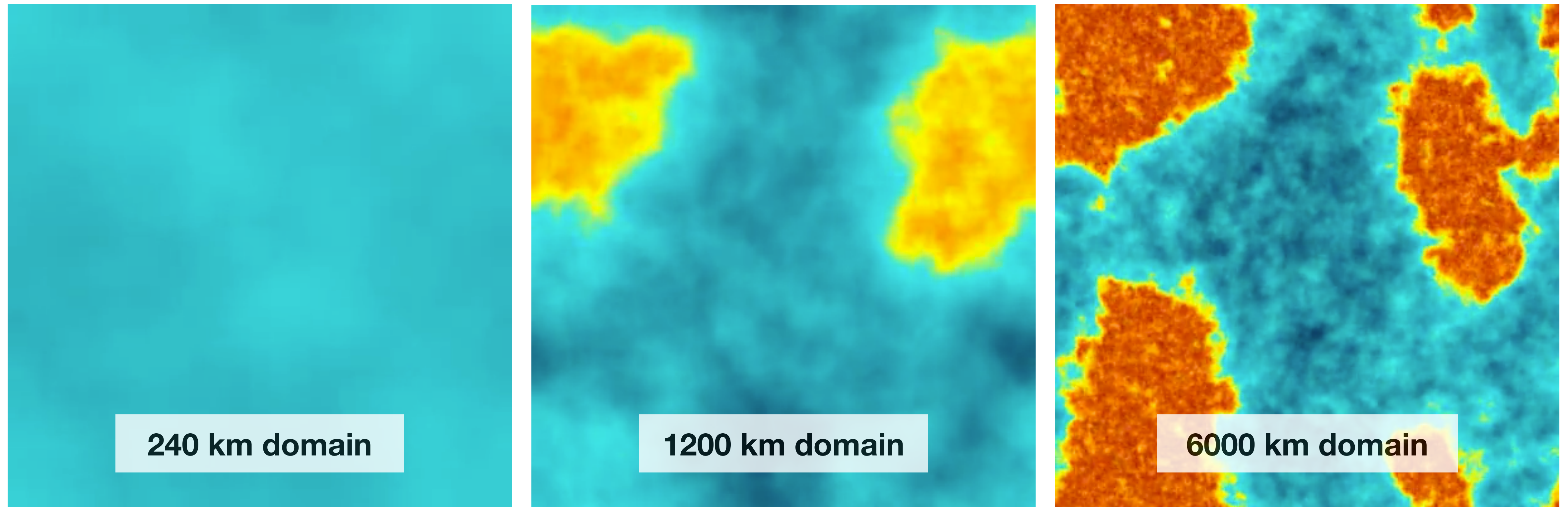
# Research highlights

- Diagnose a semi-global high-resolution (hi-res) simulation with SAM and clarify the roles of advection and diabatic terms (hi-res as in O’Gorman et al., 2020)
- Based on hi-res, propose a 2D conceptual model which displays
  - Domain-size dependence of aggregation,
  - Power-law distributions of precipitation clusters,
  - A power spectrum similar to hi-res.
- Provide analytical insights for the domain-size dependence, power spectrum, and growth exponent

