

Planning of Optimized Irrigation Decision in Weather to Extended Range using Weather Forecast with a Coupled Framework of Optimization and Ecohydrological Model

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Abstract

Optimization in irrigation scheduling using weather forecast has been proven to achieve better productivity along with reduced irrigation water requirements. We developed a farm-scale hydrological model coupled with a chance-constraint optimization to take short to medium range weather forecast and prescribe the optimal irrigation amount determined by developing the conditional probability density functions of the rainfall and subsequently the soil moisture for the days in forecast range. The stress-avoidance was ensured by maintaining the probability of crops undergoing water stress is less than a prescribed threshold (reliability factor, α). The framework was implemented for irrigation decision simulation at extended range by downscaling the forecast with Nonhomogeneous Hidden Markov Model (NHMM) as an input and produce irrigation decision in extended range (15 to 30 days). The optimization framework ensured minimal water use without significant crop water stress. The method was tested at two site locations in Nashik district in the state of Maharashtra, both being involved in grape cultivation (referred herein as Site 1 and Site 2). In short-to-medium range weather scale, the model was implemented with varied α (0.5 to 0.95) and interval between two subsequent irrigation application (1, 3 and 7 days) and significant amount of water savings with respect to the farmer's applied irrigation could be achieved. The simulation-optimization framework was only tested with $\alpha=0.95$ and once in 7 days irrigation application for extended range, and yet no significant detrimental effect on yield was observed whereas in kharif season significant potential of water savings was observed both in Site 1 and 2. While the framework in short to medium range is useful for optimal real time irrigation decision making, in the extended range, it can be implemented in planning of irrigation for the upcoming month to avoid the inconvenience of instant arrangement of water, especially in case of drought-hit regions. Considering that irrigation accounts for over 80% of the total water use worldwide, the value of such an approach as a decision-support tool for irrigation optimization is self-evident.

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Introduction

- To deal with climate change-related variation in rainfall has increasingly significantly with an increase in irrigation as well as need for food demand in the growing population (Rao et al., 2017).
- Irrigation water application in the crop water content (crop water content) is a key factor in crop yield (Rao et al., 2017; Roy & Ghosh, 2017; Roy & Narvekar, 2017).
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Simulation-Optimization Framework

Downscaling Forecast: Short-Medium Range and Extended Range

Seasonal Soil Moisture Variation with Proposed Framework

Study Area and Datasets

Water Use Optimization and Impact on Relative Yield

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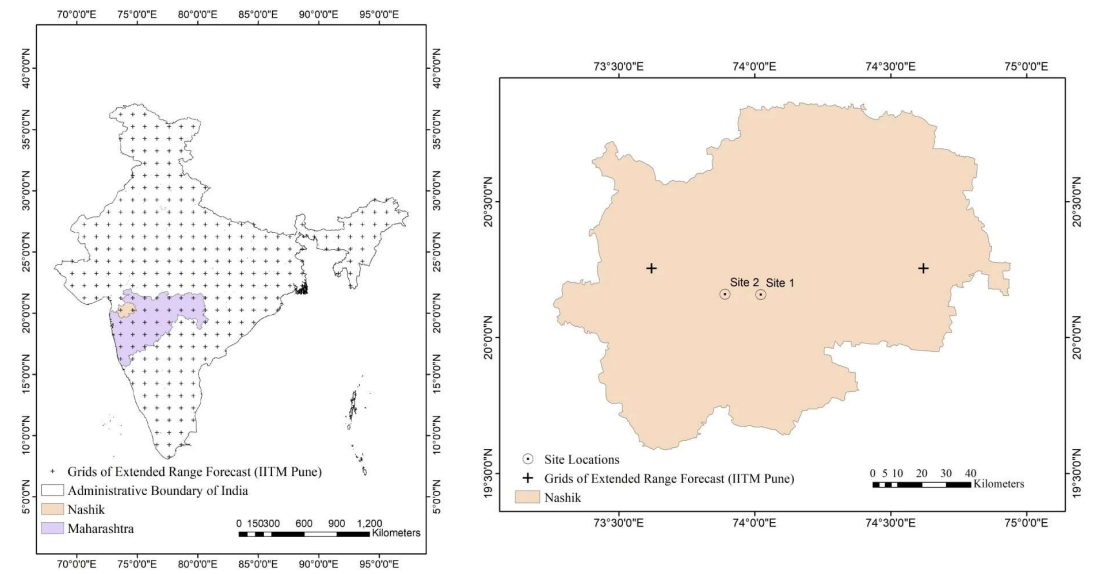
INTRODUCTION

- In last few decades, agricultural activities in India has intensified significantly with an increase in irrigation as well to meet the food demand of the growing population (Barik et al., 2017).
- Irrigation water application is the largest sector consuming water resources requiring over 80% of the total freshwater resources (Fishman et al., 2015; Vico & Porporato, 2010).
- A few studies exist where the significance of incorporating weather forecast into irrigation planning has been demonstrated at short to medium scale (Cai et al., 2011; Hejazi et al., 2014; Jamal et al., 2019; Wang & Cai, 2009). However, there exists limited studies in using the forecast in irrigation scheduling at farm or plot level, with optimization of irrigation water use to be the main decision variable.
- Here, we created a simulation-optimization framework, which can take into account the weather forecast at short, medium and extended range scale to generate optimized irrigation decision at the decided timescale and also ensuring no detrimental impact on productivity.

