

# ASHLEY: A new empirical model for the high-latitude electron precipitation and electric field

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## Abstract

In this study, a new high-latitude empirical model is introduced, named for Auroral energy Spectrum and High-Latitude Electric field variability (ASHLEY). This model aims to improve specifications of soft electron precipitations and electric field variability that are not well represented in existing high-latitude empirical models. ASHLEY consists of three components, ASHLEY-A, ASHLEY-E and ASHLEY-Evar, which are developed based on the electron precipitation and bulk ion drift measurements from the Defense Meteorological Satellite Program (DMSP) satellites during the most recent solar cycle. On the one hand, unlike most existing high-latitude electron precipitation models, which have assumptions about the energy spectrum of incident electrons, the electron precipitation component of ASHLEY, ASHLEY-A, provides the differential energy fluxes in the 19 DMSP energy channels under different geophysical conditions without making any assumptions about the energy spectrum. It has been found that the relaxation of spectral assumptions significantly improves soft electron precipitation specifications with respect to a Maxwellian spectrum (up to several orders of magnitude). On the other hand, ASHLEY provides consistent mean electric field and electric field variability under different geophysical conditions by ASHLEY-E and ASHLEY-Evar components, respectively. This is different from most existing electric field models which only focus on the large-scale mean electric field and ignore the electric field variability. Furthermore, the consistency between the electric field and electron precipitation is better taken into account in ASHLEY.

**ASHLEY: A new empirical model for the high-latitude electron precipitation and electric field**

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**Key points:**

- ASHLEY better considers the consistency between the electric field and electron precipitation than existing models.
- ASHLEY better incorporates IMF  $B_y$  polarity impacts on the electron precipitation and improves soft electron precipitation specifications.
- ASHLEY provides consistent mean electric field and electric field variability.

**Abstract:**

In this study, a new high-latitude empirical model is introduced, named for Auroral energy Spectrum and High-Latitude Electric field variability (ASHLEY). This model aims to improve specifications of soft electron precipitations and electric field variability that are not well represented in existing high-latitude empirical models. ASHLEY consists of three components, ASHLEY-A, ASHLEY-E and ASHLEY-Evar, which are developed based on the electron precipitation and bulk ion drift measurements from the Defense Meteorological Satellite Program (DMSP) satellites during the most recent solar cycle. On the one hand, unlike most existing high-latitude electron precipitation models, which have assumptions about the energy spectrum of incident electrons, the electron precipitation component of ASHLEY, ASHLEY-A, provides the differential energy fluxes in the 19 DMSP energy channels under different geophysical conditions without making any assumptions about the energy spectrum. It has been found that the relaxation of spectral assumptions significantly improves soft electron precipitation specifications with respect to a Maxwellian spectrum (up to several orders of magnitude). On the other hand, ASHLEY provides consistent mean electric field and electric field variability under different geophysical conditions by ASHLEY-E and ASHLEY-Evar components, respectively. This is different from most existing electric field models which only focus on the large-scale mean electric field and ignore the electric field variability. Furthermore, the consistency between the electric field and electron precipitation is better taken into account in ASHLEY.

## 1. Introduction

Earth's ionosphere and thermosphere (I-T) system is closely coupled with the magnetosphere, and the electromagnetic energy from magnetosphere is transferred into the I-T system through field-aligned currents (FACs). The major part of electromagnetic energy is irreversibly converted into heat through ohmic currents, and such heat is called Joule heating (Cole 1962; Thayer, 2000; Richmond, 2020). Joule heating can significantly affect the I-T system both locally and globally especially during geomagnetic storms. For example, the neutral temperature and density increase due to the enhanced Joule heating during geomagnetic storms (e.g., Fuller-Rowell et al., 1994). In addition, Joule heating can effectively change the global circulation within several hours, which markedly alters the thermospheric compositions at different latitudes and can further change the ionospheric electron density (e.g., Buonsanto, 1999; Prölss, 2011). Moreover, gravity waves can be launched due to rapid variations of Joule heating and they can propagate globally, causing large-scale traveling atmospheric disturbances and traveling ionospheric disturbances (e.g., Lu et al., 2016, 2020). A comprehensive review of Joule heating and the I-T response to Joule heating during geomagnetic storms can be found in Richmond (2020).

General circulation models (GCMs) of the I-T system are widely used to study variations of the I-T system particularly during geomagnetic storms, and accurate estimations of Joule heating are critical for reproducing observed features. Joule heating in GCMs is calculated from the electric field, conductivities associated with the solar ionization and electron precipitation together with the neutral winds (e.g., Lu et al., 1995). However, accurate estimations of Joule heating is still challenging to date since it is difficult to capture the dynamic variations of the electric field, ionospheric conductivity (mostly associated with the electron precipitation) and neutral winds (e.g., Pedatella et al., 2018; Liemohn, 2020; Billet et al., 2018). In this paper, we



focus on the improvements of the electric field and electron precipitation in GCMs. Typically, empirical models of electric field (e.g., Weimer, 2005; Heelis, 1982) and auroral electron precipitation (e.g., Fuller-Rowell and Evans, 1987; Roble and Ridley, 1987; Newell et al., 2009) are used to specify the high-latitude electric field and electron precipitation in GCMs, respectively. Alternatively, high-latitude electric field and electron precipitation patterns derived from data assimilation techniques, such as the Assimilative Mapping Ionospheric Electrodynamics (AMIE) procedure (Richmond and Kamide, 1988; Richmond, 1992), can be used. However, empirical models of electric field, electron precipitation and ionospheric conductance are still needed in those assimilative techniques as background models. The following deficiencies of the existing empirical models for high-latitude electrodynamical forcings may contribute to the inaccurate Joule heating estimations:

1) Empirical models are good at capturing large-scale patterns under certain geophysical conditions, but they may not well represent the electric field and electron precipitation patterns at a specific time. In other words, the electric field and electron precipitation variabilities are not well captured by empirical models. It has been shown that the magnitude of the electric field variability is comparable with the magnitude of the large-scale mean electric field, so the electric field variability can substantially contribute to Joule heating (e.g., Codrescu et al., 1995, 2000, 2008; Emery et al., 1999; Crowley & Hackert, 2001; Matsuo et al., 2003; Matsuo and Richmond, 2008; Cosgrove and Thayer, 2006; Fuller-Rowell et al., 2000; Rodger et al., 2001; Deng et al., 2009; Fedrizzi et al., 2012). Therefore, an electric field variability model providing the variability not captured by the large-scale mean electric field model may be needed to improve Joule heating estimations in GCMs. Moreover, it is worth noting that the large-scale mean electric field and electric field variability models need to be developed consistently, otherwise the actual

contribution of the electric field variability to Joule heating may not be well represented. Furthermore, it is also worthwhile modeling the electric field and electron precipitation variabilities consistently to improve Joule heating estimations in GCMs (e.g., Cosgrove and Codrescu, 2009; Cosgrove et al., 2011; Zhu et al., 2018; Burleigh et al., 2019).

2) Even though the electric field and electron precipitation variabilities are captured, the I-T system variations (especially in the F region) may still be imprecisely estimated. This may result from inaccurate altitudinal ionospheric conductivity profiles so that the altitudinal Joule heating distributions is incorrectly estimated in GCMs (Deng et al., 2008). It has been found that the neutral density and temperature at F region altitudes are more sensitive to the Joule heating deposited in the F-region than that deposited in lower altitudes (e.g., Deng et al., 2011; Huang et al., 2012) especially on a short time scale (<0.5-1 day). The F-region conductivity and Joule heating can be significantly underestimated owing to the underestimation of soft (<1 keV) electron precipitations which are important ionization sources of the thermosphere at the F-region altitudes (Rees, 1989). However, most existing auroral electron precipitation models typically only provide the total energy flux together with the average energy (or total number flux) and assume that the energy spectrum of incident electrons has a certain shape (e.g., Fuller-Rowell and Evans, 1987; Y. Zhang and Paxton, 2008; Newell et al., 2009), which could lead to inaccurate estimations of soft electron precipitations. For example, a Maxwellian spectrum is typically assumed because the estimated ionospheric conductance based on such assumption compares well with that calculated using measured ionospheric and thermospheric parameters (e.g., Vickrey et al., 1981; Robinson et al., 1987). Nevertheless, it was found that a Maxwellian spectrum may significantly underestimate the soft electron precipitation when comparing with the energy spectrum from measurements, sometimes by orders of magnitude (e.g., McIntosh and

P. Anderson, 2014; Wing et al., 2019). Although additional types of energy spectra different from a Maxwellian spectrum have been included in recently developed electron precipitation models (e.g., Newell et al., 2009, 2014; B. Zhang et al. 2015), soft electron precipitations may still be underestimated owing to deficient precipitation spectral identification techniques (Wing et al., 2019) and incomplete inclusion of soft electron precipitations from different sources (Khazanov and Glozer, 2020). Therefore, to better specify the altitudinal distribution of Joule heating in GCMs and improve the GCM accuracy, it is critical to develop a new electron precipitation model that can better specify the soft electron precipitations.

In this paper, a new empirical model aimed at improving the specifications of Auroral energy Spectrum and High-Latitude Electric field variability, ASHLEY, is introduced. ASHLEY is developed based on the electron precipitation and bulk ion drift measurements from the Defense Meteorological Satellite Program (DMSP) satellites. ASHLEY consists of three components: 1) an auroral electron precipitation component, ASHLEY-A, that provides the differential energy fluxes of incident electrons in the 19 DMSP energy channels without making any assumptions about the energy spectrum; 2) a high-latitude electric potential component, ASHLEY-E, that specifies the large-scale mean electric field; 3) an electric field variability component, ASHLEY-Evar, that quantifies the electric field variability not captured by ASHLEY-E. The remaining part of this paper is organized as follows: Section 2 provides an overview of the datasets used for the ASHLEY development and data processing procedures. The methodology used for the development of ASHLEY is illustrated in Section 3. Section 4 provides statistical comparisons of model to data, and Section 5 presents the outputs of ASHLEY. Section 6 discusses similarities and differences between ASHLEY and models developed in previous studies along with the directions for future improvements. The main conclusions are summarized in Section 7. More

details about fitting procedures and model reconstructions discussed in Section 3 are given in the Appendix.

## **2. Data preparation**

### **2.1 DMSP measurements**

#### **2.1.1 Electron precipitation**

The in-situ auroral electron precipitation measurements from the DMSP F16-F18 satellites during 2010-2015 are used in this study. All three satellites flew in circular Sun-synchronous orbits at an altitude of  $\sim 840$  km with an inclination of  $\sim 98.8^\circ$ . The measurements were taken by the onboard Special Sensor for Precipitating Particles, version 5 (SSJ/5), which measures incident electrons and ions from 30 eV and 30 keV every second using 19 logarithmically-spaced energy channels (Hardy et al., 2008; Redmon et al., 2017). The field of view of the SSJ/5 is a  $4^\circ$  by  $90^\circ$  fan ranging from the zenith to the horizon and the  $90^\circ$  field of view is divided into six  $15^\circ$  zones. In this study, we will focus on the electron precipitation and particularly the differential energy fluxes in 19 energy channels. The differential energy flux data are acquired from the dataset created by Redmon et al. (2017) and details about the dataset can be found in that paper. Overall, there are  $>10^5$  polar crossings ( $|\text{MLAT}| > 45^\circ$  segments of trajectories; MLAT=magnetic latitude) with good data quality used in this study, and the number of polar crossings from the Northern Hemisphere (NH) and Southern Hemisphere (SH) are roughly comparable (NH: 53348; SH: 52670).

#### **2.1.2 Bulk Ion drift**

The Special Sensor for Ions, Electrons and Scintillation (SSIIES) onboard the DMSP satellite measures the full bulk ion drift vector ( $\mathbf{V}$ ) in the spacecraft coordinate system (i.e.,  $\mathbf{V} =$

159  $V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$ , where  $\hat{x}$  is along the satellite trajectory,  $\hat{z}$  is outward of the center of the  
 160 Earth and  $\hat{y}$  completes righthanded system;  $V_x$ ,  $V_y$  and  $V_z$  are the components in directions  
 161 corresponding to their subscripts, respectively). In this study, bulk ion drift measurements from  
 162 the DMSP F15-F18 satellites during 2010-2018 are used. DMSP F16-F18 carry the latest version  
 163 of the SSIES (version 3) with a 1-s temporal resolution, whereas the DMSP F15 carries the  
 164 previous version of the SSIES (version 2) with a 4-s temporal resolution. Despite using different  
 165 versions of the SSIES, it is found that there are no significant deviations in the statistical electric  
 166 potential and electric field results in regions where all satellite flew by (not shown). In addition,  
 167 since the DMSP F15 data improve the data coverage at noon and midnight, DMSP F15 bulk ion  
 168 drift measurements are included in the dataset.