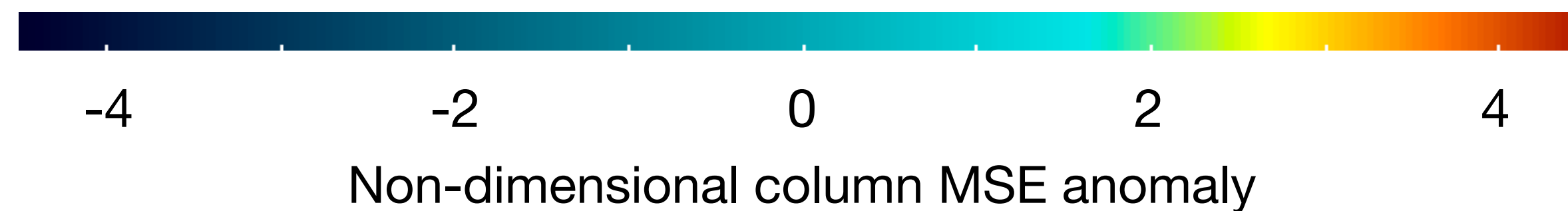




# Conceptual model reproduces the domain-size dependence of aggregation



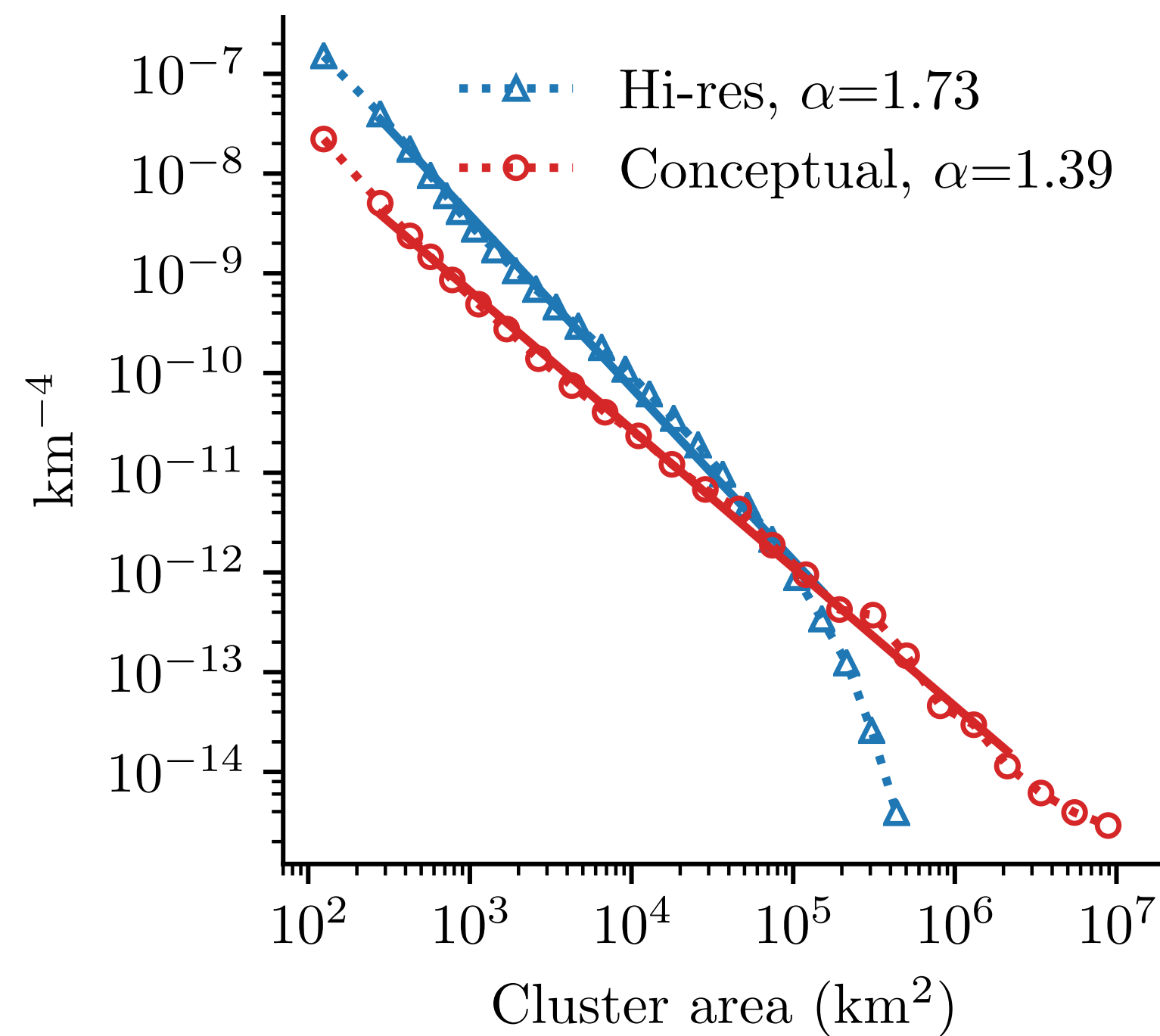
(These simulations are run with half noise to show self-aggregation)