STUDY AREA AND DATASETS

We applied the developed framework to the case studies at two farm locations in the Nashik district in the state of Maharashtra.

The two farms we selected were associated with a regional agricultural cooperative guiding the farmers by maintaining sustainable crop yield and mobilizing the supply and demand. Both the farms are used for grape cultivation and the farm locations have been shown in the study area map, named as Site 1 and Site 2, respectively.



Study Area Map (<https://drive.google.com/file/d/13NOEzG0-8KgYAUbShITtRKuqe3MGQkXu/view?usp=sharing>)

The district is one of the frequent drought-hit regions in Maharashtra despite the climatological annual rainfall of over 1100 mm. Thus, the farmers are interested in implementing a reliable irrigation optimization framework to minimize water applications without losing the crop yield.

The two sites have two different soil types and thus have led to different sets of parameterizations, which were estimated from the observed soil moisture and irrigation amounts. The geographical coordinates along with the parameters are shown in Table 1.

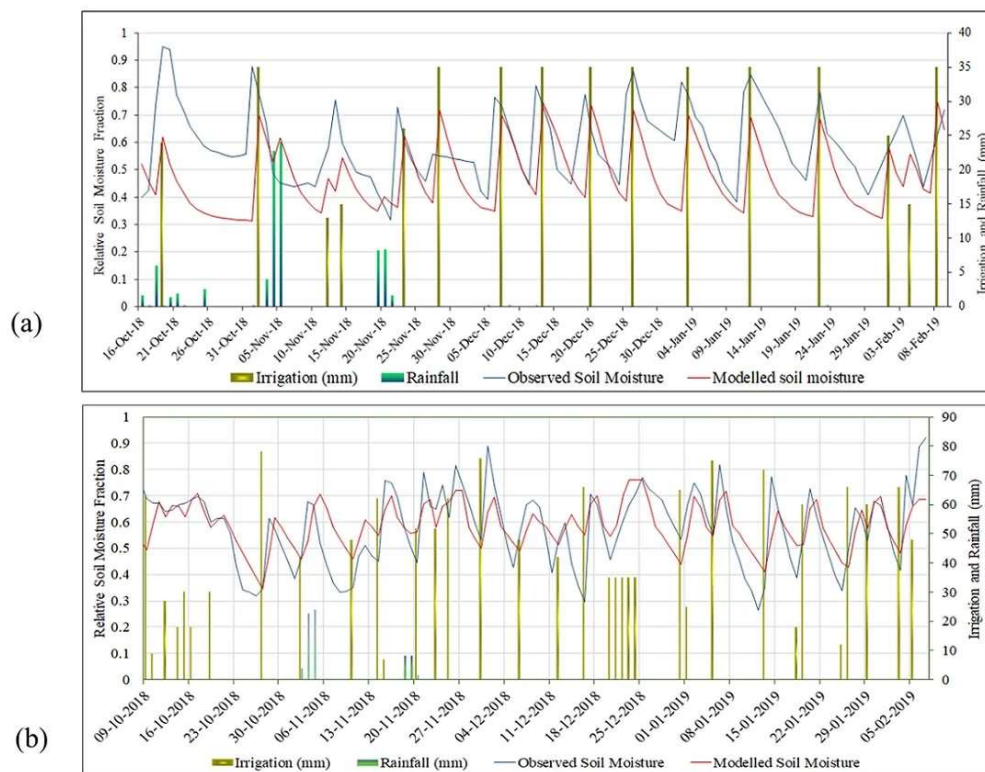
Table 1. The geographic locations and soil parameters of the site locations

Site No.	Lat. and Long.	Soil Type	s^*	s_{fc}	s_w	s_h	n	Z_r (mm)	β	K_s (mm/d)
1	20°9'29"N 74°1'19"E	Clayey loam	0.45	0.75	0.3	$0.8 \times s_w$	0.28	450	18	800
2	20°9'36"N 73°53'25"E	Sandy Loam	0.35	0.65	0.2	$0.6 \times s_w$	0.56	400	11	350

For short to medium range weather forecasts, Global Ensemble Forecast System (GEFS) Reforecast v2 product has been used.

For extended range forecast, the Multi-Model Ensemble (MME) generated by post-processing the Extended Range Prediction System (ERPAS) developed by the Indian Institute of Tropical Meteorology, Pune, was used.

To calibrate the hydrological model and to verify the outcome of the optimized model, data regarding soil moisture ground observations and actual amount of irrigation application, are required. Apart from these, we have also procured weather station data for the actual rainfall and PET computations, which have been implemented in calibrating the hydrological model by properly parameterizing for the area.