169 After removing the spacecraft velocity with respect to an Earth-centered corotating reference  
 170 frame, the residual ion drift vector has been used for the derivation of the electric potential and  
 171 electric field. Because the SSIES is sensitive to the background  $O^+$  density concentration, the  
 172 measurements are generally in poor quality when the ionospheric  $O^+$  density is low or other ion  
 173 species (such as  $H^+$ ) are dominant. In this study, only data measured when the background  $O^+$   
 174 concentration and density are relatively high (concentration:  $>90\%$ ; density:  $>4 \times 10^3 \text{ cm}^{-3}$ ) and  
 175 with the best quality flag (flag = 1) are used. If a polar crossing has many unavailable data (i.e.,  
 176 large data gap) or significant baseline issue, that polar crossing will be excluded in the final  
 177 dataset. Overall, more than half of the polar crossings in the original dataset are discarded  
 178 particularly in the local winter. The remaining dataset has more polar crossings from the northern  
 179 hemisphere than the southern hemisphere (NH: 51126; SH: 29602).

180 To calculate the electric field and electric potential, linear baseline corrections of  $V_x$ ,  $V_y$  and  
 181  $V_z$  components are applied to ensure they are zero at both ends of each polar crossing (i.e.,

182  $|\text{MLAT}|=45^\circ$  in this study). Since the  $V_x$  component is generally noisier than other components,  
 183 the standard deviations of the  $V_x$  data measured in the first and last minute of each polar crossing  
 184 are calculated prior to the baseline correction to ensure the reliability of the baseline. If both  
 185 standard deviations are smaller than 100 m/s, the  $V_x$  data are baseline corrected and included in  
 186 the dataset. Otherwise, the  $V_x$  data along that polar crossing are discarded and the electric field  
 187 vector along that track is not calculated. In addition, only the large-scale  $V_x$  data (smoothed by a  
 188 70-s sliding window) are utilized to avoid introducing unreliable small-scale and mesoscale  
 189 structures in the  $V_x$  data. If all components of the bulk ion drift vector after the baseline  
 190 correction ( $\mathbf{V}'$ ) are available, the electric field vector ( $\mathbf{E}$ ) is calculated through  $\mathbf{E} = -\mathbf{V}' \times \mathbf{B}_0$ .  
 191 Here,  $\mathbf{B}_0$  is the background geomagnetic main field vector at the satellite location from the  
 192 International Geomagnetic Reference Field-12 (IGRF-12) model (Thébault et al., 2015). The  
 193 electric field vector is then decomposed into the magnetic eastward ( $E_{d1}$ ) and equatorward ( $E_{d2}$ )  
 194 components as defined in the modified apex coordinate system using a reference height of 110  
 195 km (Richmond, 1995). More details associated with the modified apex coordinates and the  
 196 decomposition procedure can be found in Richmond (1995) and Laundal and Richmond (2017).  
 197 The electric potential is calculated following a similar procedure used in Zhu et al. (2020a):  
 198 The first step is to calculate the along-track electric field  $\mathbf{E}_x$  ( $\mathbf{E}_x = E_x \hat{\mathbf{x}}$ ), which can be  
 199 approximated through  $E_x \approx -V_y' B_{0z}$ . Here,  $V_y'$  is the horizontal cross-track ion drift vector after  
 200 applying the baseline correction and  $B_{0z}$  is the vertical component of the  $\mathbf{B}_0$  at the satellite  
 201 location. The contribution of the vertical ion drift to  $E_x$  is generally small and is therefore  
 202 neglected in our calculation. The next step is to integrate the along-track electric field to  
 203 determine the electric potential along that pass. The subsequent step is to correct the calculated

electric potential to ensure its values are zero at both ends of each polar crossing. Details of the electric potential calculation can be found in Zhu et al. (2020a).

## 2.2 IMF and solar wind data

In this study, the interplanetary magnetic field (IMF) y and z ( $B_y$  and  $B_z$ ) components in the Geocentric-Solar-Magnetospheric (GSM) coordinates are used (Note that the subscripts y and z have different meanings than those in the previous subsection). Two parameters are further calculated in this study: 1) the IMF transverse component magnitude,  $B_T$ , which represents the strength of the IMF projection onto the GSM Y-Z plane, i.e.,  $B_T = \sqrt{B_y^2 + B_z^2}$ ; 2) IMF clock angle ( $\theta_c$ ), which stands for the angle between GSM north and the IMF projection onto the GSM Y-Z plane and is given by  $\theta_c = \text{atan2}(B_y, B_z)$ . Note that a mirror correction (i.e.,  $\theta'_c = 360^\circ - \theta_c$ ) has been applied for SH polar crossings in order to take the different impacts of the IMF  $B_y$  polarity on the high-latitude electrodynamic forcings in different hemispheres into account. In addition, the solar wind flow speed ( $V_{sw}$ ) and solar wind proton density ( $N_{sw}$ ) are used. The IMF and solar wind data used in this study are 5-min averaged data from the NASA/GSFC's OMNI data set through OMNIWeb. Similar to Zhu et al. (2020a), a 30-min propagation time delay is applied to account for the traveling time from the bow shock to the ionosphere.

## 2.3 DMSP data categorization

Each DMSP polar crossing is categorized according to two parameters,  $\varepsilon_t$  and  $\theta_c$ , where:

$$\varepsilon_t = V_{sw}^{4/3} B_T^{2/3} N_{sw}^{1/6} \quad (1)$$

$\varepsilon_t$  (in the unit of  $(\text{km})^{4/3} (\text{s})^{-4/3} (\text{nT})^{2/3} (\text{cm})^{-1/2}$ ) is essentially the combination of  $B_T$  and  $V_{sw}$  terms in the Newell coupling function (Newell et al., 2007) multiplied by  $N_{sw}^{1/6}$ . As discussed in Newell et al. (2007), the term  $N_{sw}^{1/6}$  appeared in their derivation of the coupling function, but was

omitted on purpose to achieve better correlations with other parameters tested in their study. However, they found that including the term  $N_{SW}^{1/6}$  can slightly improve the correlation with the auroral power. Meanwhile, Newell and Meng (1994) suggested that the soft electron precipitation may depend on  $N_{SW}$ , so that the term  $N_{SW}^{1/6}$  was kept in the expression of  $\varepsilon_t$  since the soft electron precipitation is one major focus of this study. Moreover, the  $\sin^{\frac{8}{3}}(\frac{\theta_c}{2})$  term originally in the Newell coupling function is omitted in the expression of  $\varepsilon_t$  since the  $\sin^{\frac{8}{3}}(\frac{\theta_c}{2})$  term cannot well distinguish positive and negative IMF  $B_y$  cases. Instead, Fourier fitting will be performed to capture the IMF clock angle dependences of the electron precipitation, electric potential and electric field variability.

The averaged  $\varepsilon_t$  and  $\theta_c$  of a DMSP polar crossing are used to represent the IMF and solar wind conditions corresponding to that polar crossing (the typical averaging period is about 20 minutes). If the IMF or solar wind data are missing, the corresponding polar crossing is excluded. Moreover, polar crossings for which the standard deviation of  $\varepsilon_t$  is greater than 15% of the average of  $\varepsilon_t$  of that polar crossing or the standard deviation of  $\theta_c$  is greater than  $22.5^\circ$  are also excluded. This procedure removes polar crossings during which the IMF or solar wind data have large variations. We found about 30% polar crossings are excluded due to missing or unsteady IMF/solar wind IMF data. Distributions of the IMF and solar wind data used for ASHLEY-A and ASHLEY-E/ASHLEY-Evar developments are shown in Figures S1 and S2, respectively.

For polar crossings with good electron precipitation data, all polar crossings for which  $\varepsilon_t$  is smaller than 3,000 (roughly corresponds to the IMF  $B_T < 1$  nT case under normal solar wind conditions) are sorted into one category regardless of  $\theta_c$ . Other polar crossings for which



3,000 <  $\varepsilon_t$  < 30,000 are sorted into 8  $\varepsilon_t$  bins and 8  $\theta_c$  bins (i.e.,  $8 \times 8 + 1 = 65$  categories in total). The 360° span of  $\theta_c$  is evenly divided into 8 bins with each centered at a multiple of 45°. An upper boundary of 30,000 (roughly corresponds to the IMF  $B_T = 22$  nT case under normal solar wind conditions) is set for  $\varepsilon_t$  to exclude a small amount of polar crossings (~1%) under very strong IMF and solar wind conditions.

Likewise, for polar crossings with good electric field/potential data, all polar crossings for which  $\varepsilon_t$  is smaller than 3000 are categorized as one category regardless of  $\theta_c$ , and other polar crossings with 3,000 <  $\varepsilon_t$  < 24,000 (roughly corresponds to the IMF  $B_T = 17$  nT case under normal solar wind conditions) are sorted into 6  $\varepsilon_t$  bins and 8  $\theta_c$  bins (i.e.,  $6 \times 8 + 1 = 49$  categories in total). Fewer  $\varepsilon_t$  bins and smaller upper boundary of  $\varepsilon_t$  than those set to sort the electron precipitation data are primarily due to the smaller amount of polar crossings with good electric field/potential data. Tables 1a and 1b summarizes the lower and upper boundaries along with the median values of different  $\varepsilon_t$  bins used to sort the electron precipitation and electric field/potential data, respectively. Note that polar crossings from both hemispheres are combined together to achieve best magnetic local time (MLT) coverage since the MLT coverage is limited in a single hemisphere. In addition, polar crossings from all seasons are combined in this study to have good data coverage for the distinct parameter bins in order to achieve statistically meaningful results. In the future, the seasonal dependence will be added in the models if more data become available.

### **3. Model development**

#### **3.1 Fitting**

##### **3.1.1 Electron precipitation data**

For each  $\varepsilon_t$ - $\theta_c$  category, the differential energy flux ( $J_E$ ) in each energy channel (19 energy channels in total) and above  $50^\circ$  MLAT are binned according to their MLTs and magnetic latitudes (MLATs). The sizes of the MLT and MLAT bins are 1 hour and  $1^\circ$ , respectively. If a bin has more than 100 data points, the average of the differential energy flux is calculated. Otherwise, the linear interpolation value based on the averages of the closest two MLT bins is used to deduce the average of that bin. The distributions of the average differential energy flux pattern are further smoothed in MLT and MLAT directions afterwards by using moving average smoothing.

With the smoothed average differential energy flux pattern in each energy channel, the next steps are to capture the MLT and IMF clock angle dependences of the differential energy flux in each latitudinal bin by using Fourier fitting. First, the differential energy flux in each bin of MLAT,  $\varepsilon_t$ , and  $\theta_c$  is fitted to a Fourier series constructed by  $\phi$  ( $\phi = \frac{\text{MLT}}{12}\pi$ ). After the MLT fitting, the MLT Fourier coefficients from eight  $\theta_c$  bins in each bin of MLAT and  $\varepsilon_t$  (except for the lowest  $\varepsilon_t$  bin) are then fitted to a Fourier series constructed by  $\omega$  ( $\omega = \frac{\theta_c}{180^\circ}\pi$ ) to capture the IMF clock angle variation. The MLT and IMF clock angle fittings are detailed in Appendix A1.

