# Experimental daily ensemble streamflow forecasting system using physical model output in a Bayesian hierarchical framework

Álvaro Ossandón<sup>1</sup>, Balaji Rajagopalan<sup>2</sup>, Amar Tiwari<sup>3</sup>, and Vimal Mishra<sup>3</sup>

<sup>1</sup>University of Colorado Boulder <sup>2</sup>University of Colorado at Boulder <sup>3</sup>Indian Institute of Technology Gandhinagar

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#### Abstract

River basin floods due to summer monsoon (June-September) rainfall are the major causes of infrastructure damage and loss of human lives in India. Thus, skillful forecasts of daily streamflows are crucial for flood mitigation. We develop an experimental forecasting system that combines a deterministic physical model forecast in a Bayesian Hierarchical Framework to generate an ensemble daily streamflow forecast. The physical hydrologic model based on the land surface model – Variable Infiltration Capacity (VIC) - developed in an experimental mode to model and forecast hydrologic systems over India is used. Rainfall forecast from the Indian Meteorological Department (IMD) at several lead times (1-day, 2-day, 3-day, 4-day, and 5-day) is used to drive the VIC model to provide a single deterministic forecast trace. A Bayesian Hierarchical Model (BHM) framework is developed to post-process the VIC model forecast and generate skillful daily ensemble streamflow forecast. We demonstrate the BHM framework to daily summer (July-August) streamflow forecast at five stations in the Narmada River Basin in Central India for the period 2003-2014 and, provide preliminary assessment for the period 2015-2018. In this framework, the daily streamflow at each station is modeled as Gamma distribution with time varying parameters, which are modeled as a linear function of potential covariates that include VIC model deterministic streamflow forecast and observed spatially-averaged precipitation from the previous days. With suitable priors on the parameters, posterior distributions of the parameters and predictive posterior distributions of the daily streamflows - and thus ensembles -are obtained. The skill of the probabilistic forecast is assessed a suite of metrics (correlation coefficient, and BIAS), rank histograms, and skill scores such as CRPSS. The model skills are also assessed for various flow thresholds. The BHM framework provides a novel, flexible and powerful approach to combine forecasts from multiple models (including qualitative) and provide a combined skill ensemble forecast. This will be of immense help to enable effective disaster management and mitigation strategies.

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### Álvaro Ossandón (1,2), Balaji Rajagopalan (1,3), Amar Deep Tiwari (4), and Vimal Mishra (4)

(1) CEAE, University of Colorado, Boulder, CO, USA; (2) Departamento de Obras Civiles, Universidad Técnica Federico Santa María, Valparaíso, Chile; (3) CIRES, University of Colorado, Boulder, CO, USA; (4) Civil Engineering, Indian Institute of Technology, Gandhinagar, India; Earth Sciences, Indian Institute of Technology (IIT) Gandhinagar



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# ABSTRACT

River basin floods due to summer monsoon (June-September) rainfall are the major causes of infrastructure damage and loss of human lives in India. Thus, skillful forecasts of daily streamflows are crucial for flood mitigation. We develop an experimental forecasting system that combines a deterministic physical model forecast in a Bayesian Hierarchical Framework to generate an ensemble daily streamflow forecast. The physical hydrologic model based on the land surface model - Variable Infiltration Capacity (VIC) - developed in an experimental mode to model and forecast hydrologic systems over India is used. Rainfall forecast from the Indian Meteorological Department (IMD) at several lead times (1-day, 2-day, 3-day, 4-day, and 5-day) is used to drive the VIC model to provide a single deterministic forecast trace. A Bayesian Hierarchical Model (BHM) framework is developed to post-process the VIC model forecast and generate skillful daily ensemble streamflow forecast. We demonstrate the BHM framework to daily summer (July-August) streamflow forecast at five stations in the Narmada River Basin in Central India for the period 2003-2014 and, provide preliminary assessment for the period 2015-2018. In this framework, the daily streamflow at each station is modeled as Gamma distribution with time varying parameters, which are modeled as a linear function of potential covariates that include VIC model deterministic streamflow forecast and observed spatially-averaged precipitation from the previous days. With suitable priors on the parameters, posterior distributions of the parameters and predictive posterior distributions of the daily streamflows - and thus ensembles - are obtained. The skill of the probabilistic forecast is assessed a suite of metrics (correlation coefficient, and BIAS), rank histograms, and skill scores such as CRPSS. The model skills are also assessed for various flow thresholds. The BHM framework provides a novel, flexible and powerful approach to combine forecasts from multiple models (including qualitative) and provide a combined skill ensemble forecast. This will be of immense help to enable effective disaster management and mitigation strategies.