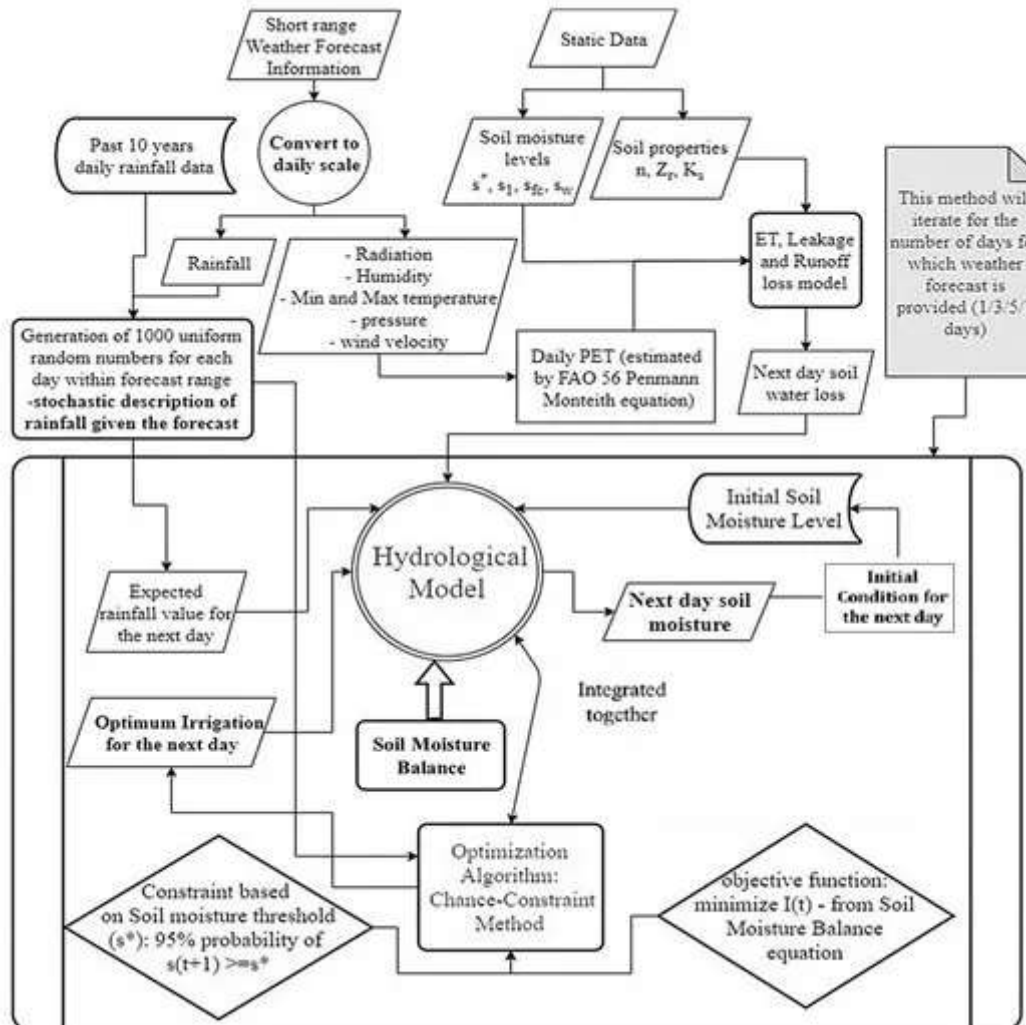


Calibration of farm scale ecohydrological model with observed the soil moisture and irrigation for the site 1 and 2 (a and b) during the cropping season after fruit pruning. The plots are only presented for the period, during which the data was available, within the season.

SIMULATION-OPTIMIZATION FRAMEWORK

Overall Framework for Short to Medium Range Irrigation Scheduling

(https://drive.google.com/file/d/1c9BufCWeiDz8LqE_LbT_JxupCWM0QMRL/view?usp=sharing)



The optimization model can be expressed as (Roy et al., 2021):

$$\text{Eq. 1: } \text{minimize } \sum_{i=1}^N I_{t+i}$$

$$P(s_{t+i} \geq s^*) \geq \alpha \quad \forall t \in T_{Seas}, i = 1, 2, 3, \dots N$$

$$\text{Eq. 2-4: } 0 \leq I_{t+i} \leq I_{max} \quad \forall t \in T_{Seas}, i = 1, 2, 3, \dots N$$

$$s_{hs} \leq \bar{s}_{t+i} \leq 1 \quad \forall t \in T_{Seas}, i = 1, 2, 3, \dots N$$

$$\text{Eq. 5: } f_{S_{t+i}}(s_{t+i}) = f^{hyd} \{F_{H=h_{t+i}|FS=f_s}(h_{t+i}), I_{t+i}, s_{t+i-1}\}$$

Equation (1) is the objective function, whereas Equation (2) enforces the main constraint in water use minimization by ensuring that the soil moisture on $(t+i)^{th}$ day is (s_{t+i}) and it does not drop below a threshold with an assured probability, designated as the reliability factor (α).

The value of α is varied within a practical range of values (0.5, 0.75, 0.85 and 0.95) and is a decision parameter for the farmer.

The assumed threshold is known as soil moisture in incipient stomatal closure (s^*) and the evapotranspiration (ET) starts to decrease beyond this minimum threshold.

Equation (3) provides an upper limit of irrigation (I_{max}) based on maximum ET rate for the days under forecast for preventing soil water loss by runoff due to over-irrigation (Roy et al., 2021).

Finally, Equation (4) provides the constraint for realistic value for the expected soil moisture on the day.

Here we considered three cases of irrigation after discussing with the local farmers.