### 3.1.2 Electric potential and electric field data

The electrostatic potential ( $\Phi$ ) can be expanded in terms of spherical harmonics in a spherical coordinate system (Jackson, 2007). Following the approach shown in Weimer (1995), if only working with the real part of the spherical harmonics,  $\Phi$  can be expressed as:

$$\Phi(\theta, \phi) = \sum_{l=0}^{12} F_{l0} P_l^0(\cos \theta) + \sum_{l=1}^{12} \sum_{m=1}^{\min(l,4)} (F_{lm} \cos m\phi + G_{lm} \sin m\phi) P_l^m(\cos \theta). \quad (2)$$

Here,  $P_l^m$  is the associated Legendre function,  $\theta$  is the polar angle converted from the MLAT  
 $(\theta = \frac{\pi - \lambda_m}{2} \pi, \text{ where } \lambda_m \text{ is the MLAT in radians})$  and  $\phi$  is the azimuthal angle which is the same  
as that defined in Section 3.1.1. The expansion is terminated at  $l=12$  and  $m=4$  to avoid unrealistic  
small-scale and mesoscale structures associated with higher order terms.

In addition to the electric potential data, the electric field ( $E_{d1}$  and  $E_{d2}$ ) data are also used in  
the fitting procedure to provide more constraints on the electric potential fitting. The  
relationships between  $\Phi$  and  $E_{d1}$  and between  $\Phi$  and  $E_{d2}$  can be found in Eqs. 4.8 and 4.9 in  
Richmond (1995), respectively. Details about the expansion of  $E_{d1}$  and  $E_{d2}$  in terms of the  
spherical harmonics can be found in Appendix A2. With all  $\Phi$ ,  $E_{d1}$  and  $E_{d2}$  data along with their  
locations in each  $\varepsilon_t$ - $\theta_c$  category,  $F_{lm}$  and  $G_{lm}$  can be obtained from a least-square fit. Details of  
the fitting procedure can be found in Appendix A2. Then  $F_{lm}$  and  $G_{lm}$  from eight  $\theta_c$  bins in each  
 $\varepsilon_t$  bin (except the lowest  $\varepsilon_t$  bin) are fitted to a Fourier series constructed by  $\omega$  to capture their  
IMF clock angle variations using the same procedures described in Section 3.1.1.

### 3.1.3 Residual electric field data

Once the ASHLEY-E component has been developed, the difference between the measured  
and modeled electric field along a polar crossing can be calculated. Each component of the  
residual electric field  $E'_{di}$  ( $i=1, 2$ ) above  $50^\circ$  MLAT are binned according to its MLT and MLAT  
for each  $\varepsilon_t$ - $\theta_c$  category. The sizes of the MLT and MLAT bins are 1 hour and  $2^\circ$ , respectively.  
Larger MLAT bin size than that used for the electron precipitation data binning is due to smaller  
amount of electric field data. The standard deviations of  $E'_{d1}$  and  $E'_{d2}$ , namely  $\sigma_1$  and  $\sigma_2$ , in each  
bin (if has  $>100$  data points) are calculated and are used to quantify the magnitudes of  $E_{d1}$  and  
 $E_{d2}$  variabilities.

After obtaining the preliminary MLAT-MLT distributions of  $\sigma_1$  and  $\sigma_2$  for each  $\varepsilon_t$ - $\theta_c$  category, the next step is to fill the data gaps in each MLAT bin by using the linear interpolation results based on the values in the adjacent two MLT bins. Then, the distributions of  $\sigma_1$  and  $\sigma_2$  are smoothed in both MLT and MLAT directions by using the sliding window smoothing. After that,  $\sigma_1$  and  $\sigma_2$  in each bin of MLAT,  $\varepsilon_t$  and  $\theta_c$  are fitted to a Fourier series constructed by MLT to capture their MLT variations. Then, the MLT Fourier coefficients from eight  $\theta_c$  bins in each bin of MLAT and  $\varepsilon_t$  (except for the lowest  $\varepsilon_t$  bin) are fitted to a Fourier series constructed by the IMF clock angle to capture the IMF clock angle variation. The MLT and IMF clock angle fitting procedures are the same as those described in Section 3.1.1, and details can be found in Appendix A1.

## **3.2 Extrapolation and expansion**

With the fitting procedures described in Section 3.1, the electron precipitation pattern can be reconstructed for any  $\varepsilon_t \leq 22770$  and any  $\theta_c$ , and the procedures are elaborated in Appendices A3. Similarly, the electric potential and electric field variability patterns can be reconstructed for any  $\varepsilon_t \leq 18357$  and any  $\theta_c$ , and the procedures are elaborated in Appendices A4. However, since the range of  $\varepsilon_t$  covered by the dataset used in this study is limited, extrapolations and expansions are performed for ASHLEY-A when  $\varepsilon_t$  exceeds 22770 and for ASHLEY-E/ASHLEY-Evar when  $\varepsilon_t$  exceeds 18357. The detailed procedures are further described in this subsection.

### **3.2.1 Extrapolation**

#### **3.2.1.1 ASHLEY-A**

The extrapolation of ASHLEY-A is done by tracking the hemispheric-integrated differential energy flux in different energy channels. The hemispheric-integrated differential energy flux is

defined as the integration of the down-going differential energy flux ( $\pi J_E$ ) over the polar hemisphere ( $|\text{MLAT}| > 45^\circ$ ) by assuming that the differential energy flux is pitch angle isotropic. Figure 1 shows hemispheric-integrated differential energy fluxes in the 19 energy channels from all 8  $\varepsilon_t$ - $\theta_c$  bins (used for the development of AHSLEY-A) where  $\theta_c$  is centered at  $225^\circ$ . As shown in Figure 1, the hemispheric-integrated differential energy fluxes in the highest 11 energy channels (central energy  $> 500$  eV) increase approximately linearly with  $\varepsilon_t$ , while the hemispheric-integrated differential energy flux tends to increase quadratically with  $\varepsilon_t$  in the lowest 8 energy channels. Similar trends can also be found when  $\theta_c$  has different values although the increment rate varies with  $\theta_c$ .

The trends shown in Figure 1 are used to extrapolate the hemispheric-integrated differential energy in the highest 11 energy channels when  $\varepsilon_t > 22,770$ . In each energy channel, the hemispheric-integrated differential energy flux at  $\varepsilon_t$  can be predicted according to the best-fit line at the given  $\theta_c$ . The slope and y-intercept of the best-fit line at  $\theta_c$  can be determined using the Fourier fitting results of the slopes and y-intercepts from the 8  $\theta_c$  bins, respectively. The ratio between the predicted hemispheric-integrated differential energy flux by the best-fit line and the hemispheric-integrated differential energy flux from the modeled pattern at  $\theta_c$  and  $\varepsilon_t = 22,770$  in each channel is calculated as the scaling factor. The extrapolated differential energy flux pattern at  $\theta_c$  and  $\varepsilon_t$  is the modeled differential energy flux pattern at  $\theta_c$  and  $\varepsilon_t = 22,770$  multiplied by the scaling factor.

For the lowest 8 energy channels, it is assumed that the increase of the hemispheric-integrated differential energy flux for  $\varepsilon_t > 22,770$  follows the same increase rate between  $\varepsilon_t = 17,590$  and  $\varepsilon_t = 22,770$  in each channel at the given  $\theta_c$ . Although such method may underestimate contributions from  $< 500$  eV electron precipitations for a very large  $\varepsilon_t$  than a quadratic

extrapolation, this method can at least provide a lower limit for <500 eV electron precipitations at a very large  $\varepsilon_t$  since the available data are limited when  $\varepsilon_t$  is very large. Again, the ratio between the predicted hemispheric-integrated differential energy flux and the hemispheric-integrated differential energy flux from the modeled pattern at  $\theta_c$  and  $\varepsilon_t=22,770$  in an energy channel is calculated as the scaling factor, which is further multiplied to the modeled differential energy flux pattern at  $\theta_c$  and  $\varepsilon_t=22,770$  to obtain the extrapolated differential energy flux pattern for that energy channel.

### 3.2.1.2 ASHLEY-E and ASHLEY-Evar

Since the electric field variability is supposed to be consistent with the background electric field model, the same extrapolation procedures are used for ASHLEY-E and ASHLEY-Evar, which are based on the extrapolation of the cross-polar-cap potential (CPCP) described in the following paragraph.

Figure 2 shows the CPCP outputs of ASHLEY-E from all 6  $\varepsilon_t$ - $\theta_c$  bins (used for the development of ASHLEY-E and ASHLEY-Evar) where  $\theta_c$  is centered at  $180^\circ$ . As expected, the CPCP increases with  $\varepsilon_t$ . However, it was found that the CPCP may be saturated at a certain point under intense solar wind and IMF conditions (e.g., Shepherd, 2007 and references therein). Therefore, to account for the saturation of the CPCP at a large  $\varepsilon_t$ , the CPCP ( $\Phi_{PC}$ ) is assumed to be linear with  $\beta$ , where

$$\beta = \frac{\varepsilon_t}{\sqrt{1 + \left(\frac{\varepsilon_t}{\varepsilon_{inf}}\right)^2}} \quad (3)$$

$\varepsilon_{inf}$  is an adjustable parameter which is set to be 40,000 to fit the trend shown in Figure 2 so that  $\Phi_{PC}$  saturates at the level of  $\sim 190$  kV comparable to the level reported in Hairston et al. (2005). The procedure is repeated for other IMF clock angles by using  $\varepsilon_{inf} = 40,000$ . It is found

that the  $\Phi_{PC}$  from ASHLEY-E is  $\sim 90$  kV under the extreme IMF and solar wind conditions reported in Mitchell et al. (2010), which is comparable to the values reported in their study (80-100 kV). Similarly, the reversal convection potential (the potential across the reversal cells) from ASHLEY-E is about  $\sim 19$  kV under the extreme IMF and solar wind conditions reported in Wilder et al. (2008), which is also comparable to the values reported in their study (15-20 kV). Therefore, our method and the choice of  $\varepsilon_{inf}$  can well capture the electric potential saturation in general. To obtain the CPCP at a given  $\varepsilon_t > 18357$  and a given  $\theta_c$ , the CPCP can be predicted according to the best-fit curve. The ratio of the predicted CPCP and the CPCP at  $\varepsilon_t = 18357$  and  $\theta_c$  is calculated as the scaling factor, which is then used to scale the electric potential and electric field variability patterns constructed at  $\varepsilon_t = 18357$  and  $\theta_c$ ,

### 3.2.2 Expansion

The electron precipitation and electric potential (electric field) patterns expand as the solar wind and IMF conditions become more intense (e.g., Feldstein and Starkov, 1967; Weimer, 2005), which is also considered in ASHLEY. In this study, expansions of the poleward auroral boundary (PAB) and convection reversal boundary (CRB) on the dawn (4-8 MLT) and dusk (16-20 MLT) sides are used to quantify the expansions of electron precipitation and electric field patterns, respectively.

Figure 3a shows the averaged co-MLAT ( $r$ ) of the PAB determined on the dawn and dusk sides along the same polar crossing by using the technique developed by Kilcommons et al. (2017) from as a function of  $\varepsilon_t$  when  $157.5^\circ < \theta_c < 202.5^\circ$ . Similarly, Figure 3b shows the averaged co-MLAT of the CRB determined on the dawn and dusk sides by using the technique developed by Zhu et al. (2020a) as a function of  $\varepsilon_t$  when  $157.5^\circ < \theta_c < 202.5^\circ$ . The CRB is found to be a good indicator of the polar cap boundary especially under southward IMF conditions

(e.g., Sotirelis et al., 2005), and the polar cap boundary was found to saturate at around  $r=21^\circ$  under southward IMF conditions (e.g., Ridley et al., 2004; Merkin et al., 2007). To take the saturation of the polar cap area into account, the co-MLAT of the CRB is assume to be linear with  $\beta$  defined in Eq. 3. The adjustable  $\varepsilon_{inf}$  is set to be 22,000 so that the best-fit curve according to the black dots shown in Figure 3b saturates at around  $r = 21^\circ$ . A similar approach is applied to capture the expansion of the PAB, and since the polar cap boundary is found to be slightly poleward of the poleward of the PAB in general (Newell et al., 2004), so it is assumed that the PAB saturates slightly equatorward of the CRB. The adjustable  $\varepsilon_{inf}$  is set to be 17,000 so that the best-fit curve according to the black dots shown in Figure 3a saturates at around  $r = 23^\circ$  and the offset between the fitted CRB and PAB is roughly constant ( $2^\circ$ ) when  $\varepsilon_t > 20000$ . The choice of  $\varepsilon_{inf}$  for the PAB can be improved in the future based on a comprehensive study of the locations of the PAB and CRB under intense southward IMF conditions.