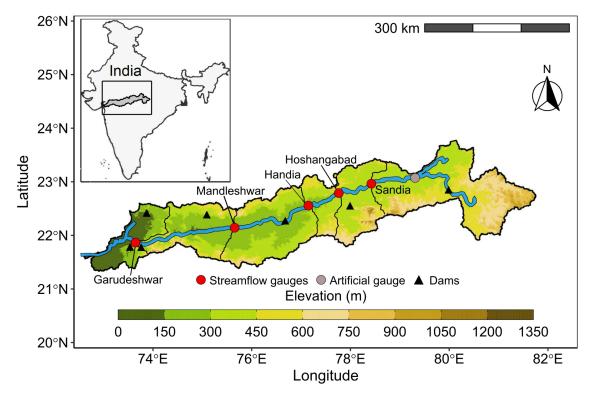
# INTRODUCTION

Improving the streamflow forecast skill is of huge importance in flood-prone river basins, especially basins driven by extreme rainfall events. Here, inspired by the framework proposed in Ossandón et al. (2021a, b), we test a Bayesian hierarchical model for post-processing for daily streamflow forecast in the Narmada River basin in India.

Physically-based hydrological models suffer from model parameterization and regionalization errors, resulting in a bias in the model output. To overcome these errors, one needs to adopt a combination of post-processing methods and data assimilation. Here, we combine the outputs of a semi-distributed hydrological model with a Bayesian Hierarchical Model (BHM).

The BHM considers the river network as a single system and uses physical model forecast outputs and other variables as potential covariates to improve streamflow skills. In addition, the Markov Chain Monte Carlo (MCMC) approach helps in reducing parameter uncertainty by providing an ensemble forecast.

# STUDY REGION AND DATA



### The Narmada River Basin, India

**Figure 1.** Map of the Narmada River basin in India, showing 150-m elevation bands, the locations of five subbasin outlets (Sandia, Hoshangabad, Handia, Mandleshwar, and Garudeshwar), and some of the major dams in the basin: Bargi, Tawa, Indirasagar, Jobat, and Sardar Sarovar (from upstream to downstream direction). The gray circle represents an artificial gauge created for the post-processing model.

- Narmada is a west-flowing river that originates from the Amarkantak ranges in Madhya Pradesh.
- The basin has an area of 98,796 km, and it extends 953 km in the east-west direction.
- It is the fifth-largest river in India and the largest west-flowing river in the country.
- The basin receives a mean annual rainfall of 1120 mm (period 1951-2018), with most of it arriving during the summer monsoon season (June September).

#### Streamflow data

• We obtained daily observed summer discharge data from 1978-2014 at five gauge stations in the Narmada River basin: Sandia, Handia, Hoshangabad, Mandleshwar, and Garudeshwar from India Water Resource Information System (IWRIS).

#### **Meteorological variables**

- We used daily gridded precipitation and temperature data from 1951-2018 from the India Meteorological Department (IMD) (Pai et al., 2014).
- We used daily gridded wind data from 1951-2018 from the NCEP-NCAR re-analysis dataset (Kistler et al. 2001).

#### **Meteorological forecast**

• We used precipitation, maximum temperature, minimum temperature, and wind forecast at three-hour temporal resolution from the Global Forecast System (GFS) for the monsoon season for the 2000-2018 period.