Case A: Some farmers have the privilege of having access to the irrigation facilities and are willing to irrigate every day as per the need. For such cases, we consider the weather forecast for the next 3 days (updating the forecast on each fourth day) and compute the irrigation water to be applied to each of the 3 days.

Case B: There are a few farmers who prefer to irrigate once every 3 days. Like Case A, the weather forecast is taken for 3 days lead time and updated on the fourth day every time.

Case C: Several farmers in India do not have access to irrigation instruments very frequently and prefer to irrigate once a week. The weather forecast is taken with 7 days lead time on every eighth day.

• Hydrological Model:

The ecohydrological model was originally developed from the idea given in a series of past literatures (Rodriguez Iturbe et al., 1999; Isham et al., 2005; Porporato et al., 2003).

It is based on a simple soil water balance as below:

$$nZ_r \frac{ds_t}{dt} = R_t + I_t - ET(s_t) - LQ(s_t)$$

The hydrological component has mainly two source-sink components: rainfall, and combined loss as the sum of ET, runoff, and leakage.

Here, the rainfall is described as a probabilistic component in the soil moisture balance equation. On the other hand, ET, runoff, and leakage rate are modeled as a function of present soil moisture and soil hydraulic properties.

The objective function for optimization is defined by minimizing irrigation water use, applying constraints of not permitting plants to undergo water stress by restricting the soil moisture level to stay above the soil moisture level of incipient stomatal closure.

DOWNSCALING FORECAST: SHORT-MEDIUM RANGE AND EXTENDED RANGE

Short to Medium Range Rainfall Forecast:

We employ forecasts to model the distribution of rainfall at short to medium range (days 1-7).

We attempt to describe the probability distribution of temporal precipitation pattern with two parameters:

- the probability of rainfall depth for the following days to be greater than 0, conditional to the forecast (p)

$$p = P(R > 0 \mid FS = fs)$$

- the expected rainfall depth during the next day, given that non-zero rainfall amount is predicted to occur (γ).

The depth of the nonzero rainfall, scaled by rooting depth and soil porosity, n^*Z_r , denoted by random variable H ($= R / n^*Z_r$), which is assumed to be exponentially distributed.

Combining these two parameters, the CDF of rainfall given the forecast, was obtained, as shown in the equation below:

$$F_{H=h \mid F=fc}(h) = \begin{cases} 1 - p + p * (1 - e^{-\gamma h}), & \text{when } R > 0 \\ 1 - p, & \text{when } R = 0 \end{cases}$$

This CDF was used to generate rainfall values on a day, given a forecast for the day. The generated values were used in Monte-Carlo simulations to take care of the uncertainty in the forecasts.

Extended Range Forecast Downscaling:

We implemented a Bayesian Nonhomogeneous Hidden Markov Model (NHMM) to predict the forecast at farmscale level. For extended range forecast, IITM Pune ERPAS data for 3 weeks lead time was taken.

Selected predictors for NHMM were:

- (i) the forecasted daily rainfall, (ii) relative humidity (rh_{um}), (iii) mean sea level pressure (mslp), (iv) northward wind velocity at 10 m (v_{10}), (v) eastward wind velocity at 10 m (u_{10}), (vi) geopotential height at 850 hPa (Z_{850}), (vii) geopotential height at 200 hPa (Z_{200}), (viii) westerly wind shear (WS), and (ix) southerly wind shear (SS).

NHMM predicts next day's rainfall based on weather state of the current day. The assumptions are as shown here.

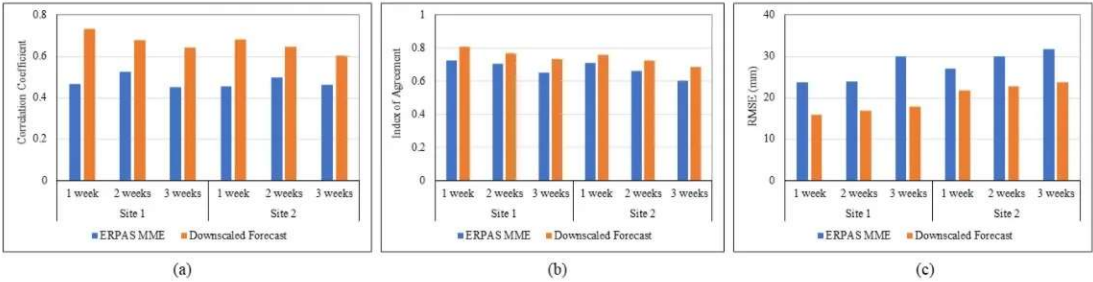
$$P(R_t \mid WS_1^T, R_1^{t-1}, X_1^T) = P(R_t \mid WS_t)$$