The expansions of the PAB and CRB in other  $\theta_c$  bins are captured in a similar approach using the same  $\varepsilon_{inf}$  values determined in the  $157.5^\circ < \theta_c < 202.5^\circ$  bin, except for the CRB in  $\theta_c$  bins which are centered at  $315^\circ$ ,  $0^\circ$  and  $45^\circ$  since the CRB is typically difficult to be identified from the observation under IMF  $B_z$  northward and dominant conditions. The IMF clock angle dependences of the slope and y-intercept of the best-fit  $r$ - $\beta$  line are then determined by a Fourier expansion with respect to the IMF clock angle.

The expansion rate of the PAB can be determined by the ratio of the values of  $r$  on the best-fit  $r$ - $\beta$  line of the PAB at the given  $\varepsilon_t$  and  $\varepsilon_t = 22,770$  and at the given  $\theta_c$ . Once the expansion rate of the PAB is determined, the extrapolated differential energy flux pattern is radially expanded according to the expansion rate. However, it is worth noting that the differential energy flux needs to be scaled down by the square of the expansion rate of the PAB in order to maintain the



same hemispheric-integrated differential energy flux. Likewise, the expansion rate of the CRB can be determined by the ratio of the values of  $r$  on the best-fit  $r$ - $\beta$  line of the CRB at the given  $\varepsilon_t$  and  $\varepsilon_t = 18,357$  and at the given  $\theta_c$  between  $90^\circ$  and  $270^\circ$ . For the expansion rate of the CRB for  $\theta_c < 90$ , it is assumed that the expansion rate is 1 (i.e., no expansion) at  $\theta_c = 0^\circ$  and is a linear function of  $\sin^2(\frac{\omega}{2})$  between  $\theta_c = 0^\circ$  and  $\theta_c = 90^\circ$  ( $\omega = \frac{\theta_c}{180^\circ} \pi$ ). Similarly, for the expansion rate of the CRB for  $\theta_c > 270^\circ$ , it is assumed that the expansion rate is 1 at  $\theta_c = 360^\circ$  and is a linear function of  $\sin^2(\frac{\omega}{2})$  between  $\theta_c = 270^\circ$  and  $\theta_c = 360^\circ$ . Once the expansion rate of the CRB is determined, the extrapolated electric potential and electric field variability patterns are radially expanded according to the expansion rate. However, the modeled electric field variability needs to be downscaled by the expansion rate of the CRB to ensure that the ratio between the electric field variability and the background mean electric field does not change as the electric potential pattern expands radially.

#### 4. Statistical comparisons of model to data

The modeled results along each polar crossing used in the ASHLEY development are calculated under its corresponding  $\varepsilon_t$  and  $\theta_c$ , and the modeled and measured data in each  $\varepsilon_t$ - $\theta_c$  bin are binned according to their MLATs and MLTs. Comparisons between the binning results of the modeled and measured data from some specific  $\varepsilon_t$ - $\theta_c$  bins can be found in supplement Figures S3-S5. Here we focus on comparisons from all  $\varepsilon_t$ - $\theta_c$  bins used for the ASHLEY development.

Figure 4 compares averages of the modeled and measured differential energy flux data from all MLAT-MLT and  $\varepsilon_t$ - $\theta_c$  bins used for the ASHLEY-A development. The sizes of MLAT and MLT bins are  $1^\circ$  and 1 h, respectively, so that the numbers of the MLT, MLAT and  $\varepsilon_t$ - $\theta_c$  bin are

24, 40 (50°-90° MLAT) and 65, respectively. Likewise, Figures 5a-5c compare averages of the modeled and measured electric potential,  $E_{d1}$  and  $E_{d2}$  from all MLAT-MLT and  $\varepsilon_t$ - $\theta_c$  bins used for the ASHLEY-E development, respectively. Figures 5d and 5e serve as validations of ASHLEY-Evar. In each plot, the x-axis value of a grey dot represents the standard deviations of measured  $E_{di}$  ( $i=1,2$ ) in a MLAT-MLT bin of an  $\varepsilon_t$ - $\theta_c$  bin used for the ASHLEY-Evar development, and the y-axis value of a grey dot denotes the root mean squares of modeled  $\sigma_i$  ( $i=1,2$ ) in the same MLAT-MLT bin. For Figure 5, the MLAT bin size is 2° and the MLT bin size is 1 h, so that the numbers of the MLT, MLAT and  $\varepsilon_t$ - $\theta_c$  bins are 24, 20 (50°-90° MLAT) and 49, respectively.

Overall, all the grey dots are concentrated around the  $y=x$  line (blue-dashed line) and the best-fit line (red-thick line) according to the grey dots does not significantly deviate from the  $y=x$  line. Figures 4 and 5 along with Figures S3-S5 manifest that all components of the ASHLEY model generally work well in a statistical sense.

## 5. Model outputs

### 5.1 ASHLEY-A outputs

Figure 6 shows the ASHLEY-A outputs of the differential energy flux in the 19 DMSP energy channels when the IMF is purely southward (IMF  $B_z=-8$  nT,  $V_{sw}=450$  km/s and  $N_{sw}=4$  cm<sup>-3</sup>). Figure 6 indicates that >500 eV electrons mainly precipitate on the night side while <500 eV electrons are more likely to precipitate on the day side and are located at higher MLATs than >500 eV electron precipitations. Meanwhile, a salient peak can be found near the magnetic noon and between 70° and 75° MLAT in channels of which the central energy is around 100 eV. The peak location may correspond to the dayside cusp location since the electrons precipitating

470 into the cusp are typically found to have the average energy around 100 eV (Newell and Meng,  
471 1988).

472 Figure 7 compares the modeled differential energy fluxes in three DMSP energy channels  
473 when the IMF is purely northward, eastward, westward and southward (IMF  $B_T = -8$  nT,  $V_{SW} = 450$   
474 km/s and  $N_{SW} = 4$  cm<sup>-3</sup>). For  $>1$  keV electrons, the precipitation is most intense and equatorward  
475 for the southward IMF case. By contrast, the precipitation is weakest and occurs most poleward  
476 for the northward IMF case. Moreover, the electron precipitation does not differ significantly  
477 under positive and negative IMF  $B_y$  conditions. However, unlike  $>1$  keV electron precipitations,  
478 the magnitude of the dayside peak shown in the  $\sim 100$  eV channel is weakest under purely  
479 southward IMF conditions and is strongest under purely northward IMF conditions. In addition,  
480 the location of the dayside peak appears to depend on the IMF  $B_y$  polarity. The peak location  
481 tends to shift to the dawn side as the IMF  $B_y$  becomes more negative, indicating that the cusp  
482 shifts to the dawn side as the IMF  $B_y$  becomes more negative, which is consistent with previous  
483 findings (e.g., Candidi et al., 1983; Newell et al., 1989).

484 Figure 8a serves as an example to illustrate how the modeled energy spectrum deviates from a  
485 Maxwellian spectrum determined from the total energy flux ( $Q_0$ ) and average energy ( $\bar{E}$ ) of the  
486 modeled spectrum. The average energy can be calculated from the modeled spectrum by using  
487 the Eq. 2 in Robinson et al. (1987), and the lower and upper boundaries of the integral in the  
488 numerator and denominator of that equation are 500 eV and 30 keV, respectively. The total  
489 energy flux is calculated by multiplying a factor of  $\pi$  to the numerator of that equation by  
490 assuming the downward differential energy flux is isotropic. The IMF and solar wind conditions  
491 for the case shown in Figure 8a are: the IMF  $B_y = 0$ , the IMF  $B_z = -8$  nT,  $V_{SW} = 450$  km/s and  
492  $N_{SW} = 4$  cm<sup>-3</sup>, and the location is on the dawn side (MLT = 4.5 h, MLAT = 64.5°). For the modeled

spectrum (red dots) shown in Figure 8a,  $Q_0 = 4.87 \text{ mW/m}^2$  and  $\bar{E} = 5.08 \text{ keV}$ , and the derived Maxwellian spectrum is indicated by blue crosses. As compared with the modeled spectrum (red dots), the Maxwellian spectrum overestimates 1-10 keV electrons and underestimates both  $<1 \text{ keV}$  and  $>10 \text{ keV}$  electrons. More importantly, the Maxwellian spectrum markedly underestimates  $<1 \text{ keV}$  electron precipitations. In particular, the difference is approximately 2 orders of magnitude for  $\sim 100 \text{ eV}$  electrons. Hence, the contribution of soft electron precipitations to the I-T system can be significantly underestimated if a Maxwellian energy spectrum is assumed.

## 5.2 ASHLEY-E and ASHLEY-Evar outputs

Figure 9 shows the electric potential outputs from ASHLEY-E for 8 different IMF clock angles, and other IMF and solar wind parameters for the cases shown in Figure 9 are: the IMF  $B_T = 8 \text{ nT}$ ,  $V_{SW} = 450 \text{ km/s}$  and  $N_{SW} = 4 \text{ cm}^{-3}$ . In general, the electric potential displays a two-cell pattern except for the northward IMF  $B_z$  case, where a multiple-cell pattern appears. In addition, the negative cell on the dusk side and the positive cell on the dawn side are shaped into round and crescent cells, respectively, when the IMF  $B_y$  is positive. The opposite is true for the negative IMF  $B_y$  case. Meanwhile, the round cell typically has a larger absolute extremum than the crescent cell. Moreover, as shown in Figure 11a, the CPCP varies with the IMF clock angle, which maximizes and minimizes when the IMF  $B_z$  is purely southward and northward, respectively. Overall, the outputs from ASHLEY-E are consistent with previous studies (e.g., Thomas and Shepherd, 2018 and references therein).

Figure 10 compares the mean electric field magnitude ( $E_1 = \sqrt{\bar{E}_{d1}^2 + \bar{E}_{d2}^2}$ ) and electric field variability magnitude ( $E_2 = \sqrt{\sigma_1^2 + \sigma_2^2}$ ) for different IMF clock angles. For the cases shown in Figure 10, the conditions are: the IMF  $B_T = 8 \text{ nT}$ ,  $V_{SW} = 450 \text{ km/s}$  and  $N_{SW} = 4 \text{ cm}^{-3}$ .  $\bar{E}_{d1}$  and  $\bar{E}_{d2}$

are calculated from the electric potential outputs of ASHLEY-E by using Eqs. 4.8 and 4.9 in Richmond (1995), and  $\sigma_1$  and  $\sigma_2$  are direct outputs of ASHLEY-Evar. As shown in Figure 10a,  $E_1$  typically displays a three-peak structure and a more complex pattern appears when the IMF  $B_z$  is purely northward. Figure 10b shows that  $E_2$  tends to peak on the dawn and dusk sides when the IMF is purely southward and the peak on the dawn side has a higher magnitude, while it tends to have a single peak on the day side when the IMF is purely northward. In addition, the distribution of  $E_2$  depends on the IMF  $B_y$  polarity:  $E_2$  tends to peak on the morning side when the IMF  $B_y$  is positive with a relatively wider MLT span, whereas it tends to peak near noon when the IMF  $B_y$  is negative with a weaker magnitude and a narrower MLT span. However, when the IMF has a southward component, the MLT spans of the  $E_2$  peak seems to be comparable for positive and negative IMF  $B_y$  cases. Figure 11b further compares the IMF clock angle dependences of the averaged  $E_1$  and  $E_2$  over the  $|\text{MLAT}| > 60^\circ$  region. In general, both of the averaged  $E_1$  and  $E_2$  maximize when the IMF is purely southward and the polar average of  $E_2$  is generally comparable with the polar average of  $E_1$  when the IMF has a southward component. However, the polar average of  $E_2$  is much larger than the polar average of  $E_1$  when the IMF is northward. The results shown in Figures 10 and 11b are consistent with results shown in Matsuo et al. (2003) in general.