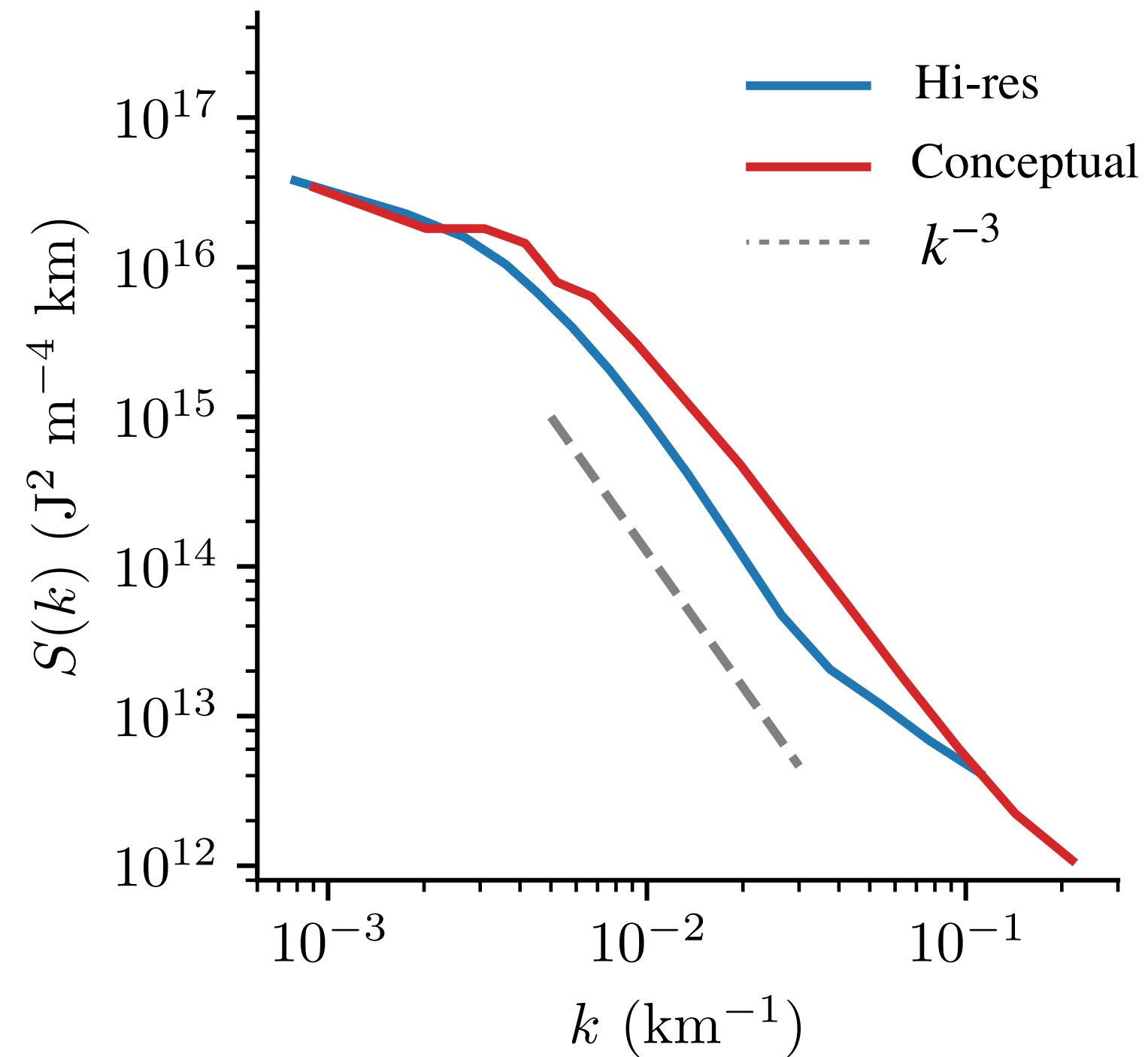


# Conceptual model reproduces similar precipitation cluster distributions, MSE spectrum, and growth exponent

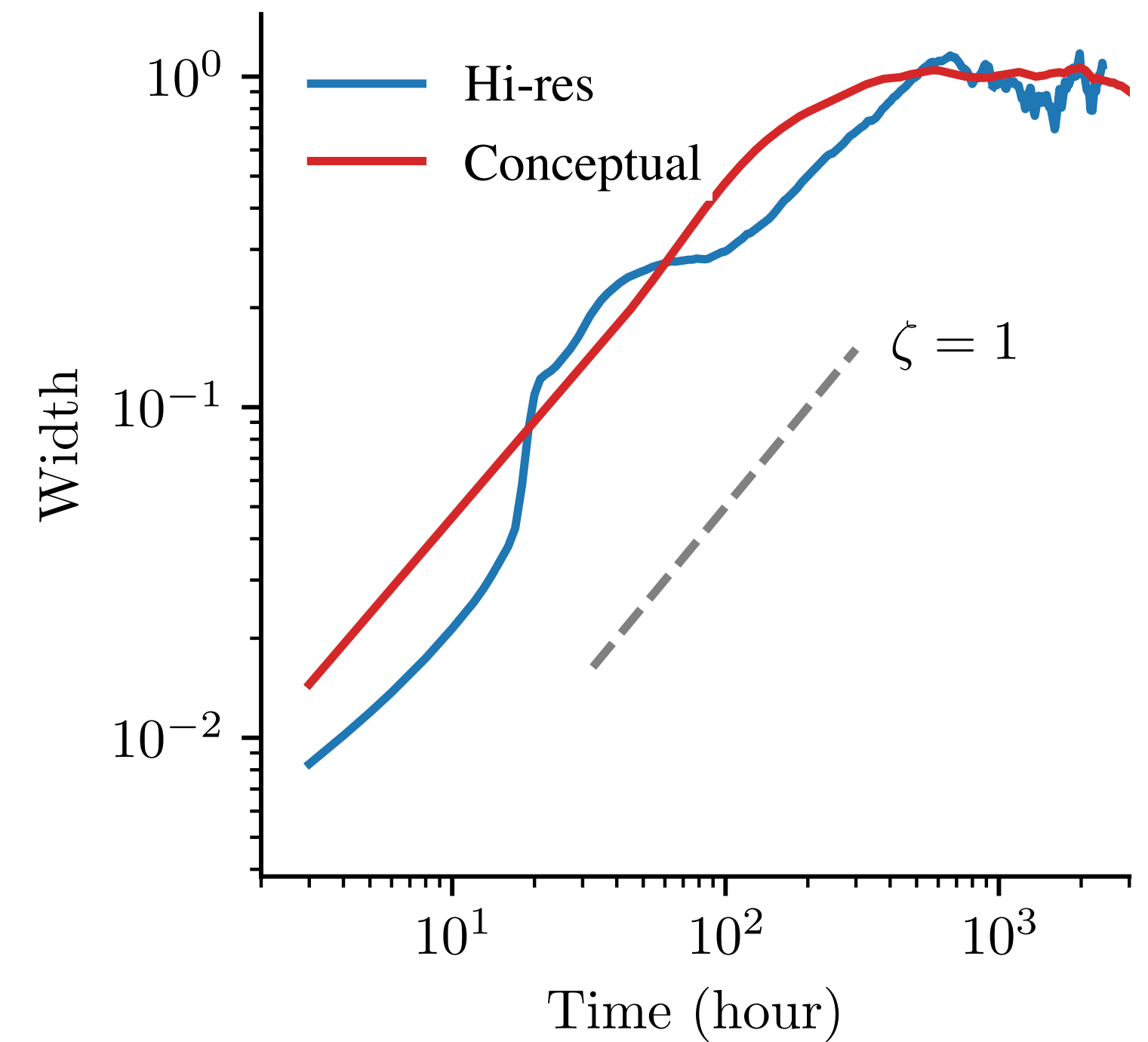
1. Power-law precipitation cluster distributions (Li et al., 2021)



2. Similar -3 MSE spectrum (Wing and Cronin, 2015)



3. Similar *growth exponent*,  $\zeta$ , with column MSE