$$P(WS_t \mid WS_1^{t-1}, X_1^T) = P(WS_t \mid WS_{t-1}, X_t)$$

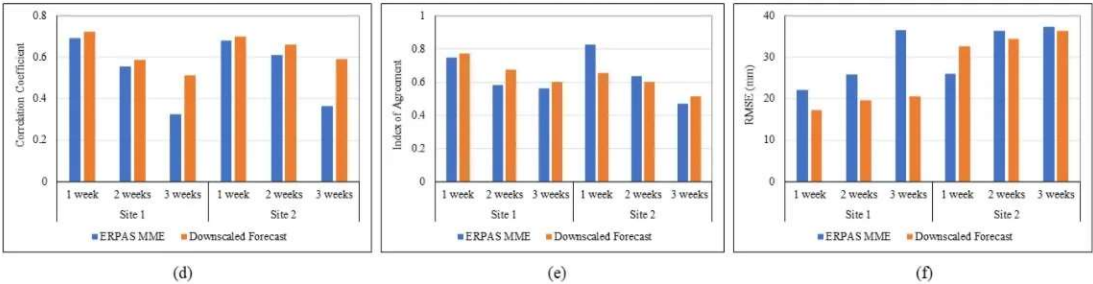
The first assumption states that the current day's rainfall is only dependent on current day's weather state, and second assumption states that the weather state on t -th day is conditional on previous day's weather state and current day's external atmospheric predictors

We have generated 1000 ensembles of rainfall given an extended range prediction for a period of 3 weeks. The same optimization framework is attached with this weather generator based rainfall prediction (1000 ensembles)

Improvement in Prediction Performance after Downscaling in Training Period (2010 – 2016)



Improvement in Prediction Performance after Downscaling in Testing Period (2017 – 2019)



SEASONAL SOIL MOISTURE VARIATION WITH PROPOSED FRAMEWORK

The soil moisture profiles with the irrigation scheduling at extended range are shown in the figure below for both site locations (dark blue line). Results for 1-, 2- and 3-weeks lead time are shown separately for Rabi and Kharif season during 2015-2016.

The generate soil moisture profile obtained using farmer's applied irrigation and observed rainfall are also plotted alongside for comparison (light blue line).

The solid black line represents the threshold soil moisture level (s^*) below which crop ET will start decreasing.

[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638351747/agu-fm2021/B4-79-7D-E3-DC-D7-F1-12-72-DC-59-00-A2-11-CF-FF/Image/ezgif-6-65e1f3762591_y5q4me.mp4

With decrease in Reliability Factor, the water savings is increased, while the instances of crossing down the threshold soil moisture level for ensuring water stress, is increased.

The water savings decreased with increase in lead time.

In non-monsoon days (Rabi season), more soil moisture threshold crossing down events are noticed, even with higher α values.

For short-to-Medium Range, the soil moisture profiles with the 3 different cases mentioned, are shown in the figure below. The plots are shown for Reliability Factor of 0.95 for both the Sites in figures below.

[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638366580/agu-fm2021/B4-79-7D-E3-DC-D7-F1-12-72-DC-59-00-A2-11-CF-FF/Image/Site1_3Cases_SMVariation_95_vepaor.mp4

Seasonal Soil Moisture Variation for Site 1 with 3 Cases of Irrigation with RF 0.95

(<https://drive.google.com/file/d/1eyn7K2bhBDBQrqGXbXfTTespIXdR0z4y/view?usp=sharing>)

[VIDEO] https://res.cloudinary.com/amuze-interactive/image/upload/f_auto,q_auto/v1638366600/agu-fm2021/B4-79-7D-E3-DC-D7-F1-12-72-DC-59-00-A2-11-CF-FF/Image/Site2_3Cases_SMVariation_95_lelfot.mp4

Seasonal Soil Moisture Variation for Site 2 with 3 Cases of Irrigation with RF 0.95

(https://drive.google.com/file/d/1GrqQ_5Z43yohBlTqquVujLeoihLYC3_y/view?usp=sharing)

WATER USE OPTIMIZATION AND IMPACT ON RELATIVE YIELD

- With an increase in the interval between two consecutive irrigation applications, the water use is increased (becoming less conservative), for both Kharif and Rabi seasons.
- With different reliability factor (α) values, irrigation water application was varied in a consistent pattern. In general, it was observed that in the season having almost no rainfall, irrigation requirement is higher as expected.
- We found that at Site 1, for Case A, during the Rabi season, the seasonal average irrigation amount was reduced by 10%–37%, and during the Kharif season, the amount of water savings increased to 27%–46%, depending on the value of α , compared to the farmer's adopted method of irrigation.
- For Case B, the water savings achieved are 4%–28% and 8%–30% during the Rabi and Kharif seasons, respectively.
- Similarly, for Case C, on an average, –14% to 13% and 1%–22% of irrigation water optimization were possible for the Rabi and Kharif seasons, respectively.

By analyzing the Yield Loss (YL) and change in Relative Yield (RY) defined by FAO-I&D Paper No. 66 (2012), we observed that in most cases, yield can be maintained with no water stress condition, with the proposed optimized irrigation scheme.