## 6. Discussion

### 6.1 Similarities and differences with previous empirical models

The large-scale high-latitude electric field and electron precipitation have been studied for several decades and several empirical models have been established for the electric field (e.g., Papitashvili and Rich, 2002; Weimer, 2005; Cousins and Shepherd, 2010) and electron precipitations (e.g., Hardy et al., 1985, 1987; Fuller-Rowell and Evans, 1987; Y. Zhang and

Paxton, 2008; Newell et al., 2009, 2014) based on different measurements. However, to our knowledge, existing electric field models and electron precipitation models have been developed separately. As a consequence, the consistency between the electric field and electron precipitation models is lacking. For example, Sheng et al. (2019) found that the CRB from the Weimer (2005) convection model is significantly equatorward (up to  $>10^\circ$  in MLAT) of the PAB from the Fuller-Rowell and Evans (1987) electron precipitation model under intense southward IMF and solar wind conditions, which may contradict the understanding established in previous studies (e.g., Sotirelis et al., 2005). In addition, the simulations conducted in Sheng et al. (2019) indicated that the large offsets between the CRB and PAB result in significant underestimations of Joule heating. A primary advantage of ASHLEY is that the electron precipitation and electric field components have been developed concurrently and, as much as possible, consistently. For example, in addition to using electric field and electron precipitation data from the same platform (DMSP satellite) and same solar cycle (solar cycle 24), the consistency between the CRB and PAB has also been taken into account under intense IMF and solar wind conditions (see Section 3.2.2). Apart from improving the consistency between the electron precipitation and electric field components, ASHLEY also improves specifications of the soft electron precipitation and electric field variability.

#### **6.1.1 Soft electron precipitation**

Although several electron precipitation models have been developed (see Section 1), most of them only provide the total energy flux, total number flux and average energy of an assumed Maxwellian energy spectrum. Apart from those models, Hardy et al. (1985) established distributions of the average spectrum in 7 Kp bins (Kp range: 0-6) based on 2.5 years of DMSP SSJ3 measurements. Although the datasets used in this study and used in Hardy et al. (1985) are

from two different solar cycles and two different versions of SSJ, our results are qualitatively consistent with Hardy et al. (1985). However, the Kp index is a low-resolution (3-h) geomagnetic index, and the IMF and solar wind conditions can be considerably different even though the Kp index is similar. Thus, the electron precipitation evolutions may not be well captured in a Kp-based electron precipitation model. Therefore, a Kp-based electron precipitation model may provide same electron precipitation patterns for 3 hours while an electron precipitation model based on the IMF and solar wind may better capture the evolution of the electron precipitation in such case. Moreover, a positive IMF  $B_y$  condition probably gives a very similar Kp as a negative IMF  $B_y$  condition as long as the magnitude of  $B_y$  and solar wind conditions are similar (Newell et al., 2008). However, as shown in Figure 8, the differences in the soft electron precipitation are significant when the direction of the IMF  $B_y$  is opposite although differences in the keV electrons are less significant. Therefore, the IMF  $B_y$  dependence of the soft electron precipitation may not be well specified in the statistical patterns built by Hardy et al. (1985) as compared with those provided by ASHLEY-A. Furthermore, ASHLEY-A can provide distributions of the energy spectrum under intense IMF and solar wind conditions based on reasonable extrapolations and expansions. Therefore, ASHLEY-A can be more useful in studying the I-T system during intense geomagnetic storms when coupling into GCMs.

In addition to the Hardy model, the Ovation Prime (OP) models developed by Newell et al. (2009, 2014) also improve the energy spectrum specification in empirical models. The major characteristic of the OP models is that they provide the total energy flux, total number flux and probability of three types of electron precipitations: diffuse, mono-energetic and broadband. However, it is still challenging to correctly identify the precipitation type to date (e.g., Dombeck et al., 2018; Wing et al., 2019). For example, as pointed out by Wing et al (2019), it is highly

possible that an energy spectrum matches none of the above three types and is labeled as the diffuse type for simplicity and convenience, so that the diffuse precipitation may still dominate in the OP models. Moreover, like the Kp index, the Newell coupling function used to drive OP models does not distinguish the IMF B<sub>y</sub> polarity either. Furthermore, the total energy flux, total number flux and probability in each MLAT-MLT bin from the OP models is assumed to be linear with the Newell coupling function. However, a linear fitting may underestimate the evolution of <500 eV electron precipitations as implied by Figure 1. Therefore, the contribution of the soft electron precipitation may still not be accurately estimated in the OP models.

#### **6.1.2 Electric field variability**

While most electric field models only provide large-scale high-latitude mean electric fields, there are some efforts in studying the statistical distribution of the high-latitude electric field variability. For example, Codrescu et al. (2000) established the electric field variability pattern in 10 auroral activity index bins and in different seasons according the Millstone Hill incoherent scatter radar (ISR) measurements. Similarly, Cosgrove and Thayer (2006) established a Kp-based statistic pattern based on Sondrestrom ISR measurements in a limited latitudinal region. Matsuo et al. (2003) studied the distributions of the mean electric field and electric field variability at high latitudes under several different IMF conditions and in different seasons based on the Dynamic Explorer 2 (DE-2) satellite ion bulk drift measurements and the Weimer (2001) empirical electric potential model which is also developed based on the DE-2 data. Moreover, Matsuo and Richmond (2008) further analyzed the distribution of the electric field variability on different scales under several different IMF conditions and in different seasons, and they found that the large-scale electric field variability tends to be larger than the small-scale and mesoscale



electric field variabilities. Similar conclusion has been reached by Cosgrove et al. (2011) based on Sondrestrom ISR measurements. Later, Cousins and Shepherd (2012) developed several statistical maps of the small-scale and mesoscale electric field variabilities for different interplanetary electric fields, in different seasons and in different hemispheres based on Super Dual Auroral Radar Network (SuperDARN) radar measurements. Although statistical patterns of high-latitude electric field variability have been established under different geophysical conditions, a dynamic empirical electric field variability model that is consistent with the background large-scale mean electric field model is still lacking to date. To our knowledge, the empirical model used in Deng et al. (2009) is the only existing empirical model provide consistent mean electric field and electric field variability which is based on the DE-2 ion bulk drift measurements. The methodology used to develop that model is implemented in the development of ASHLEY-E and ASHLEY-Evar. In comparison to the model used in Deng et al. (2009), the solar wind dependences of the electric field and electric field variability are implemented in ASHLEY-E and ASHLEY-Evar while the seasonal dependences of the electric field and electric field variability are not taken into account. Meanwhile, the expansions of the electric potential and electric field variability patterns are considered in ASHLEY-E and ASHLEY-Evar under intense IMF and solar wind conditions.

## **6.2 Low-energy tail of the energy spectrum**

The strong low-energy tail shown in the ASHLEY-A energy spectrum (Figure 8a) is frequently seen in observations (e.g., Evans, 1974; Fung and Hoffman, 1988; Hardy et al., 1985; McIntosh and Anderson, 2014; Wing et al., 2019) and its sources are considerably complex since the electron precipitation is not a simple one-way transport of electrons from the magnetosphere

to the ionosphere (Khazanov and Glocer, 2020, and references therein). For example, if a field-aligned potential drop is present, the upgoing electrons without sufficient kinetic energy to overcome such potential drop will be reflected downward and subsequently are observed as downward precipitation flux (Evans, 1994; Evans and Moore, 1979; Richards, 2013). In addition, it is also possible that the upgoing superthermal electrons from the conjugate hemisphere contribute to the formation of the low-energy tail (Khazanov and Glocer, 2020).

Meier et al. (1989) developed an empirical formula (hereafter, M89 formula) to account for the low-energy tail which was later used in the model developed by Strickland et al. (1993). The blue dashed line in Figure 8b shows the spectrum constructed by the M89 formula using  $Q_0 = 4.87 \text{ mW/m}^2$ ,  $\bar{E} = 5.08 \text{ keV}$  (hereafter, M89-I spectrum). Although the low-energy tail has been significantly improved in contrast to a simple Maxwellian energy spectrum, the magnitude of the low-energy tail is still underestimated by 50% in comparison with that of the ASHLEY energy spectrum in general. However, it is worth noting that the M89 formula is based on the total energy and average energy of the whole energy spectrum while the total energy and average energy outputs of ASHLEY-A are calculated by using the  $>500 \text{ eV}$  portion of the energy spectrum (Section 5.1). The total energy and average energy calculated from the whole energy spectrum shown in Figure 8a are  $Q'_0 = 5.01 \text{ mW/m}^2$  and  $\bar{E}' = 2.92 \text{ keV}$ , respectively, and the corresponding spectrum calculated from the M89 formula is indicated by the green dashed line in Figure 8b (hereafter, M89-II spectrum). It is clear that the low-energy tail calculated by using  $Q'_0$  and  $\bar{E}'$  is more comparable with that of the ASHLEY-A energy spectrum as compared with the low-energy tail calculated by using  $Q_0$  and  $\bar{E}$ . However, the discrepancies of 1-10 keV electrons between the M89-II and ASHLEY-A spectra are larger than those between the M89-I and ASHLEY-A spectra. The discrepancy shown in Figure 8b is a general case in the auroral

oval although it may vary quantitatively with the location. Therefore, the ionospheric conductances may be significantly overestimated in the auroral zone when the M89-II spectrum is utilized to drive a GCM, and it might be necessary to propose a new empirical formula for the incident electron energy spectrum in order to obtain the I-T responses at both E-region and F-region altitudes correctly. In addition to the empirical formula, physical-based models such as the SuperThermal Electron Transport (STET) model developed by Khazanov et al. (2014) may also be useful to reconstruct the low-energy tail. It would be interesting to compare the performance of different methods in representing the low-energy tail in the future.

The downward low-energy precipitation flux can lead to ionizations of the thermosphere at the F-region altitudes, which increases the F-region conductivity but may not significantly change the height-integrated conductivity (i.e., conductance). Therefore, the altitudinal Joule heating distribution will be significantly changed, which may cause significant changes of the I-T system. The impacts of soft electron precipitations on the I-T system will be more comprehensively investigated in the future by coupling the ASHLEY model to a GCM. There are also other mechanisms altering the altitudinal Joule heating, such as Alfvén waves incident from the magnetosphere (e.g., Lotko and Zhang, 2018; Verkhoglyadova et al., 2018; Hogan et al., 2020). The relative significance of two different mechanisms to the I-T system under different conditions will also be an interesting topic that deserves future explorations.

### **6.3 Future improvements**

As more data become available, we plan to incorporate seasonal variation in ASHLEY. For example, it is found that the distribution and magnitude of the electron precipitation display seasonal dependence (e.g., Newell et al., 2010). Therefore, given that the DMSP SSJ data are in

equally good quality in different seasons, it would be interesting to investigate the seasonal dependence of the differential energy flux in each DMSP SSJ energy channel. Besides, we will include the electron precipitation variability together with its correlation with the electric field variability on different scales, so that the estimation of the localized Joule heating can be improved (Zhu et al., 2018). Moreover, a boundary-oriented binning technique (Zhu et al., 2020a) will be utilized instead of the static-binning method utilized in this study, which can help resolve the smoothing issue caused by the static-binning method and further improve the total Joule heating estimation.

## **7. Summary**

In this study, we have developed a new empirical model, ASHLEY, that can improve specifications of the electron precipitation energy spectrum and high-latitude electric field variability in GCMs based on the DMSP electron precipitation and bulk ion drift measurements in the solar cycle 24. In addition to having better consistency between the electron precipitation and electric field, ASHLEY also has several advantages over other existing empirical models, which are summarized as follows:

- 1) The auroral electron precipitation component, ASHLEY-A, provides the averaged differential energy flux in the 19 DMSP energy channels under different IMF and solar wind conditions without making any assumptions about the energy spectrum. It is found that soft electron precipitation specifications can be remarkably improved as compared with the typically assumed Maxwellian energy spectrum having the same total energy flux and average energy. The outputs of ASHLEY-A indicate that the distributions of  $>500$  eV and  $<500$  eV electrons can be significantly different:  $>500$  eV electrons mainly precipitate on the night side whereas  $<500$

electrons mainly precipitate on the day side. Moreover, the differential energy flux displays a salient peak near the local noon in channels with their central energy around 100 eV, which may correspond to the dayside cusp. Furthermore, the impact of the IMF  $B_y$  polarity on the electron precipitation is better taken into account in ASHLEY-A than existing electron precipitation models. It is found that the polarity of the IMF  $B_y$  component can significantly affect the distributions of  $<500$  eV electron precipitations.