#### VIC hydrological model

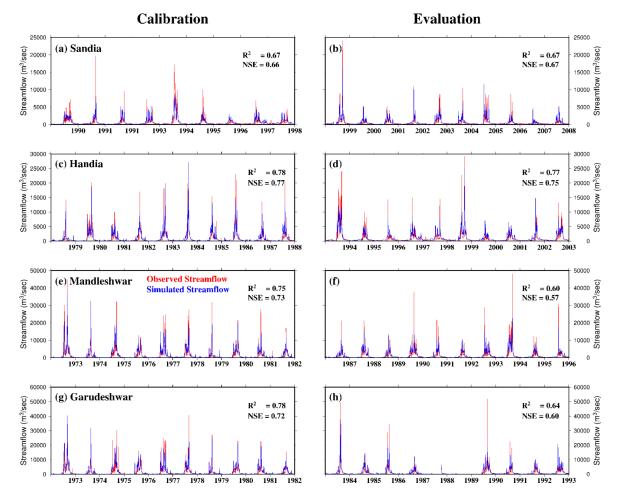
- We used the Variable Infiltration Capacity (VIC) hydrological model (Liang et al., 1994, 1996) at 0.25° resolution for modeling the Narmada basin.
- VIC is a semi-distributed hydrological model that solves water and energy balance at each grid cell.
- It generates 3-layer soil moisture and runoff values at each grid point.
- We integrate the VIC's reservoir module with the routing model (Lohmann et al. 1996) to simulate streamflow, reservoir storage, inflow to the reservoir, and water release from the reservoir.

## METHODS

### VIC model

#### Calibration

- We calibrated the VIC model against observed daily streamflow at four gauge stations, Sandia, Handia, Mandleswar, and Garudeshwar for the period 1990-1998, 1979-1988, 1973-1982, and 1973-1982 respectively.
- We used Nash–Sutcliffe efficiency (NSE; Nash and Sutcliffe, 1970) and coefficient of determination (R<sup>2</sup>) as calibration metrics.



**Figure 2.** Calibration and evaluation of the VIC model against the observed daily streamflow for (a, b) Sandia, (c, d) Handia, (e, f) Mandleshwar, and (g, h) Garudeshwar stations in the Narmada river basin. The time period of calibration and evaluation was taken according to the availability of observed daily streamflow.

#### Forecast

- We developed daily meteorological forcing of 5 days forecast for the monsoon (June-September) season for the 2003-2018 period.
- For each forecast date during the monsoon season, we simulated initial hydrologic conditions using the observed forcing from IMD and the VIC model.
- Using the initial hydrologic conditions simulated with the observed forcing and meteorological forecasts from GFS, the VIC model simulations were conducted for each forecast day during the monsoon season.

#### Table 1. Deterministic metrics values for VIC forecast.

Lead	Garudeshwar			Mandleshwar		Handia			Hoshangabad			Sandia			
time	NSE	R	BIAS (%)	) NSE	R	BIAS (%)	) NSE	R	BIAS (%)	NSE	R	BIAS (%)	NSE	R	BIAS (%)
1-day	-0.01	0.77	121	0.38	0.71	45	0.55	0.8	37	0.56	0.81	42	0.58	0.79	29
2-day	-0.08	0.75	124	0.47	0.75	47	0.55	0.8	42	0.56	0.81	45	0.55	0.78	32
3-day	-0.24	0.65	125	0.35	0.67	47	0.31	0.64	43	0.34	0.66	46	0.33	0.62	32
4-day	-0.34	0.55	120	0.21	0.56	45	0.14	0.49	38	0.17	0.51	41	0.2	0.48	26
5-day	-0.35	0.47	110	0.12	0.47	39	0.02	0.35	32	0.05	0.38	35	0.08	0.35	20