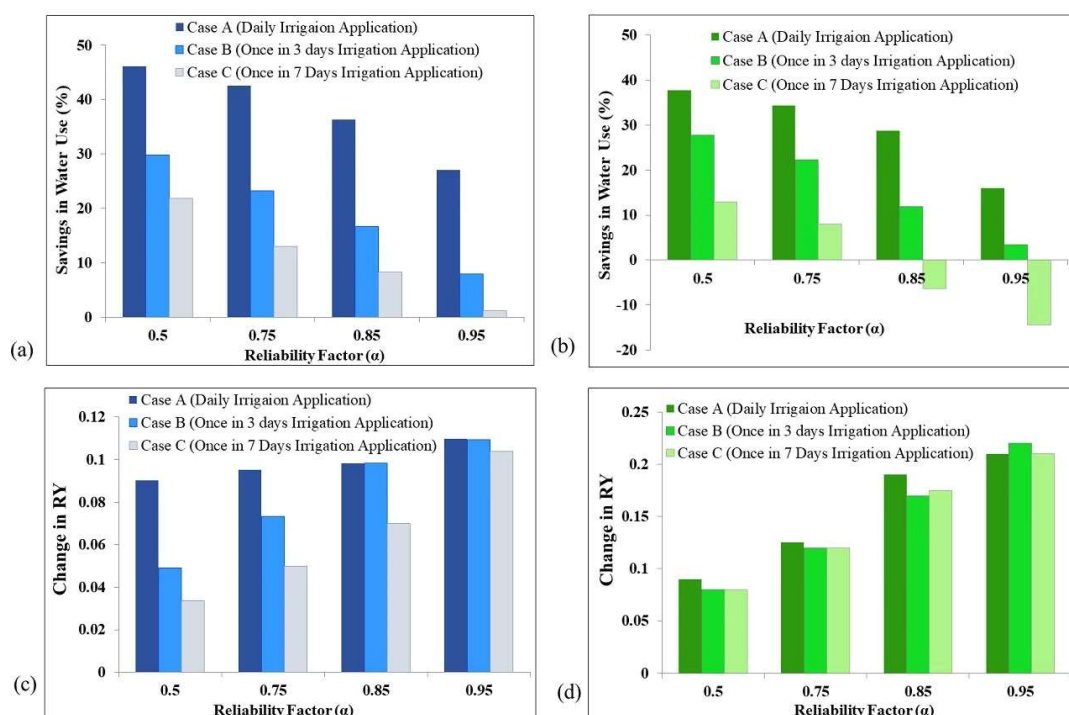
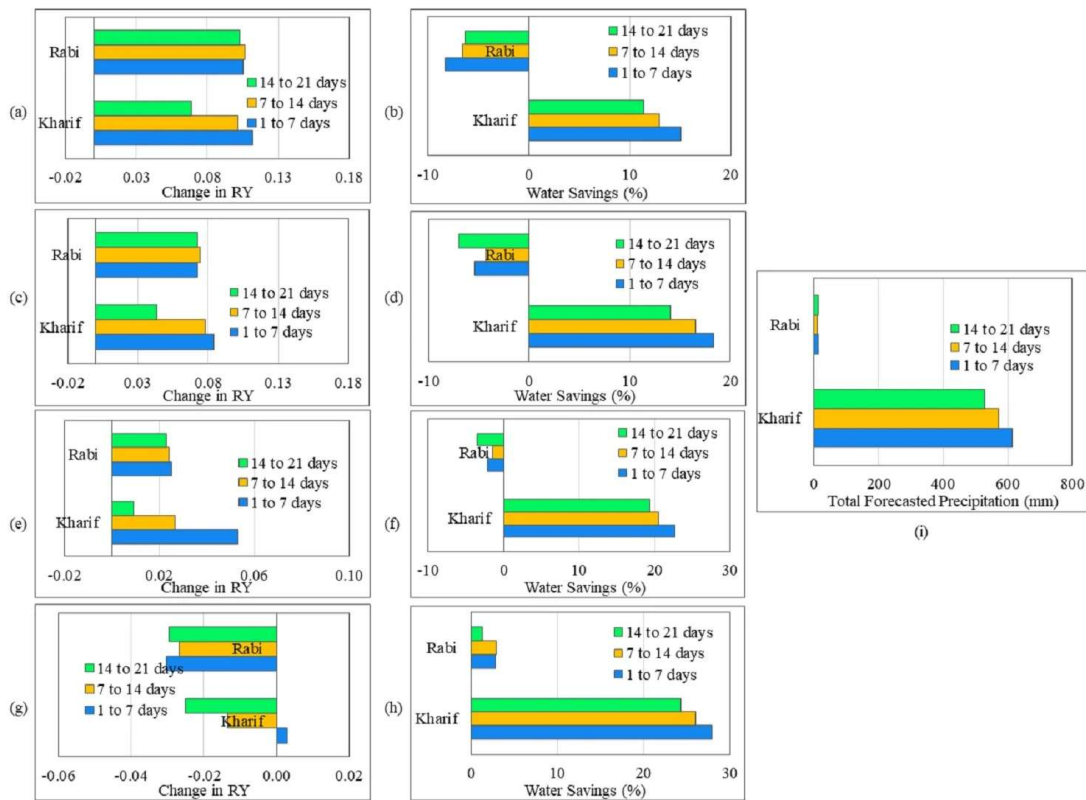


Figure 9. Change in Minimization of Irrigation Water Use for varied Reliability Factor during (a) Kharif Season and (b) Rabi Season. Change in Relative Yield for varied Reliability Factor during (c) Kharif Season and (d) Rabi Seasons. All the 3 Cases of Irrigation (Case A, B and C) are presented