2) ASHLEY provides consistent high-latitude mean electric field and electric field variability under different IMF and solar wind conditions through ASHLEY-E and ASHLEY-Evar, respectively. The modeled electric potential and electric field variability distributions are generally consistent with previous statistical results.

## Appendix:

### A1. MLT and IMF clock angle fitting

For the differential energy flux ( $J_E$ ) or  $\sigma_1$  or  $\sigma_2$  in each bin of MLAT,  $\varepsilon_t$ , and  $\theta_c$ , a fourth-order Fourier series has been used to capture their MLT variations. Let us call the quantity to be fitted  $y$ , thus:

$$y = \sum_{m=0}^4 (A_m \cos(m\phi) + B_m \sin(m\phi)) \quad (A1)$$

Here,  $\phi = \frac{\text{MLT}}{12}\pi$ , and  $A_m$  and  $B_m$  are MLT fitting coefficients at the order of  $m$ , and are a function of MLAT,  $\varepsilon_t$  and  $\theta_c$ . The maximum order of 4 is determined after trial-and-error tests, where it has been found that a higher-order Fourier series would not improve the fitting results yet would introduce unrealistic small-scale structures. For each bin of MLAT and  $\varepsilon_t$  (median  $\varepsilon_t > 3000$ ),  $A_m$  and  $B_m$  are fitted to a fourth-order Fourier series constructed by  $\omega$  ( $\omega = \frac{\theta_c}{180^\circ}\pi$ ):

$$A_m(\text{or } B_m) = \sum_{n=0}^4 (C_n \cos(n\omega) + D_n \sin(n\omega)) \quad (\text{A2})$$

Here,  $A_m$  and  $B_m$  in each  $\theta_c$  bin are assumed to represent the fitting coefficients at its central  $\theta_c$ , i.e., a multiple of  $45^\circ$  (i.e., a multiple of  $\frac{\pi}{4}$  for  $\omega$ ), and in order to implement a fourth-order fitting,  $A_m$  and  $B_m$  are linearly interpolated to  $\omega$  equal to multiples of  $\frac{\pi}{8}$ . The Fourier fitting is done using `numpy.linalg.lstsq` in Python's NumPy package (Harris et al., 2020).

## **A2. Expansions of $E_{d1}$ and $E_{d2}$ in terms of spherical harmonics and electric potential fitting**

According to Eq. 2 in this paper and Eqs. 4.8 and 4.9 in Richmond (1995), the expansions of  $E_{d1}$  and  $E_{d2}$  can be expressed as follows:

$$E_{d1}(\theta, \phi) = \sum_{l=1}^{12} \sum_{m=1}^{\min(l,4)} \frac{m}{R \cos \lambda_m} (F_{lm} \sin m\phi - G_{lm} \cos m\phi) P_l^m(\cos \theta) \quad (\text{A3})$$

$$E_{d2}(\theta, \phi) = \frac{4 \sin \theta}{R \sin I_m} \left( \sum_{l=0}^{12} F_{l0} \frac{\partial P_l^0(x)}{\partial x} \Big|_{x=\cos \theta} + \right. \quad (\text{A4})$$

$$\left. \sum_{l=1}^{12} \sum_{m=1}^{\min(l,4)} (F_{lm} \cos m\phi + G_{lm} \sin m\phi) \frac{\partial P_l^m(x)}{\partial x} \Big|_{x=\cos \theta} \right)$$

The coefficients in Eqs. A3 and A4 are the same as those in Eq. 2,  $R$  in Eqs. A3 and A4 is 6482 (6372+110) km and  $\sin I_m$  in Eq. A4 can be calculated by using Eq. 3.7 in Richmond (1995).

By using the electric potential data and their locations, we can construct  $\mathbf{A}_1 \mathbf{X} = \mathbf{B}_1$  from Eq 2, where  $\mathbf{X}$  is constructed by  $F_{lm}$  and  $G_{lm}$ . Similarly, by using  $E_{d1}$  and  $E_{d2}$  data and their locations, we can construct  $\mathbf{A}_2 \mathbf{X} = \mathbf{B}_2$  and  $\mathbf{A}_3 \mathbf{X} = \mathbf{B}_3$  from Eqs. A3 and A4, respectively. The three equations can further be combined to  $\mathbf{A} \mathbf{X} = \mathbf{B}$ , where  $\mathbf{A}^T = [\mathbf{A}_1^T, \mathbf{A}_2^T, \mathbf{A}_3^T]$  and  $\mathbf{B}^T = [\mathbf{B}_1^T, \mathbf{B}_2^T, \mathbf{B}_3^T]$ . Similar to

the Fourier fitting, we also use `numpy.linalg.lstsq` in Python's NumPy package (Harris et al., 2020) to achieve the fitting.

### **A3. Reconstruct the electron precipitation pattern for $\varepsilon_t \leq 22770$**

Case 1: If  $\varepsilon_t \leq 2579$ , then the MLT Fourier fitting coefficients in the first  $\varepsilon_t$  bin are used to reconstruct the differential energy flux in different MLAT bins and energy channels (In this subsection, the  $\varepsilon_t$  bins correspond to those listed in Table 1a). Case 2: If  $\varepsilon_t > 2579$ , the two  $\varepsilon_t$  bins with the median value of  $\varepsilon_t$  closest to the given  $\varepsilon_t$  are determined at first by using Table 1a. Then the differential energy flux patterns from those two  $\varepsilon_t$  bins are combined according to their weights  $w_1$  and  $w_2$ , which can be calculated using following procedures: Assume the closest two median values of  $\varepsilon_t$  are  $\varepsilon_1$  and  $\varepsilon_2$ , respectively ( $\varepsilon_1 < \varepsilon_t \leq \varepsilon_2$ ), then  $w_1 = \frac{\varepsilon_2 - \varepsilon_t}{\varepsilon_2 - \varepsilon_1}$  and  $w_2 = 1 - w_1$ . For the  $\varepsilon_t$  bin with the median value of  $\varepsilon_t$  greater than 3000, the MLT fitting coefficients in each MLAT bin and in each channel are reconstructed according to the IMF clock angle Fourier fitting coefficients (determined in Section 3.1.1) and the given IMF clock angle  $\theta_c$ . Then the differential energy flux is calculated using the MLT Fourier fitting coefficients.

### **A4. Reconstruct the electric potential and electric field variability patterns for $\varepsilon_t \leq 18357$**

Case 1: If  $\varepsilon_t \leq 2583$ , then the spherical harmonics fitting coefficients in the first  $\varepsilon_t$  bin are used to reconstruct the electric potential (In this subsection, the  $\varepsilon_t$  bins discussed are those listed in Table 1b). Similarly, the MLT Fourier fitting coefficients in different MLAT bins and in the first  $\varepsilon_t$  bin are used to reconstruct  $\sigma_1$  and  $\sigma_2$  in different MLAT bins. Case 2: If  $\varepsilon_t > 2583$ , the two  $\varepsilon_t$  bins with the median value of  $\varepsilon_t$  closest to the given  $\varepsilon_t$  are determined at first by using Table 1b. Then the patterns from those two  $\varepsilon_t$  bins are combined according to their weights  $w_1$  and  $w_2$ , which can be calculated using following procedures: Assume the closest two median

values of  $\varepsilon_t$  are  $\varepsilon_1$  and  $\varepsilon_2$ , respectively, then  $w_1 = \frac{\beta_2 - \beta}{\beta_2 - \beta_1}$  and  $w_2 = 1 - w_1$ . Here,  $\beta_1$ ,  $\beta_2$  and  $\beta$  are calculated from  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_t$ , respectively ( $\varepsilon_1 < \varepsilon_t \leq \varepsilon_2$ ), using Eq.3 and  $\varepsilon_{inf} = 40,000$ . For the  $\varepsilon_t$  bin with the median value of  $\varepsilon_t$  greater than 3000, the spherical harmonics fitting coefficients are reconstructed according to the IMF clock angle Fourier fitting coefficients determined in Section 3.1.2 and the given IMF clock angle  $\theta_c$ , and the electric potential can be determined using the spherical harmonics fitting coefficients. Similarly, for the  $\varepsilon_t$  bin with the median value of  $\varepsilon_t$  greater than 3000, the MLT Fourier fitting coefficients in each MLAT bin are reconstructed according to the IMF clock angle Fourier fitting coefficients determined in Section 3.1.3 and the given IMF clock angle  $\theta_c$ . Then  $\sigma_1$  and  $\sigma_2$  in different MLAT bins can be determined using the MLT Fourier fitting coefficients.

#### **A5. Variables defined in Sections 2-5**

Table 2 lists all variables defined in Sections 2-5, including their definitions, units, calculations and places of first shown, for better references.

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783 (<https://omniweb.gsfc.nasa.gov>). The DMSP electron precipitation and auroral boundary data  
784 can be found at NASA SPDF CDAWeb (<https://cdaweb.sci.gsfc.nasa.gov/index.html/>) and the  
785 DMSP ion drift data can be obtained at NOAA NCEI (<https://satdat.ngdc.noaa.gov/dmsp/data/>).  
786 The codes of auroral boundary identification technique developed by Kilcommons et al. (2017)  
787 are available at Kilcommons and Burrell (2019) and the link is  
788 <http://doi.org/10.5281/zenodo.3267415>. The data used to generate the figures are available at  
789 Zhu et al., (2020b) and the link is <http://doi.org/10.5281/zenodo.4151717>. The codes of  
790 ASHLEY are available at Zhu et al., (2020c) and the link is  
791 <https://doi.org/10.5281/zenodo.4152364>.

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## References

- Billett, D. D., Grocott, A., Wild, J. A., Walach, M. T., & Kosch, M. J. (2018). Diurnal variations in global Joule heating morphology and magnitude due to neutral winds. *Journal of Geophysical Research: Space Physics*, 123, 2398–2411. <https://doi.org/10.1002/2017JA025141>
- Buonsanto, M. J. (1999). Ionospheric Storms—A Review. *Space Science Reviews*, 88(3), 563–601. <https://doi.org/10.1023/A:1005107532631>
- Burleigh, M., Zettergren, M., Lynch, K., Lessard, M., Moen, J., Clausen, L., Kenward, D., Hysell, D., & Liemohn, M. (2019). Transient Ionospheric Upflow Driven by Poleward Moving Auroral forms Observed During the Rocket Experiment for Neutral Upwelling 2 (RENU2) Campaign. *Geophysical Research Letters*, 46(12), 6297–6305. <https://doi.org/10.1029/2018GL081886>
- Candidi, M., H. W. Kroehl, and C. I. Meng (1983), Intensity distribution of dayside polar soft electron precipitation and the IMF, *Planetary and Space Science*, 31(5), 489–498, doi:10.1016/0032-0633(83)90040-5.
- Cole, K. D. (1962). Joule heating of the upper atmosphere. *Australian Journal of Physics*, 15, 223.
- Codrescu, M. V., T. J. Fuller-Rowell, and J. C. Foster (1995), On the importance of E-field variability for Joule heating in the high-latitude thermosphere, *Geophys. Res. Lett.*, 22, 2393–2396, doi: 10.1029/95GL01909.
- Codrescu, M. V., T. J. Fuller-Rowell, J. C. Foster, J. M. Holt, and S. J. Cariglia (2000), Electric field variability associated with the Millstone Hill electric field model, *J. Geophys. Res.*, 105(A3), 5265–5274, doi:10.1029/1999JA900463.
- Codrescu, M. V., T. J. Fuller-Rowell, V. Munteanu, C. F. Minter, and G. H. Millward (2008), Validation of the Coupled Thermosphere Ionosphere Plasmasphere Electrodynamics model: CTIPE-Mass Spectrometer Incoherent Scatter temperature comparison, *Space Weather*, 6(9), 09005, doi:10.1029/2007SW000364.
- Cosgrove, R. B., and J. P. Thayer (2006), Parametric dependence of electric field variability in the sondrestrom database: A linear relation with kp, *Journal of Geophysical Research: Space Physics*, 111(A10), doi:10.1029/2006JA011658.
- Cosgrove, R. B., & Codrescu, M. (2009). Electric field variability and model uncertainty: A classification of source terms in estimating the squared electric field from an electric field model. *Journal of Geophysical Research: Space Physics*, 114(A6). <https://doi.org/10.1029/2008JA013929>
- Cosgrove, R., M. McCready, R. Tsunoda, and A. Stromme (2011), The bias on the joule heating estimate: Small-scale variability versus resolved-scale model uncertainty and the correlation of

electric field and conductance, *Journal of Geophysical Research: Space Physics*, 116(A9),  
doi:10.1029/2011JA016665.