### **Bayesian hierarchical model**

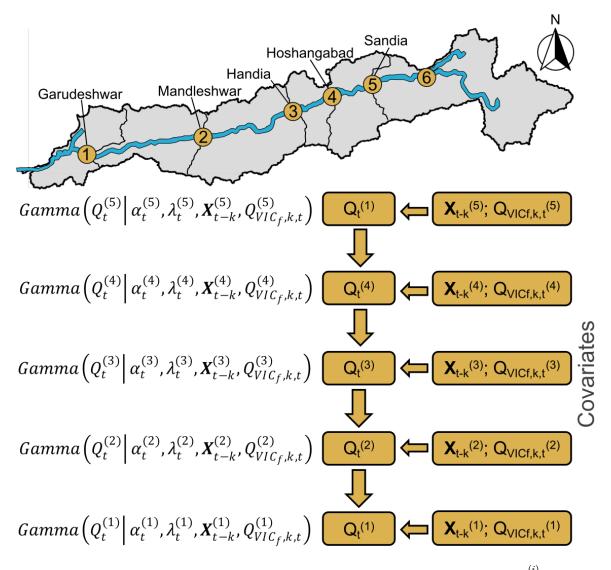
#### **Potential covariates**

For the k-day lead time forecast at the gauge i and day t, as potential covariates we considered:

- +  $Q_{VIC_f,k,t}^{(i)}$ : k-day lead daily VIC forecasted streamflow at the gauge i.
- $P_{f,pd,t-k}^{(i)}$ : k-day lead p-day spatial average forecasted precipitation from the area between stations i and i+1.
- +  $Q_{VIC_s,t-k}^{(i+1)}$ k-day lagged daily VIC simulated streamflow at the upstream gauge i+1.
- $P^{(i)}_{ob,pd,t-k}$ : k-day lagged p-day spatial average observed precipitation from the area between stations i and i+1.

#### **Model structure**

For the structure of the Bayesian hierarchical model (BHM) for the Narmada Basin, we considered that observed streamflow at each gauge station follows a gamma distribution with time-varying parameters. Figure 3 displays the conceptual sketch of the BHM implemented here for a k-day lead time forecast.



**Figure 3.** Conceptual sketch of the Bayesian hierarchical model for the Narmada River basin.  $Q_t^{(i)}$  corresponds to the observed streamflow at gauge i and day t,  $\mathbf{X}_{t-k}^{(i)}$  To the vector of other covariates considered in the BHM for the gauge i at day t, and  $Q_{VIC_f,k,t}^{(i)}$  to the k-day lead daily VIC forecasted streamflow at the gauge i and day t.

This gives the model structure represented by the following equations

$$Gamma\left(Q_t^{(i)} \mid lpha_t^{(i)}, \lambda_t^{(i)}, \mathbf{X}_{t-k}^{(i)}, Q_{VIC_f,k,t}^{(i)}
ight), \quad i=1,2,\dots,5$$

$$\alpha_{t}^{(i)} = \frac{\left(\mu_{t}^{(i)}\right)^{2}}{\left(\sigma_{t}^{(i)}\right)^{2}}, \quad \lambda_{t}^{(i)} = \frac{\mu_{t}^{(i)}}{\left(\sigma_{t}^{(i)}\right)^{2}}$$

$$\mu_{t}^{(i)} = \begin{cases} \beta_{1}^{(i)} + \beta_{2}^{(i)} Q_{VIC_{f},k,t}^{(i)} + \boldsymbol{\beta}_{3}^{(i)} \mathbf{X}_{t-k}^{(i)} & Q_{VIC_{f},k,t}^{(i)} \leq Q_{VIC_{f},80th}^{(i)} \\ \beta_{m+2}^{(i)} + \beta_{m+3}^{(i)} \left( Q_{VIC_{f},k,t}^{(i)} \right)^{n} + \boldsymbol{\beta}_{m+4}^{(i)} \mathbf{X}_{t-k}^{(i)} & Q_{VIC_{f},k,t}^{(i)} > Q_{VIC_{f},80th}^{(i)} \end{cases}$$

$$\sigma_t^{(i)} = \begin{cases} \phi_1^{(i)} + \phi_2^{(i)} Q_{VIC_f,k,t}^{(i)} + \phi_3^{(i)} \mathbf{X}_{t-k}^{(i)} & Q_{VIC_f,k,t}^{(i)} \leq Q_{VIC_f,80th}^{(i)} \\ \phi_{m+2}^{(i)} + \phi_{m+3}^{(i)} Q_{VIC_f,k,t}^{(i)} + \phi_{m+4}^{(i)} \mathbf{X}_{t-k}^{(i)} & Q_{VIC_f,k,t}^{(i)} > Q_{VIC_f,80th}^{(i)} \end{cases}$$