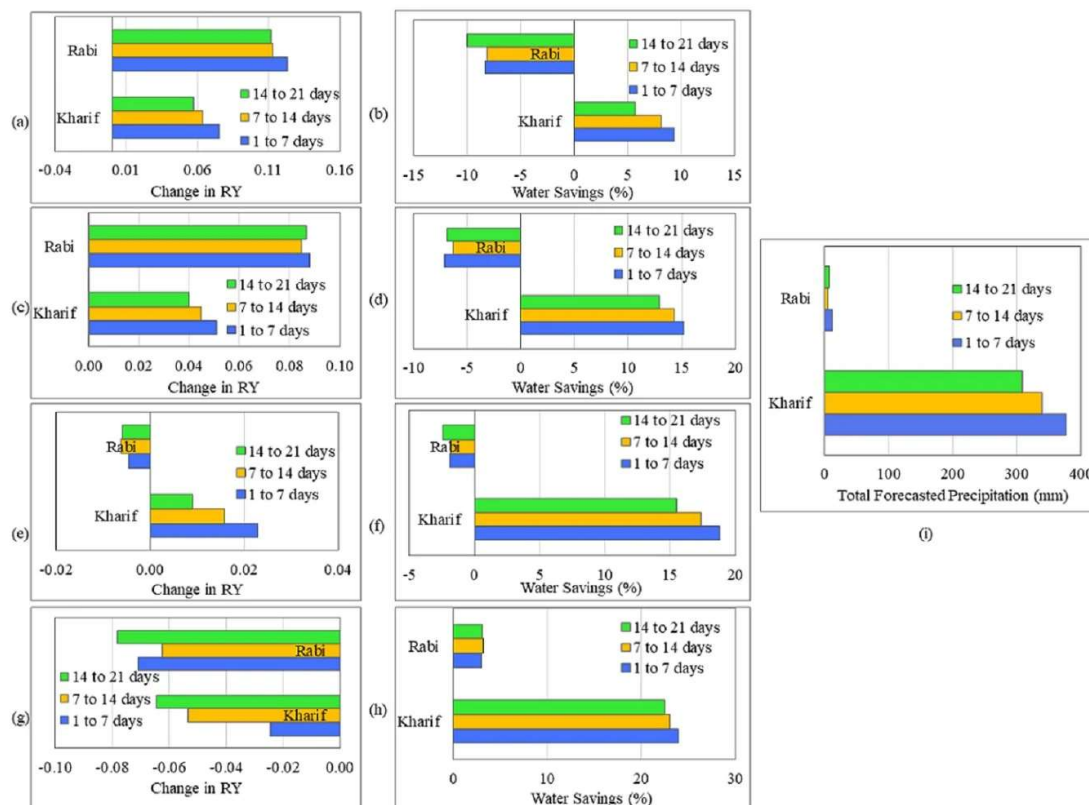
- For implementation in extended range in Site 1
 - The water savings for Kharif seasons at Site 1 range from 15.1% to 27.9% for 1 week lead time forecasts with α varying from 0.95 to 0.5 (when averaged for the 3 seasons); the same for 2 weeks and 3 weeks lead time forecasts lie in the range of 12.9% to 25.9% and 11.3% to 24.3%, respectively.
 - For Rabi seasons, the seasonal total water savings ranges are -8.3% to 1.8%, -6.6% to 3%, and -6.3% to 1.3% for 1-, 2-, and 3-weeks lead times, respectively.
- For Site 2 the extent of water use optimization obtained by the proposed framework is almost similar to that at Site 1

- o Range of seasonal total water savings for Kharif seasons being 9.3% to 23.9%, 8.1% to 23%, and 5.7% to 22.5%
- o For Rabi seasons being -8.4% to 3%, -8.2% to 3.3% and -9.9% to 3.2%, with the three different forecast lead times mentioned.



Water Savings and Change in Relative Yield in Site 1 with 1-, 2- and 3-weeks lead time for different α values

[(a) & (b) : $\alpha=0.95$; (c) & (d) : $\alpha=0.85$; (e) & (f): $\alpha=0.75$; (g) & (h): $\alpha=0.5$]



Water Savings and Change in Relative Yield in Site 2 with 1-, 2- and 3-weeks lead time for different α values

[(a) & (b) : $\alpha=0.95$; (c) & (d) : $\alpha=0.85$; (e) & (f): $\alpha=0.75$; (g) & (h): $\alpha=0.5$]

- With ERPAS forecasts for intraseasonal variations of the rainfall events, water use in irrigation could be optimized better in Kharif seasons than in Rabi seasons.
- The proposed approach results in excess water usage during the Rabi season, specifically when the forecast is 3 weeks in advance (irrigation applied after 2 weeks and sustained for the 3rd week).
- Performance of the simulation-optimization approach is not significantly inferior for 3 weeks lead time compared to 1 week lead time. The conditional ensemble approach developed in the algorithm with a skillful ERPAS makes the irrigation management system beneficial at a longer lead time.
- In addition, the approach allows the farmers to make the water requirement planning 3 weeks in advance. After making the arrangements, the farmers may use the water depending on real-time soil moisture and weather scale forecasts (Roy et al., 2021).

The combination of both will result in efficient water arrangements as well savings.

This is consistent with the Ready-Set-Go framework envisioned under the S2S project in terms of combining forecasts at different timescales for the best outcomes in various sectors ("S2S Prediction", 2019).