# Building the conceptual model: the column MSE budget

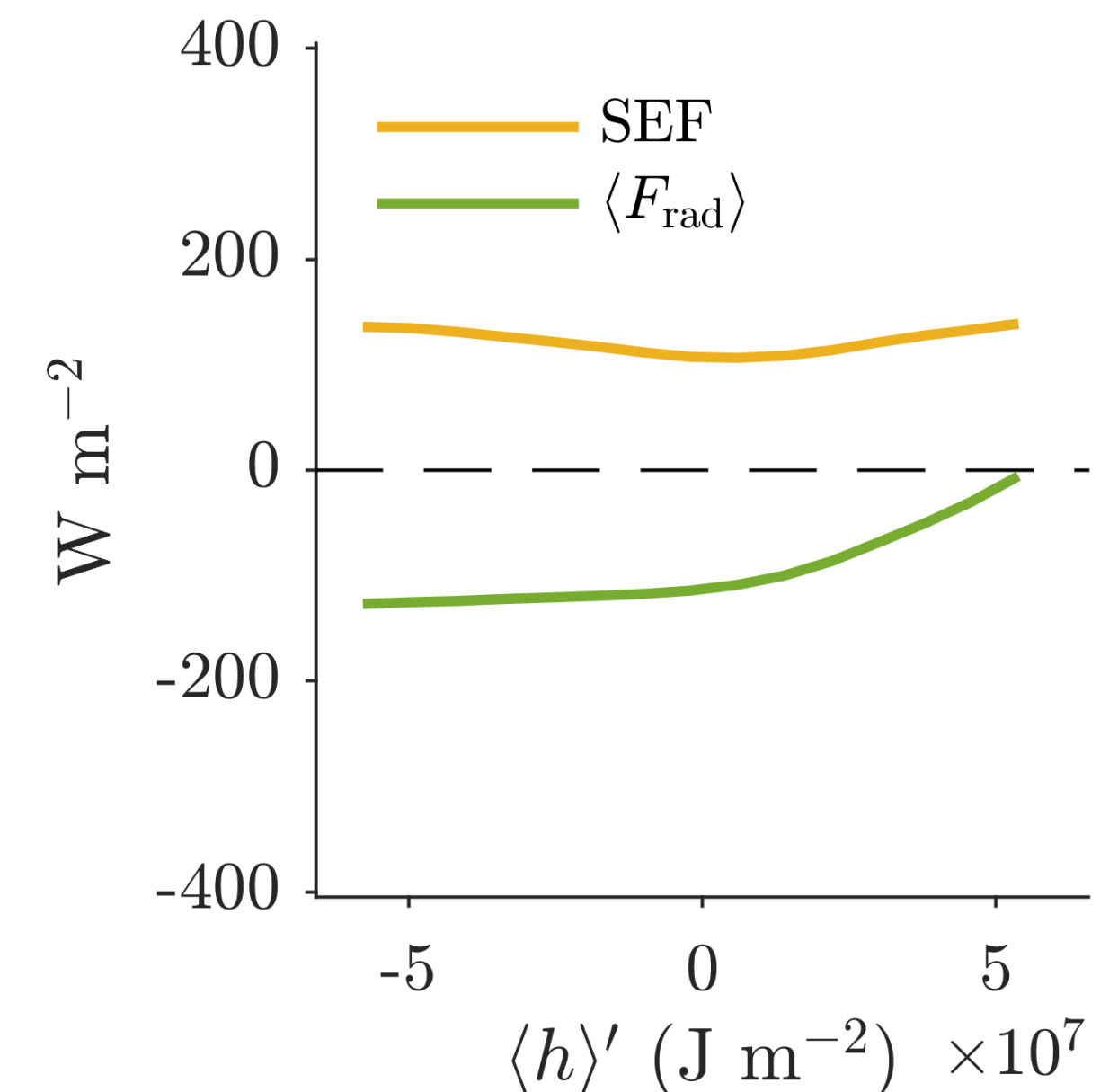
$$\frac{\partial \langle h \rangle}{\partial t} = \underbrace{-\langle \mathbf{u}_h^{\text{res}} \cdot \nabla_h h \rangle'}_{\text{Horizontal advection}} \underbrace{-\langle w \frac{\partial h}{\partial z} \rangle'}_{\text{Vertical advection}} + \underbrace{\langle F \rangle_{\text{rad}}}_{\text{Radiative heating}} + \underbrace{SEF}_{\text{Surface enthalpy flux}}$$