Cousins, E. D. P., and S. G. Shepherd (2010), A dynamical model of high-latitude convection  
derived from SuperDARN plasma drift measurements, *J. Geophys. Res.*, 115, A12329,  
doi:10.1029/2010JA016017.

Cousins, E. D. P., and S. G. Shepherd (2012), Statistical maps of small-scale electric field  
variability in the high-latitude ionosphere, *Journal of Geophysical Research (Space Physics)*,  
117(A12), A12304, doi:10.1029/2012JA017929.

Crowley, G., & Hackert, C. L. (2001). Quantification of high latitude electric field variability.  
*Geophysical Research Letters*, 28(14), 2783–2786. <https://doi.org/10.1029/2000GL012624>

Deng, Y., Maute, A., Richmond, A. D., and Roble, R. G. (2008), Analysis of thermospheric  
response to magnetospheric inputs, *J. Geophys. Res.*, 113, A04301, doi:10.1029/2007JA012840.

Deng, Y., A. Maute, A. D. Richmond, and R. G. Roble (2009), Impact of electric field variability  
on Joule heating and thermospheric temperature and density, *Geophys. Res. Lett.*, 36, L08105,  
doi: 10.1029/2008GL036916.

Deng, Y., T. J. Fuller-Rowell, R. A. Akmaev, and A. J. Ridley (2011), Impact of the altitudinal  
Joule heating distribution on the thermosphere, *Journal of Geophysical Research (Space  
Physics)*, 116, A05313, doi:10.1029/2010JA016019.

Emery, B. A., Lathuillere, C., Richards, P. G., Roble, R. G., Buonsanto, M. J., Knipp, D. J., et al.  
(1999). Time dependent thermospheric neutral response to the 2–11 November 1993 storm  
period. *Journal of Atmospheric and Terrestrial Physics*, 61(3-4), 329–350. [https://doi.org/10.1016/S1364-6826\(98\)00137-0](https://doi.org/10.1016/S1364-6826(98)00137-0).

Evans, D. S. (1974), Precipitating electron fluxes formed by a magnetic field aligned potential  
difference, *J. Geophys. Res.*, 79(19), 2853, doi:10.1029/JA079i019p02853.

Evans, D. S., and T. E. Moore (1979), Precipitating electrons associated with the diffuse aurora:  
evidence for electrons of atmospheric origin in the plasma sheet, *J. Geophys. Res.*, 84(A11),  
6451–6457, doi:10.1029/JA084iA11p06451.

Fedrizzi, M., Fuller-Rowell, T. J., & Codrescu, M. V. (2012). Global Joule heating index derived  
from thermospheric density physics-based modeling and observations. *Space Weather*, 10(3).  
<https://doi.org/10.1029/2011SW000724>

Feldstein, Y. I., & Starkov, G. V. (1967). Dynamics of auroral belt and polar geo magnetic  
disturbances. *Planetary and Space Science*, 15, 209–229. [https://doi.org/10.1016/0032-0633\(67\)90190-0](https://doi.org/10.1016/0032-0633(67)90190-0)

Fuller-Rowell, T. J., and D. S. Evans (1987), Height-integrated Pedersen and Hall conductivity

patterns inferred from the TIROS-NOAA satellite data, *J. Geophys. Res.*, 92, 7606–7618, doi: 10.1029/JA092iA07p07606.

Fuller-Rowell, T. J., Codrescu, M. V., Moffett, R. J., & Quegan, S. (1994). Response of the thermosphere and ionosphere to geomagnetic storms. *Journal of Geophysical Research: Space Physics*, 99(A3), 3893–3914. <https://doi.org/10.1029/93JA02015>

Fuller-Rowell, T. J., Codrescu, M. C., and Wilkinson, P.: Quantitative modeling of the ionospheric response to geomagnetic activity, *Ann. Geophys.*, 18, 766–781, <https://doi.org/10.1007/s00585-000-0766-7>, 2000.

Fung, S. F., and R. A. Hoffman (1988), On the spectrum of the secondary auroral electrons, *J. Geophys. Res.*, 93(A4), 2715–2724, doi:10.1029/JA093iA04p02715.

Hairston, M. R., K. A. Drake, and R. Skoug (2005), Saturation of the ionospheric polar cap potential during the October–November 2003 superstorms, *Journal of Geophysical Research (Space Physics)*, 110(A9), A09S26, doi:10.1029/2004JA010864.

Hardy, D. A., M. S. Gussenhoven, and E. Holeman (1985), A statistical model of auroral electron precipitation, *J. Geophys. Res.*, 90(A5), 4229–4248, doi: 10.1029/JA090iA05p04229.

Hardy, D. A., Gussenhoven, M. S., Raistrick, R., & McNeil, W. J. (1987). Statistical and functional representations of the pattern of auroral energy flux, number flux, and conductivity. *Journal of Geophysical Research: Space Physics*, 92(A11), 12275–12294. <https://doi.org/10.1029/JA092iA11p12275>

Hardy, D. A., Holeman, E. G., Burke, W. J., Gentile, L. C., & Bounar, K. H. (2008). Probability distributions of electron precipitation at high magnetic latitudes. *Journal of Geophysical Research: Space Physics*, 113(A6). <https://doi.org/10.1029/2007JA012746>

Harris, C.R., Millman, K.J., van der Walt, S.J. et al. Array programming with NumPy. *Nature* 585, 357–362 (2020). DOI: [0.1038/s41586-020-2649-2](https://doi.org/10.1038/s41586-020-2649-2).

Heelis, R. A., Lowell, J. K., & Spiro, R. W. (1982). A model of the high-latitude ionospheric convection pattern. *Journal of Geophysical Research: Space Physics*, 87(A8), 6339–6345. <https://doi.org/10.1029/JA087iA08p06339>

Hogan, B., Lotko, W., & Pham, K. (2020). Alfvénic Thermospheric Upwelling in a Global Geospace Model. *Journal of Geophysical Research: Space Physics*, 125(12), e2020JA028059. <https://doi.org/10.1029/2020JA028059>

Huang, Y., Richmond, A. D., Deng, Y., and Roble, R. (2012), Height distribution of Joule heating and its influence on the thermosphere, *J. Geophys. Res.*, 117, A08334, doi:10.1029/2012JA017885.

Jackson, J. D. (2007), *Classical electrodynamics*, John Wiley & Sons.

- Khazanov, G. V., Glocer, A., & Himwich, E. W. (2014). Magnetosphere-ionosphere energy interchange in the electron diffuse aurora. *Journal of Geophysical Research: Space Physics*, 119(1), 171–184. <https://doi.org/10.1002/2013JA019325>
- Khazanov, G. V., & Glocer, A. (2020). How Magnetically Conjugate Atmospheres and the Magnetosphere Participate in the Formation of Low-Energy Electron Precipitation in the Region of Diffuse Aurora. *Journal of Geophysical Research: Space Physics*, 125(8), e2020JA028057.
- Kilcommons, L. M., R. J. Redmon, and D. J. Knipp (2017), A new DMSP magnetometer and auroral boundary data set and estimates of field-aligned currents in dynamic auroral boundary coordinates, *J. Geophys. Res. Space Physics*, 122, 9068–9079, doi:10.1002/2016JA023342.
- Kilcommons, L. M, and A. G. Burrell (2019). `lkilcommons/ssj_auroral_boundary`: Version 1 (Version v1.0.0). Zenodo. <http://doi.org/10.5281/zenodo.3267415>
- Laundal, K.M., Richmond, A.D. Magnetic Coordinate Systems. *Space Sci Rev* 206, 27–59 (2017). <https://doi-org/10.1007/s11214-016-0275-y>
- Liemohn, M. W. (2020). The Case for Improving the Robinson Formulas. *Journal of Geophysical Research: Space Physics*, 125(10), e2020JA028332. <https://doi.org/10.1029/2020JA028332>
- Lotko, W., & Zhang, B. (2018). Alfvénic Heating in the Cusp Ionosphere-Thermosphere. *Journal of Geophysical Research: Space Physics*, 123(12), 10,368–10,383. <https://doi.org/10.1029/2018JA025990>
- Lu, G., A. D. Richmond, B. A. Emery, and R. G. Roble (1995), Magnetosphere-ionosphere-thermosphere coupling: Effect of neutral winds on energy transfer and field-aligned current, *J. Geophys. Res.*, 100, 19,643–19,660, doi:10.1029/95JA00766.
- Lu, G., A. D. Richmond, H. Lühr, and L. Paxton (2016), High-latitude energy input and its impact on the thermosphere, *Journal of Geophysical Research (Space Physics)*, 121(7), 7108–7124, doi:10.1002/2015JA022294.
- Lu, G., Zakharenkova, I., Cherniak, I., & Dang, T. (2020). Large-Scale Ionospheric Disturbances During the 17 March 2015 Storm: A Model-Data Comparative Study. *Journal of Geophysical Research: Space Physics*, 125(5), e2019JA027726. <https://doi.org/10.1029/2019JA027726>
- Matsuo, T., A. D. Richmond, and K. Hensel (2003), High-latitude ionospheric electric field variability and electric potential derived from DE-2 plasma drift measurements: Dependence on IMF and dipole tilt, *Journal of Geophysical Research (Space Physics)*, 108, 1005, doi:10.1029/2002JA009429
- Matsuo, T., & Richmond, A. D. (2008). Effects of high-latitude ionospheric electric field variability on global thermospheric Joule heating and mechanical energy transfer rate. *Journal of*

Geophysical Research: Space Physics, 113(A7). <https://doi.org/10.1029/2007JA012993>

Merkin, V. G., and C. C. Goodrich (2007), Does the polar cap area saturate? *Geophys. Res. Lett.*, 34(9), L09107, doi:10.1029/2007GL029357.

McIntosh, R. C., & Anderson, P. C. (2014). Maps of precipitating electron spectra characterized by Maxwellian and kappa distributions. *Journal of Geophysical Research: Space Physics*, 119(12), 10,116–10,132. <https://doi.org/10.1002/2014JA020080>

Meier, R. R., D. J. Strickland, J. H. Hecht, and A. B. Christensen (1989), Deducing composition and incident electron spectra from ground-based auroral optical measurements: A study of auroral red line processes, *J. Geophys. Res.*, 94(A10), 13,541–13,552, doi: 10.1029/JA094iA10p13541.

Mitchell, E. J., R. E. Lopez, R. J. Bruntz, M. Wiltberger, J. G. Lyon, R. C. Allen, S. J. Cockrell, and P. L. Whittlesey (2010), Saturation of transpolar potential for large Y component interplanetary magnetic field, *Journal of Geophysical Research (Space Physics)*, 115(A6), A06201, doi:10.1029/2009JA015119.

Newell, P. T., and C.-I. Meng (1988), The cusp and the cleft/boundary layer: Low-altitude identification and statistical local time variation, *J. Geophys. Res.*, 93(A12), 14,549– 14,556, doi:10.1029/JA093iA12p14549.

Newell, P. T., C.-I. Meng, D. G. Sibeck, and R. Lepping (1989), Some low-altitude cusp dependencies on the interplanetary magnetic field, *J. Geophys. Res.*, 94(A7), 8921–8927, doi:10.1029/JA094iA07p08921.

Newell, P. T., and C.-I. Meng (1994), Ionospheric projections of magnetospheric regions under low and high solar wind pressure conditions, *J. Geophys. Res.*, 99(A1), 273–286, doi:10.1029/93JA02273.

Newell, P. T., Ruohoniemi, J. M., & Meng, C.-I. (2004). Maps of precipitation by source region, binned by IMF, with inertial convection streamlines. *Journal of Geophysical Research: Space Physics*, 109(A10). <https://doi.org/10.1029/2004JA010499>

Newell, P. T., T. Sotirelis, K. Liou, C. I. Meng, and F. J. Rich (2007), A nearly universal solar wind-magnetosphere coupling function inferred from 10 magnetospheric state variables, *Journal of Geophysical Research (Space Physics)*, 112(A1), A01206, doi: 10.1029/2006JA012015.