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ABSTRACT

Optimization in irrigation scheduling using weather forecast has been proven to achieve better productivity along with reduced irrigation water requirements. We developed a farm-scale hydrological model coupled with a chance-constraint optimization to take short to medium range weather forecast and prescribe the optimal irrigation amount determined by developing the conditional probability density functions of the rainfall and subsequently the soil moisture for the days in forecast range. The stress-avoidance was ensured by maintaining the probability of crops undergoing water stress is less than a prescribed threshold (reliability factor, α). The framework was implemented for irrigation decision simulation at extended range by downscaling the forecast with Nonhomogeneous Hidden Markov Model (NHMM) as an input and produce irrigation decision in extended range (15 to 30 days). The optimization framework ensured minimal water use without significant crop water stress. The method was tested at two site locations in Nashik district in the state of Maharashtra, both being involved in grape cultivation (referred herein as Site 1 and Site 2). In short-to-medium range weather scale, the model was implemented with varied α (0.5 to 0.95) and interval between two subsequent irrigation application (1, 3 and 7 days) and significant amount of water savings with respect to the farmer's applied irrigation could be achieved. The simulation-optimization framework was only tested with $\alpha=0.95$ and once in 7 days irrigation application for extended range, and yet no significant detrimental effect on yield was observed whereas in kharif season significant potential of water savings was observed both in Site 1 and 2. While the framework in short to medium range is useful for optimal real time irrigation decision making, in the extended range, it can be implemented in planning of irrigation for the upcoming month to avoid the inconvenience of instant arrangement of water, especially in case of drought-hit regions. Considering that irrigation accounts for over 80% of the total water use worldwide, the value of such an approach as a decision-support tool for irrigation optimization is self-evident.

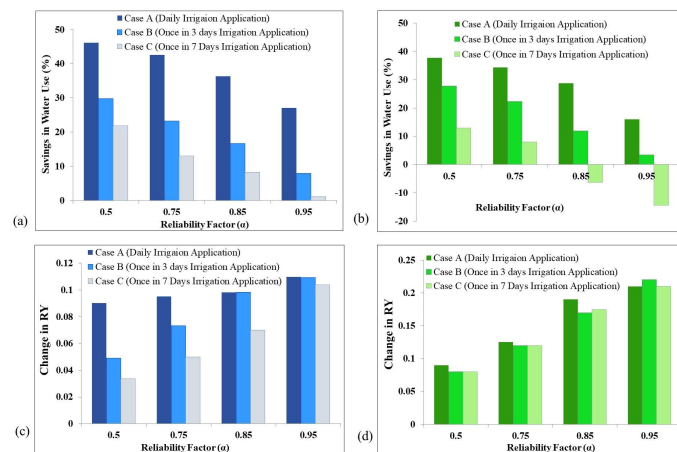


Figure 1. Change in Minimization of Irrigation Water Use for varied Reliability Factor during (a) Kharif Season and (b) Rabi Season. Change in Relative Yield for varied Reliability Factor during (c) Kharif Season and (d) Rabi Seasons. The results are for Short to Medium range

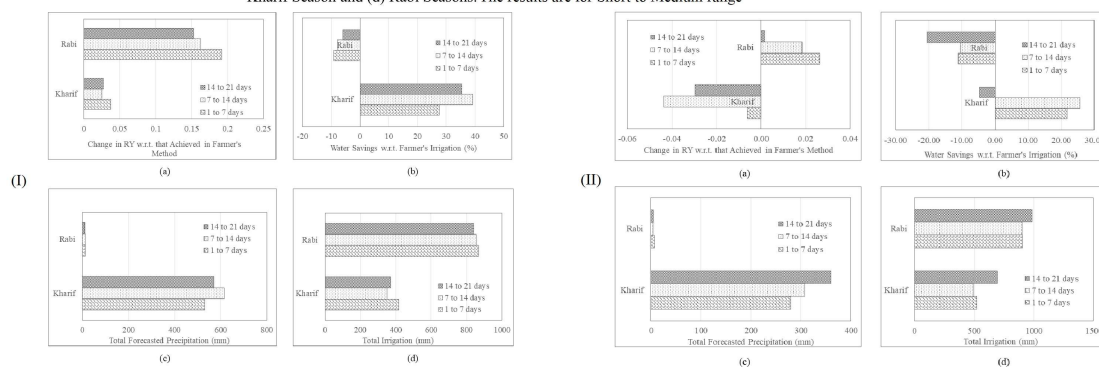


Figure 2. Plots of (a) Change in RY and (b) Savings in irrigation water use (%) w.r.t. the farmer's method of irrigation scheduling, (c) Total seasonal precipitation (mm) and (d) Total seasonal irrigation amount (mm) scheduled using forecast for (t+1)th to (t+7)th day, (t+8)th to (t+14)th day and (t+15)th to (t+21)th day. The values are obtained by taking the average of 3 consecutive Kharif (10th Apr to 08th Oct) and Rabi (10th Oct-08th Apr) seasons each, for (I) Site 1 and (II) Site 2.

(https://agu.confex.com/data/abstract/agu/fm21/0/6/Paper_796560_abstract_751707_0.jpg)

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