$h = c_p T + gz + L_v q$   
 $\langle \cdot \rangle$  is column integration

- Design the 2D conceptual model based on diagnosis from hi-res

$$\frac{\partial \tilde{h}}{\partial t} = \underbrace{D \nabla^2 \tilde{h}}_{\text{Horizontal advection}} \underbrace{-\Gamma M(\tilde{h}')}_{\text{Vertical advection}} + \underbrace{\langle F \rangle_{\text{rad}}}_{\text{Radiative heating}} + \underbrace{SEF}_{\text{Surface enthalpy flux}} + \underbrace{\xi}_{\text{Temporal red noise}} \quad (\tilde{h}: \text{non-dimensional CMSE})$$

- Use column-tracking to add memory to gross moist stability  $\Gamma$  and radiative heating  $\langle F \rangle_{\text{rad}}$





# Diffusion and reaction terms set minimum domain scale for self-aggregation

$$\frac{\partial \tilde{h}}{\partial t} = D \nabla^2 \tilde{h} \boxed{-\Gamma M(\tilde{h}') + \langle F \rangle_{\text{rad}} + SEF} + \boxed{\mathcal{E}}$$

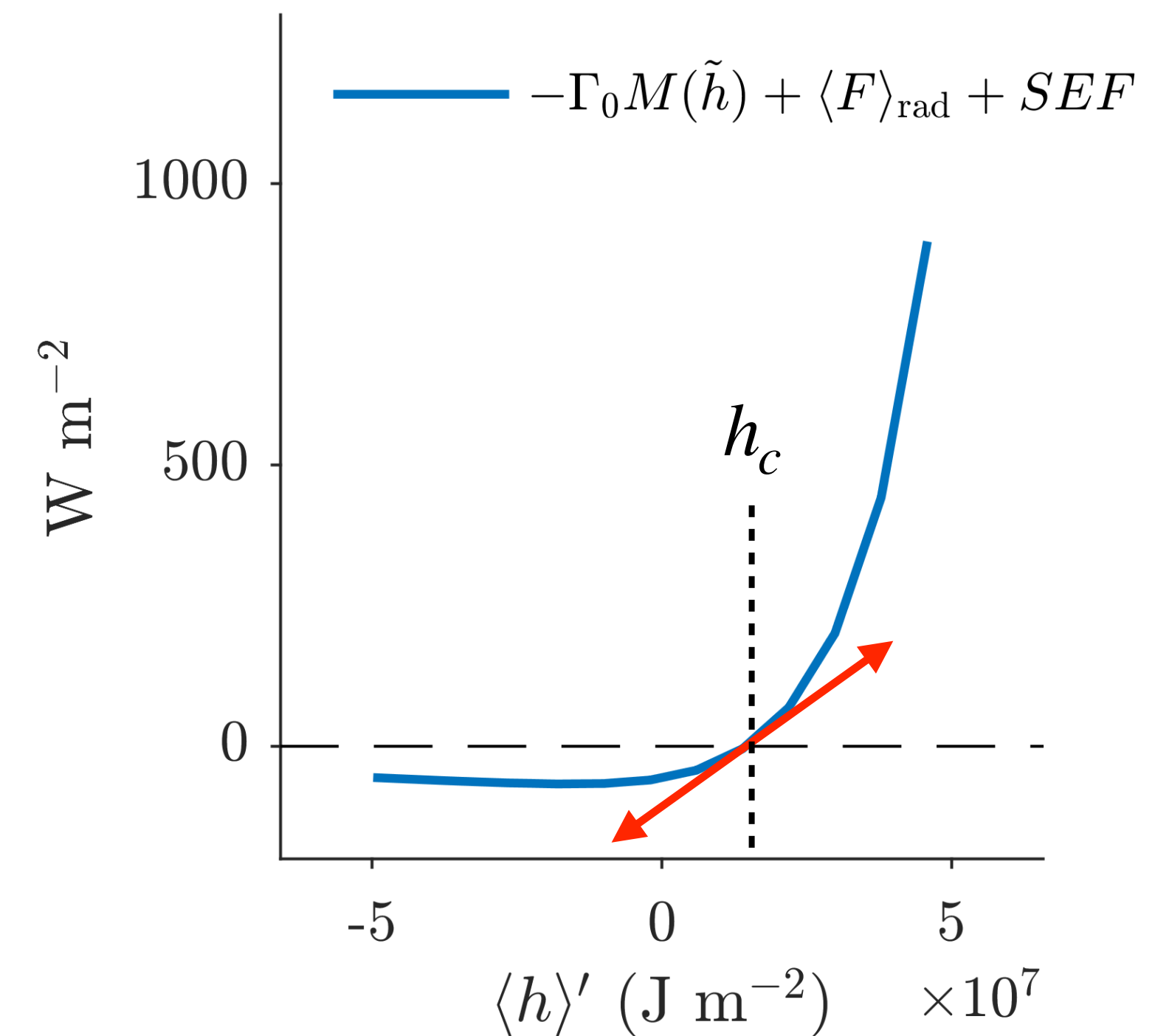
- Consider linear spectral growth

$$\frac{\partial \hat{h}}{\partial t} = (-Dk^2 + \mathcal{E})\hat{h},$$

$\mathcal{E}$  is gradient of vertical advection + diabatic heating at  $h_c$

$$\begin{cases} k < \sqrt{\frac{\mathcal{E}}{D}} \longrightarrow \text{Grow} \\ k > \sqrt{\frac{\mathcal{E}}{D}} \longrightarrow \text{Decay due to diffusion} \end{cases}$$

$$\text{Critical scale} \sim 2\pi\sqrt{\frac{D}{\mathcal{E}}} \sim 500 \text{ km}$$





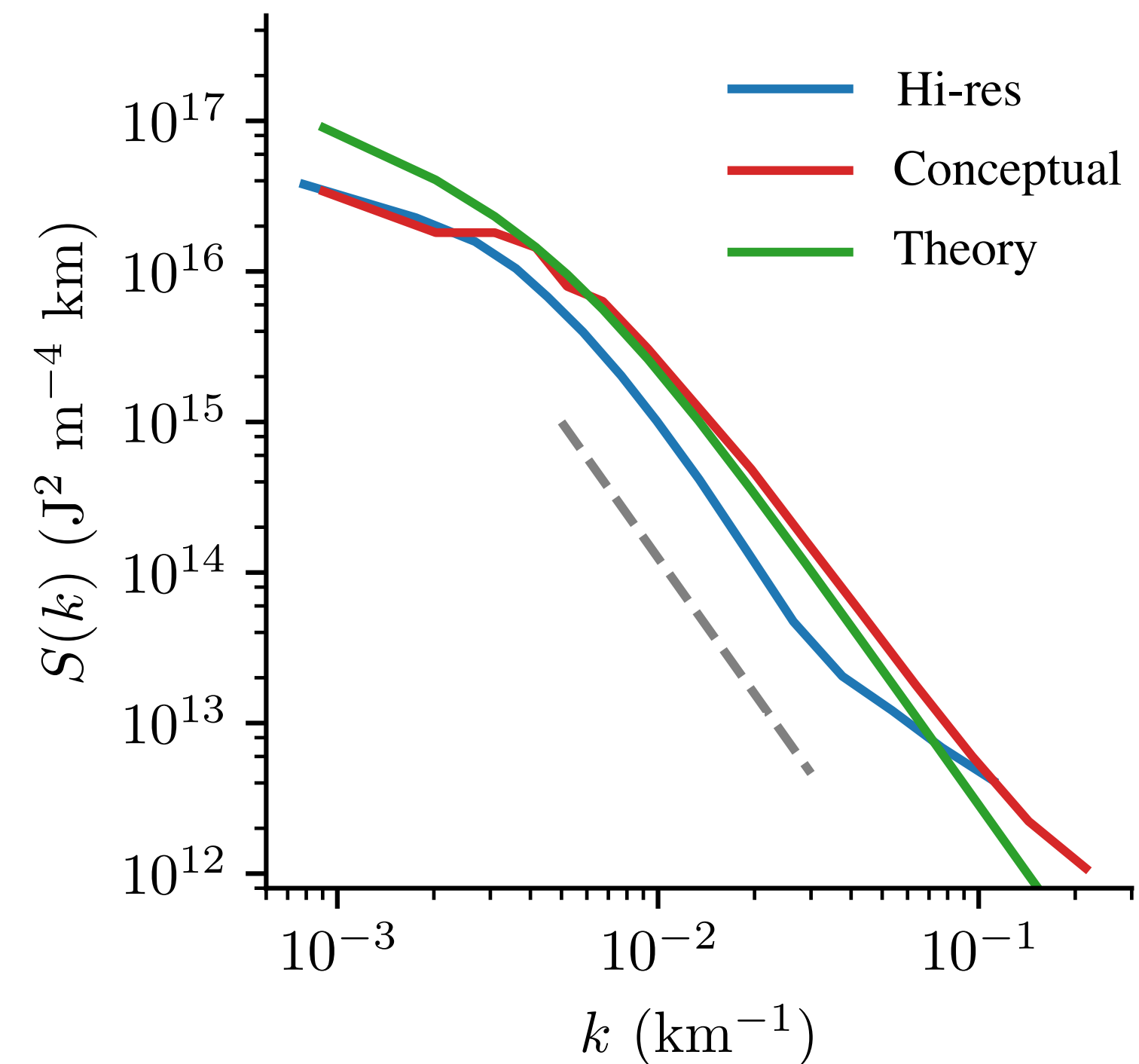
# Diffusion and red noise reconstruct power spectrum similar to hi-res

$$\frac{\partial \tilde{h}}{\partial t} = D \nabla^2 \tilde{h} \left[ -\Gamma M(\tilde{h}') + \langle F \rangle_{\text{rad}} + SEF \right] + \xi, \quad (d\xi = -\beta \xi + \sigma dW_t)$$