- Posterior distributions of the parameters and streamflow (ensembles) were estimated using the the No-U-Turn Sampler (NUTS; Hoffman and Gelman 2014) for the Markov Chain Monte Carlo method (Gelman and Hill 2006).
- BHM candidates were calibrated for the period 2003-2014.
- For each lead time, we selected the best covariates for each gauge based on the lowest value of the leave-one-out cross-validation information criteria (LOOIC; Vehtari et al. 2017)
- To assess the probabilistic performance of the proposed BHM, we considered the continuous ranked probability score (CRPSS; Gneiting & Raftery, 2007)

Lead time	Covariates												
	1: Garudeshwar	2: Mandleshwar	3: Handia	4: Hoshangabad	5: Sandia	LOICC							
1-day	$\left(Q_{VIC_f,1,t}^{(1)}\right)^2$ , $Q_{VIC_s,t-1}^{(2)}$	$\left(Q_{VIC_f,1,t}^{(2)}\right)^2$ , $Q_{VIC_s,t-1}^{(3)}$	$\left(Q_{VIC_f,1,t}^{(3)}\right)^2$ , $Q_{VIC_s,t-1}^{(4)}$	$\left(Q_{VIC_{f},1,t}^{(4)}\right)^{2}$ , $Q_{VIC_{s},t-1}^{(5)}$	$\left(Q_{VIC_{f},1,t}^{(5)}\right)^{2}$ , $Q_{VIC_{s},t-1}^{(6)}$	7983							
2-day	$\left( Q_{VIC_{f},2,t}^{(1)}  ight)^{2.5}$ , $Q_{VIC_{s},t-2}^{(2)}$	$\left( Q_{VIC_{f},2,t}^{(2)}  ight)^{2.5}$ , $Q_{VIC_{s},t-2}^{(3)}$	$\left( Q_{VIC_{f},2,t}^{(3)}  ight)^{1.5}$ , $Q_{VIC_{s},t-2}^{(4)}$	$\left( Q_{VIC_{f},2,t}^{(4)}  ight)^{1.5}$ , $Q_{VIC_{s},t-2}^{(5)}$	$\left(Q_{VIC_{f},2,t}^{(5)} ight)^{1.5}$ , $Q_{VIC_{s},t-2}^{(6)}$	8086							
3-day	$\left( Q_{VIC_{f},3,t}^{(1)}  ight)^{3.5}$ , $Q_{VIC_{s},t-3}^{(2)}$	$\left( Q_{VIC_{f},3,t}^{(2)}  ight)^{3.5}$ , $Q_{VIC_{s},t-3}^{(3)}$	$\left( Q_{VIC_{f},3,t}^{(3)}  ight)^{3.5}$ , $Q_{VIC_{s},t-3}^{(4)}$	$\left( Q_{VIC_{f},3,t}^{(4)}  ight)^{3.5}$ , $Q_{VIC_{s},t-3}^{(5)}$	$\left(Q_{VIC_{f},3,t}^{(5)}\right)^{2.5}$ , $Q_{VIC_{s},t-3}^{(6)}$	8986							
4-day	$\left( Q_{VIC_{f},4,t}^{(1)} ight) ^{2.5}$ , $Q_{VIC_{s},t-4}^{(2)}$	$\left( Q_{VIC_{f},4,t}^{(2)}  ight)^{2.5}$ , $Q_{VIC_{s},t-4}^{(3)}$	$\left( Q_{VIC_{f},4,t}^{(3)}  ight)^{2.5}$ , $Q_{VIC_{s},t-4}^{(4)}$	$\left( Q_{VIC_{f},4,t}^{(4)}  ight)^{2.5}$ , $Q_{VIC_{s},t-4}^{(5)}$	$\left(Q_{VIC_{f},4,t}^{(5)} ight)^{1.5}$ , $Q_{VIC_{s},t-4}^{(6)}$	9855							
5-day	$\left(Q_{VIC_{f},5,t}^{(1)}\right)^{2}$ , $Q_{VIC_{s},t-5}^{(2)}$	$\left(Q_{VIC_f,5,t}^{(2)}\right)^{1.5}, Q_{VIC_s,t-5}^{(3)}$	$\left(Q_{VIC_{f},5,t}^{(3)}\right)^{2}$ , $Q_{VIC_{s},t-5}^{(4)}$	$\left(Q_{VIC_{f},5,t}^{(4)}\right)^{2}$ , $Q_{VIC_{s},t-5}^{(5)}$	$\left(Q_{VIC_{f},5,t}^{(5)}\right)^{2}$ , $Q_{VIC_{s},t-5}^{(6)}$	10470							