- With a bit of math, we show that

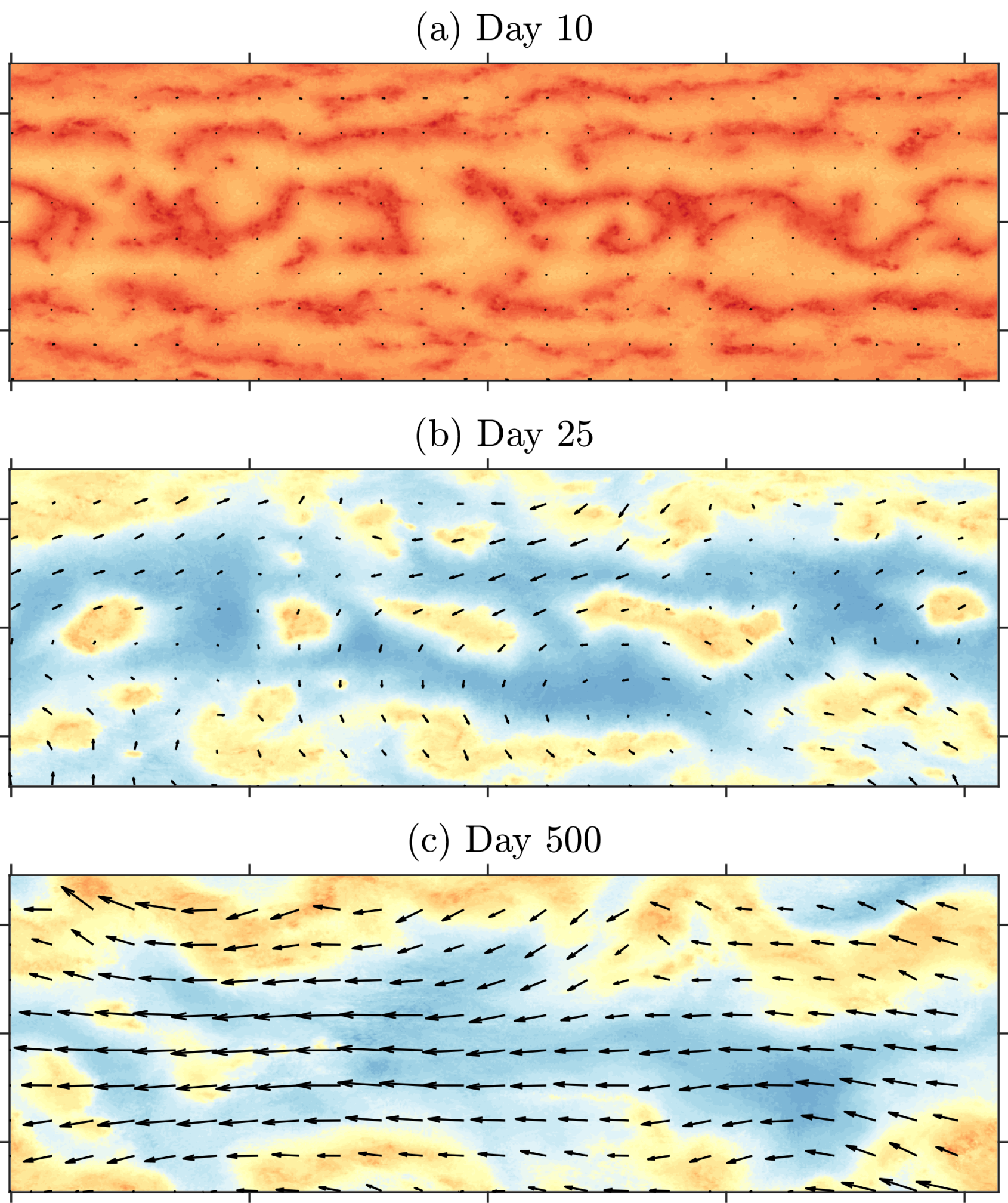
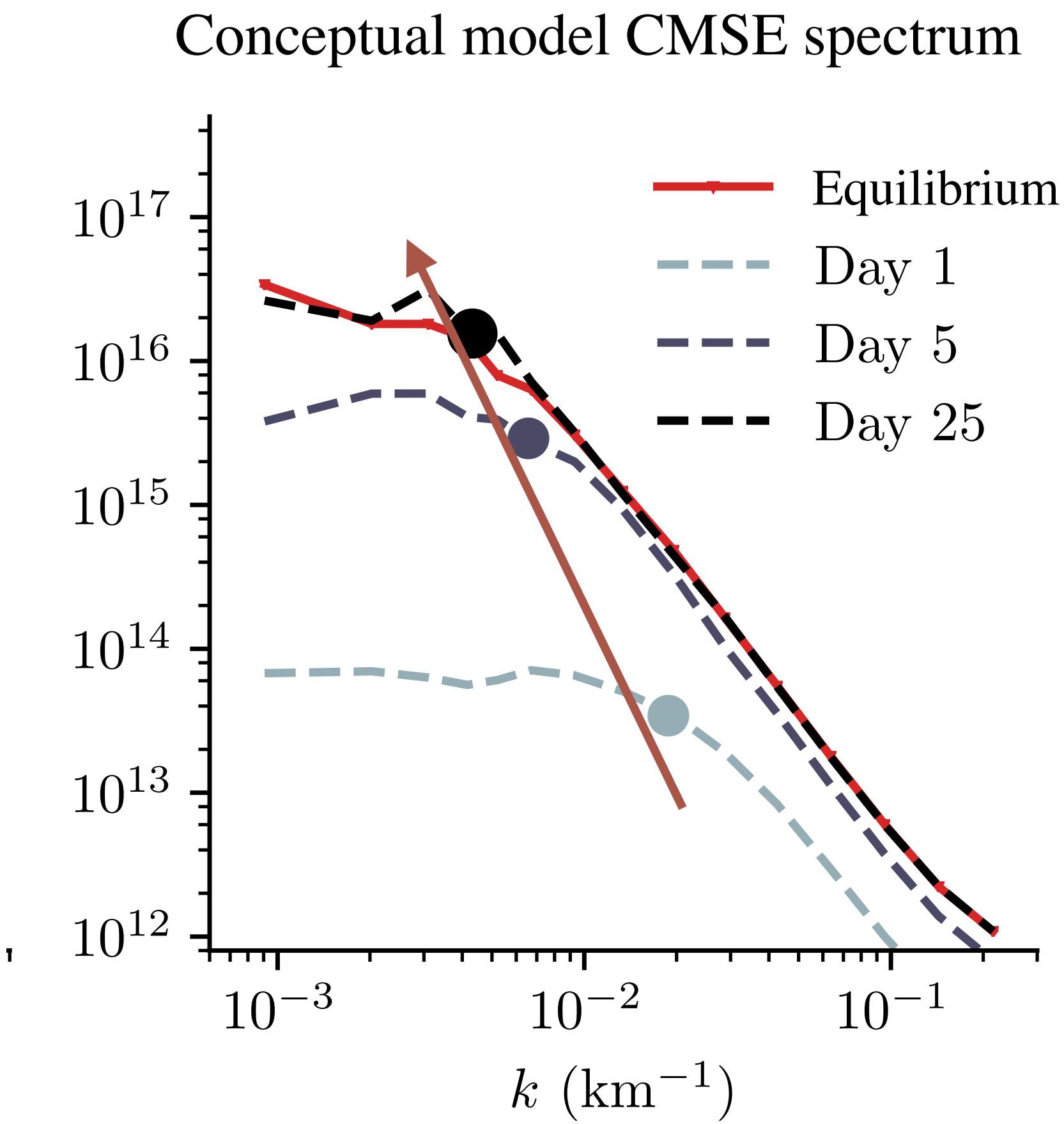
$$S(k) \propto \frac{1}{(Dk^2 + \beta)Dk}$$

$$\begin{cases} S(k) \rightarrow k^{-1} & \text{when } k \rightarrow 0 \\ S(k) \rightarrow k^{-3} & \text{when } k \rightarrow \infty \end{cases}$$





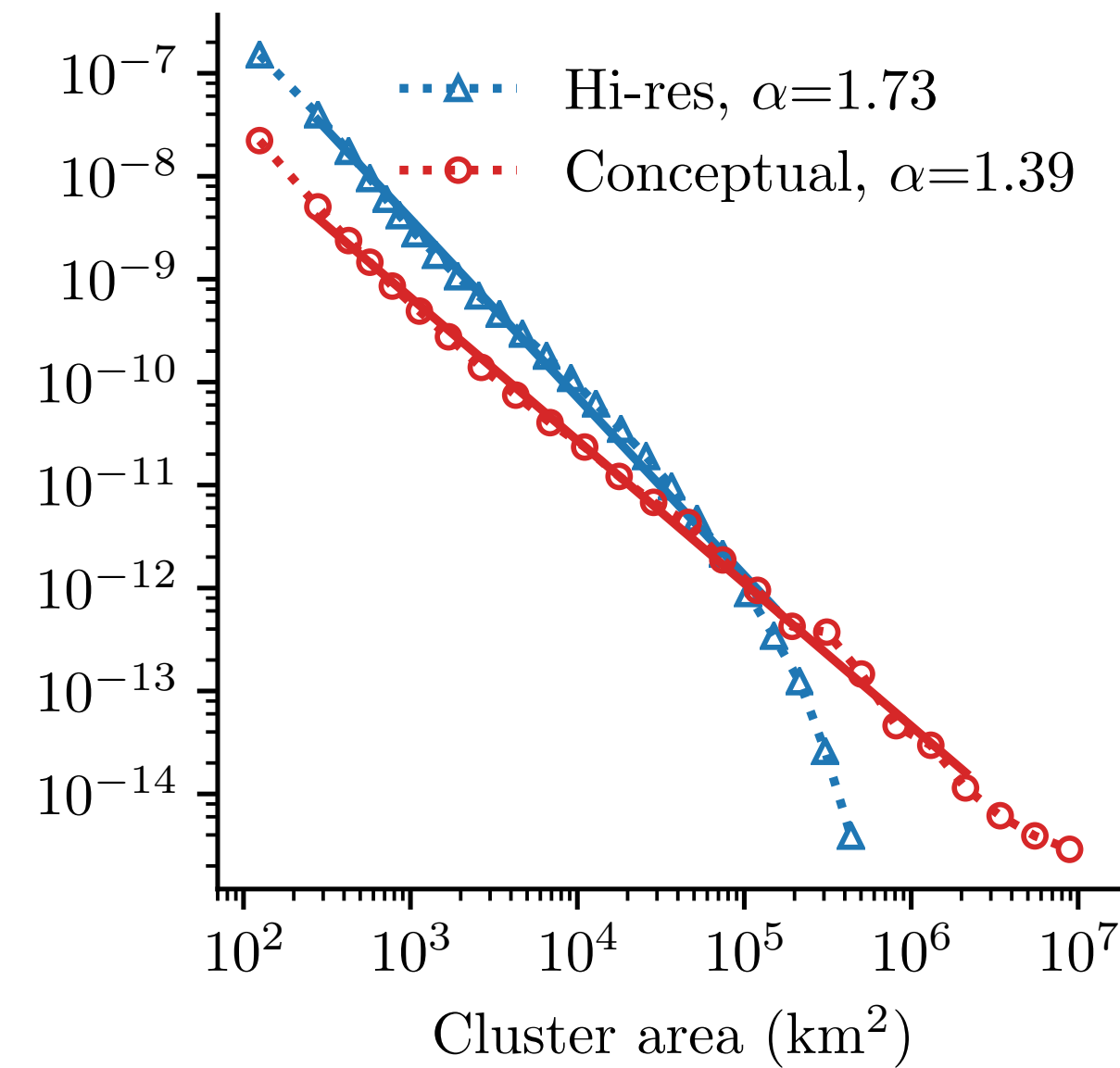
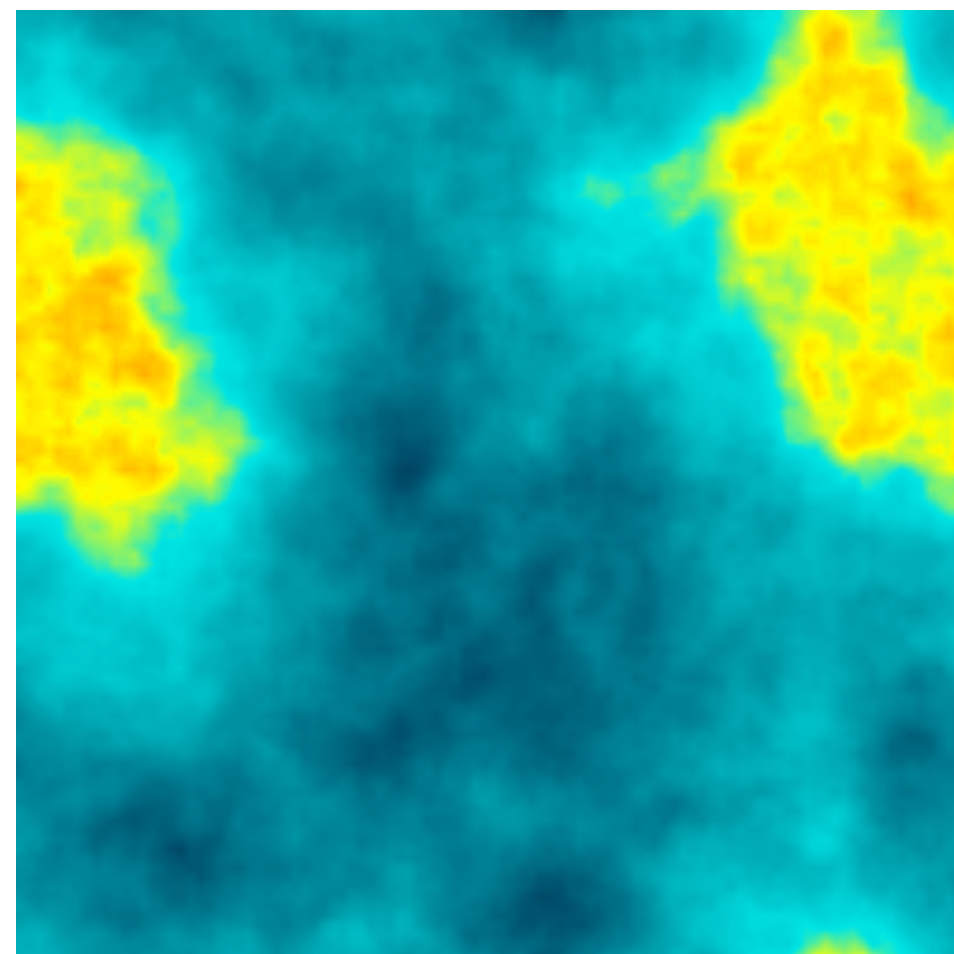
# Decrease in roll off wavenumber implies coarsening in convective organization