**Table 2.** LOOIC values for the best BHM for different lead times.

### RESULTS

### **BHM Calibration**

For each lead time, the BHM was calibrated for the period 2003-2014. 3000 simulations from posterior distributions of the model parameters, and consequently, streamflow ensembles were obtained.

Table 3. Deterministic metrics values for the calibration of the BHM forecast.

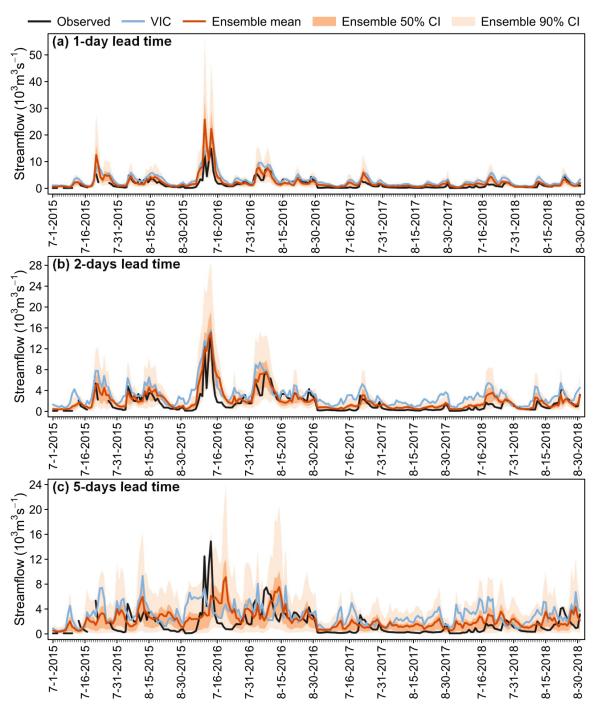
Lead	Garudeshwar			Ма	Mandleshwar		Handia			Hoshangabad			Sandia		
time	NSE	R	BIAS (%	%) NSE	R	BIAS (%)	NSE	R	BIAS (%)	NSE	R	BIAS (%)	NSE	R	BIAS (%)
1-day	0.56	0.75	4.3	0.63	0.8	1.0	0.68	0.83	-0.4	0.67	0.82	-2.6	0.69	0.83	-1.3
2-day	0.66	0.82	1.3	0.65	0.81	0.6	0.69	0.83	-2.2	0.69	0.83	-2.1	0.66	0.82	-1.9
3-day	0.54	0.74	0.9	0.61	0.79	1.1	0.49	0.7	-3.8	0.52	0.72	-2.5	0.46	0.68	-2.2
4-day	0.34	0.59	2.2	0.37	0.61	0.6	0.29	0.54	0.3	0.32	0.56	-0.6	0.28	0.54	0.8
5-day	0.2	0.49	5.6	0.23	0.49	4.2	0.13	0.4	4.3	0.18	0.44	4.1	0.16	0.43	5.9

For the calibration of the BHM forecast, there is a significant increase of all deterministic performance metrics at all the gauges compared to the VIC forecast (Table 1).

The BHM performance decreases as the lead time increases.