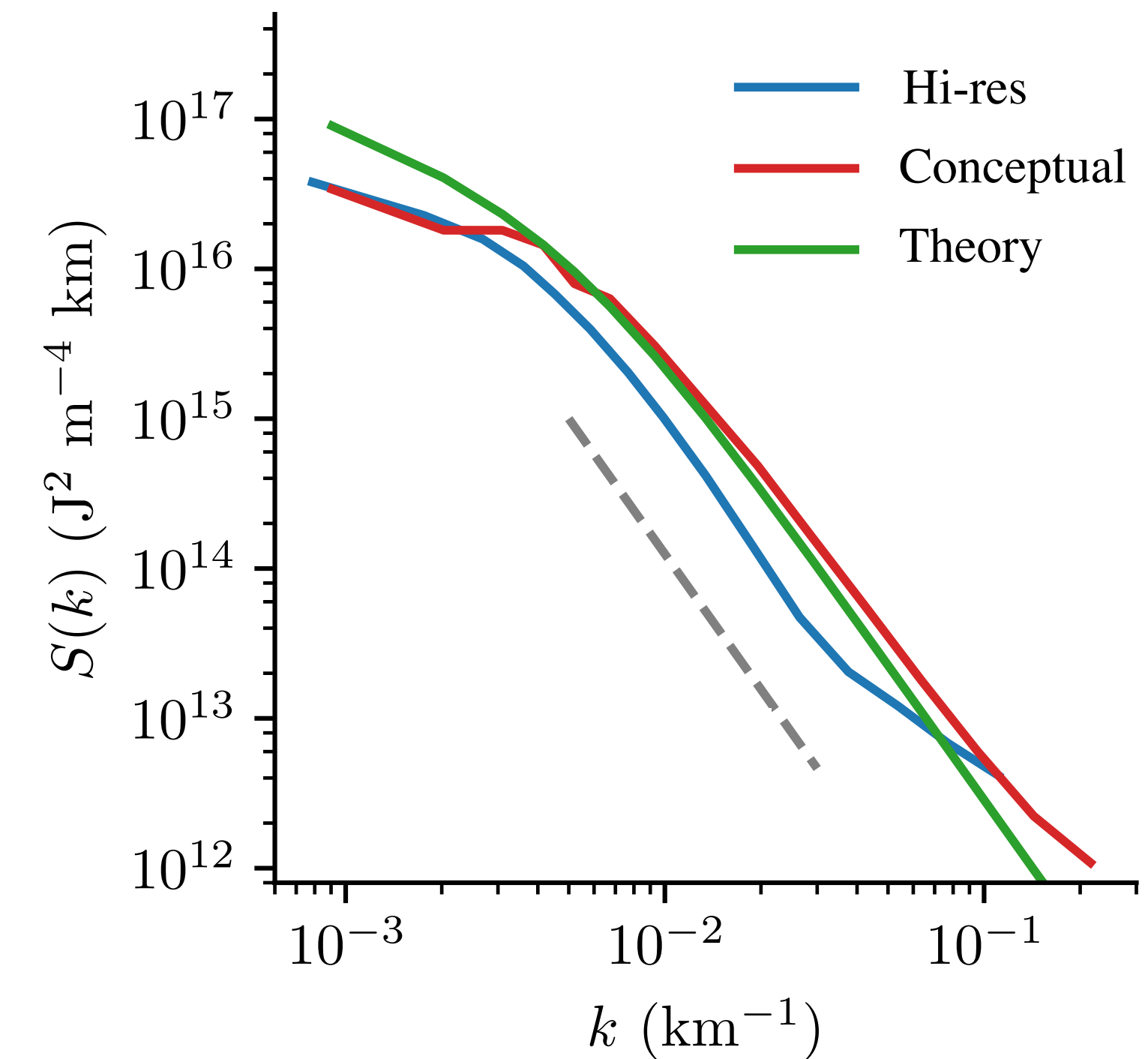


# Conclusions

- Designed a conceptual model that exhibits self-aggregation and power-law distributions of precipitation clusters



- Diffusion with red noise produces the CMSE spectrum with slope -3



- Competition between horizontal and vertical advection results in a minimum domain size for self-aggregation

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