### **BHM Validation**

Streamflow ensembles were obtained for the period 2014-2018.



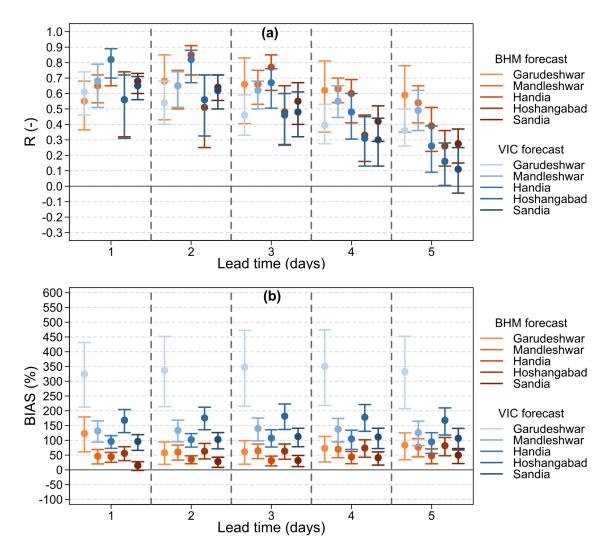
**Figure 4.** Times series of observed, VIC forecast, and BHM ensembles forecast of daily monsoon streamflow for period 2015-2018 at Handia gauge for (a) 1-day, (b) 2-day, and (c) 5-day lead time. Black lines correspond to observed streamflow, light blue lines to VIC forecast, orange lines to BHM ensemble mean, orange bands to 50% confidence of the ensembles, and light orange bands to 90% confidence of the ensembles.

 All the observed values are captured by the BHM ensemble spread at Handia gauge for 1- and 2-day lead times.

For BHM, there is a reduction of the positive and negative bias for 1 and 2-day lead times compared to the VIC forecast.

There is a significant reduction of the performance for a 5-day lead time.

#### **Deterministic accuracy metrics**

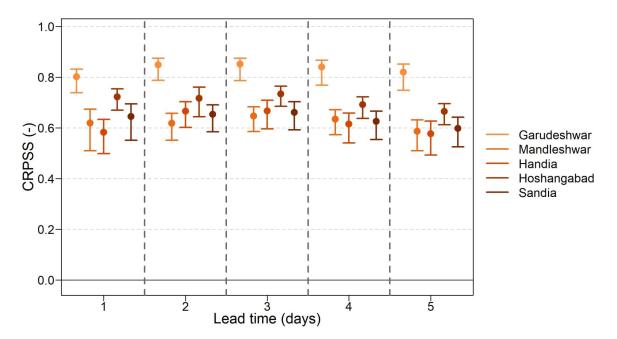


**Figure 5.** (a) correlation coefficient, R, and (b) Bias (%) at different lead times and five gauges of the Narmada River Basin for VIC forecast (blue colors) and BHM ensembles forecast (orange colors). The error bars define 95% confidence limits obtained through bootstrapping.

Although an increase in the correlation is not seen for all the gauges and lead times, the BHM model can preserve the correlation and for some gauges and lead times even improve the correlation.

As for the calibration, the validation of the BHM shows a significant reduction of the BIAS for all the lead times compared to the VIC forecast.

#### **Probabilistic accuracy metrics**



**Figure 6.** CRPSS at different lead times and five gauges of the Narmada River Basin for BHM ensembles forecast (orange colors). The error bars define 95% confidence limits obtained through bootstrapping. VIC forecast was considered as the reference forecast model.

BHM shows a significantly better performance than the VIC forecast (CRPSS values above 0.5).

# CONCLUSIONS

The proposed Bayesian Hierarchical model for postprocessing deterministic (VIC) physical model forecast outputs offers:

Skillful daily ensemble streamflow forecast for 1 to 5 days lead relative to VIC.

easy coupling with meteorical forecast

Reduction of non-systematic (positive and negative) bias relative to VIC.

For lead times longer than 3 days, the forecast ensemble information can be useful for flood risk assessment and mitigation.

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Ministry of Earth Sciences Government of India





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