Newell, P. T., T. Sotirelis, K. Liou, and F. J. Rich (2008), Pairs of solar wind-magnetosphere coupling functions: Combining a merging term with a viscous term works best, *Journal of Geophysical Research (Space Physics)*, 113(A4), A04218, doi: 10.1029/2007JA012825.

Newell, P. T., T. Sotirelis, and S. Wing (2009), Diffuse, monoenergetic, and broadband aurora: The global precipitation budget, *J. Geophys. Res.*, 114, A09207, doi:10.1029/2009JA014326.

- Newell, Patrick T., Sotirelis, T., & Wing, S. (2010). Seasonal variations in diffuse, monoenergetic, and broadband aurora. *Journal of Geophysical Research: Space Physics*, 115(A3). <https://doi.org/10.1029/2009JA014805>
- Newell, P. T., Liou, K., Zhang, Y., Sotirelis, T., Paxton, L. J., & Mitchell, E. J. (2014). OVATION Prime-2013: Extension of auroral precipitation model to higher disturbance levels. *Space Weather*, 12(6), 368–379. <https://doi.org/10.1002/2014SW001056>
- Papitashvili, V. O., and F. J. Rich (2002), High-latitude ionospheric convection models derived from Defense Meteorological Satellite Program ion drift observations and parameterized by the interplanetary magnetic field strength and direction, *Journal of Geophysical Research (Space Physics)*, 107(A8), 1198, doi:10.1029/2001JA000264.
- Pedatella, N. M., Lu, G., & Richmond, A. D. (2018). Effects of high-latitude forcing uncertainty on the low-latitude and midlatitude ionosphere. *Journal of Geophysical Research: Space Physics*, 123, 862–882. <https://doi.org/10.1002/2017JA024683>
- Prölss, G. W. (2011). Density Perturbations in the Upper Atmosphere Caused by the Dissipation of Solar Wind Energy. *Surveys in Geophysics*, 32(2), 101–195. <https://doi.org/10.1007/s10712-010-9104-0>
- Rees, M. H. (1989). *Physics and chemistry of the upper atmosphere* (Vol. 1). Cambridge University Press.
- Redmon, R. J., W. F. Denig, L. M. Kilcommons, and D. J. Knipp (2017), New DMSP database of precipitating auroral electrons and ions, *J. Geophys. Res. Space Physics*, 122, 9056–9067, doi:10.1002/2016JA023339.
- Richards, P. G. (2013), Reevaluation of thermosphere heating by auroral electrons, *Advances in Space Research*, 51(4), 610–619, doi:10.1016/j.asr.2011.09.004.
- Richmond, A. D., and Y. Kamide (1988), Mapping electrodynamic features of the high-latitude ionosphere from localized observations: Technique, *J. Geophys. Res.*, 93(A6), 5741–5759, doi:10.1029/JA093iA06p05741.
- Richmond, A. D. (1992), Assimilative mapping of ionospheric electrodynamics, *Advances in Space Research*, 12(6), 59–68, doi:10.1016/0273-1177(92)90040-5.
- Richmond, A. D. (1995), Ionospheric Electrodynamics Using Magnetic Apex Coordinates., *Journal of Geomagnetism and Geoelectricity*, 47(2), 191–212, doi:10.5636/jgg.47.191.
- Richmond, A.D. (accepted). Joule heating in the thermosphere. In W. Wang, Y. Zhang (Eds.), *Advances in Upper Atmosphere Research: Dynamics and Energetics*. Washington, DC: American Geophysical Union.
- Ridley, A. J., and E. A. Kihn (2004), Polar cap index comparisons with AMIE cross polar cap

potential, electric field, and polar cap area, *Geophys. Res. Lett.*, 31(7), L07801, doi: 10.1029/2003GL019113.

Robinson, R. M., Vondrak, R. R., Miller, K., Dabbs, T., & Hardy, D. (1987). On calculating ionospheric conductances from the flux and energy of precipitating electrons. *Journal of Geophysical Research: Space Physics*, 92(A3), 2565–2569. <https://doi.org/10.1029/JA092iA03p02565>

Roble, R. G., & Ridley, E. C. (1987). An auroral model for the NCAR thermospheric general circulation model (TGCM). *AnGeo*, 5, 369-382.

Rodger, A. S., Wells, G. D., Moffett, R. J., & Bailey, G. J. (2001). The variability of Joule heating, and its effects on the ionosphere and thermosphere. *Annales Geophysicae*, 19(7), 773–781. <https://doi.org/10.5194/angeo-19-773-2001>

Sheng, C., Deng, Y., Chen, Y.-J., Heelis, R. A., & Huang, Y. (2019). Effects of Alignment Between Particle Precipitation and Ion Convection Patterns on Joule Heating. *Journal of Geophysical Research: Space Physics*, 124(6), 4905–4915. <https://doi.org/10.1029/2018JA026446>

Shepherd, S. G. (2007). Polar cap potential saturation: Observations, theory, and modeling. *Journal of Atmospheric and Solar-Terrestrial Physics*, 69(3), 234–248. <https://doi.org/10.1016/j.jastp.2006.07.022>

Sotirelis, T., J. M. Ruohoniemi, R. J. Barnes, P. T. Newell, R. A. Greenwald, J. P. Skura, and C.-I. Meng (2005), Comparison of SuperDARN radar boundaries with DMSP particle precipitation boundaries, *J. Geophys. Res.*, 110, A06302, doi:10.1029/2004JA010732.

Strickland, D. J., J. Daniell, R. E., J. R. Jasperse, and B. Basu (1993), Transport-theoretic model for the electron-proton-hydrogen atom aurora 2. Model results, *J. Geophys. Res.*, 98(A12), 21,533–21,548, doi:10.1029/93JA01645.

Thayer, J. P. (2000). High-latitude currents and their energy exchange with the ionosphere-thermosphere system. *Journal of Geophysical Research: Space Physics*, 105(A10), 23015–23024. <https://doi.org/10.1029/1999JA000409>

Thébault, E., Finlay, C. C., Beggan, C. D., Alken, P., Aubert, J., Barrois, O., Bertrand, F., Bondar, T., Boness, A., Brocco, L., Canet, E., Chambodut, A., Chulliat, A., Coisson, P., Civet, F., Du, A., Fournier, A., Fratter, I., Gillet, N., ... Zvereva, T. (2015). International Geomagnetic Reference Field: The 12th generation. *Earth, Planets and Space*, 67(1), 79. <https://doi.org/10.1186/s40623-015-0228-9>

Thomas, E. G., & Shepherd, S. G. (2018). Statistical Patterns of Ionospheric Convection Derived From Mid-latitude, High-Latitude, and Polar SuperDARN HF Radar Observations. *Journal of Geophysical Research: Space Physics*, 123(4), 3196–3216. <https://doi.org/10.1002/2018JA025280>



Verkhoglyadova, O. P., Meng, X., Mannucci, A. J., & McGranaghan, R. M. (2018). Semianalytical Estimation of Energy Deposition in the Ionosphere by Monochromatic Alfvén Waves. *Journal of Geophysical Research: Space Physics*, 123(6), 5210–5222. <https://doi.org/10.1029/2017JA025097>

Vickrey, J. F., Vondrak, R. R., & Matthews, S. J. (1981). The diurnal and latitudinal variation of auroral zone ionospheric conductivity. *Journal of Geophysical Research: Space Physics*, 86(A1), 65–75. <https://doi.org/10.1029/JA086iA01p00065>

Weimer, D. R. (1995), Models of high-latitude electric potentials derived with a least error fit of spherical harmonic coefficients, *J. Geophys. Res.*, 100(A10), 19,595–19,608, doi: 10.1029/95JA01755.

Weimer, D. R. (2005), Improved ionospheric electrodynamic models and application to calculating Joule heating rates, *Journal of Geophysical Research (Space Physics)*, 110, A05306, doi: 10.1029/2004JA010884.

Wilder, F. D., C. R. Clauer, and J. B. H. Baker (2008), Reverse convection potential saturation during northward IMF, *Geophys. Res. Lett.*, 35(12), L12103, doi: 10.1029/2008GL034040.

Wing, S., Khazanov, G. V., Sibeck, D. G., & Zesta, E. (2019). Low Energy Precipitating Electrons in the Diffuse Aurorae. *Geophysical Research Letters*, 46(7), 3582–3589. <https://doi.org/10.1029/2019GL082383>

Zhang, B., Lotko, W., Brambles, O., Wiltberger, M., & Lyon, J. (2015). Electron precipitation models in global magnetosphere simulations. *Journal of Geophysical Research: Space Physics*, 120(2), 1035–1056. <https://doi.org/10.1002/2014JA020615>

Zhang, Y., and L. J. Paxton (2008), An empirical Kp-dependent global auroral model based on TIMED/GUVI FUV data, *Journal of Atmospheric and Solar-Terrestrial Physics*, 70, 1231–1242, doi: 10.1016/j.jastp.2008.03.008.

Zhu, Q., Deng, Y., Richmond, A., & Maute, A. (2018). Small-Scale and Mesoscale Variabilities in the Electric Field and Particle Precipitation and Their Impacts on Joule Heating. *Journal of Geophysical Research: Space Physics*, 123(11), 9862–9872. <https://doi.org/10.1029/2018JA025771>

Zhu, Q., Deng, Y., Richmond, A., Maute, A., Chen, Y. J., Hairston, M., ... & Mitchell, E. (2020a). Impacts of Binning Methods on High-Latitude Electrodynamical Forcing: Static Versus Boundary-Oriented Binning Methods. *Journal of Geophysical Research: Space Physics*, 125(1), e2019JA027270.

Zhu, Qingyu, Deng, Yue, Maute, Astrid, Kilcommons, Liam, Knipp, Delores, & Hairston, Marc. (2020b). ASHLEY: A new empirical model for the high-latitude electron precipitation and electric field (Version V1). <http://doi.org/10.5281/zenodo.4151717>.

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1162 Zhu, Qingyu, Deng, Yue, Maute, Astrid, Kilcommons, Liam, Knipp, Delores, & Hairston, Marc.  
1163 (2020c). qyzhuta/ASHLEY\_v1: beta (Version v1). Zenodo.  
1164 <https://doi.org/10.5281/zenodo.4152364>.

# Tables and Figures

Table 1a. Electron precipitation

	1	2	3	4	5	6	7	8	9
Lower boundary	0	3000	5000	7000	9000	11000	13000	16000	20000
Upper boundary	3000	5000	7000	9000	11000	13000	16000	20000	30000
Median	2579	4283	6073	7956	9930	11942	14254	17590	22770

Table 1b. Electric field/potential

	1	2	3	4	5	6	7
Lower boundary	0	3000	5500	7500	9500	12000	16000
Upper boundary	3000	5500	7500	9500	12000	16000	24000
Median	2583	4615	6539	8456	10614	13524	18357

Table 1. Summary of the lower and upper boundaries along with the median value of each  $\varepsilon_t$  bin used in binning (a) the electron precipitation and (b) electric field/potential data.

Variable(s)	Meaning	Unit	Calculation	First shown
$\mathbf{V}$ ( $\mathbf{V}'$ )	bulk ion drift vector before (after) baseline correction	m/s	–	Section 2.1
$V_x, V_y, V_z$ ( $V'_x, V'_y, V'_z$ )	Components of $\mathbf{V}$ ( $\mathbf{V}'$ ) in the spacecraft coordinate system	m/s	–	Section 2.1
$\mathbf{B}_0, B_{0z}$	Background geomagnetic main field vector and its vertical component at the satellite location, respectively	nT	–	Section 2.1
$\mathbf{E}$	Electric field vector	mV/m	$-\mathbf{V}' \times \mathbf{B}_0$	Section 2.1
$E_x$	Along-track electric field component	mV/m	$\approx -V'_y B_{0z}$	Section 2.1
$B_y, B_z$	IMF y and z components in the GSM coordinates	nT	–	Section 2.2
$B_T$	IMF transverse component magnitude	nT	$\sqrt{B_y^2 + B_z^2}$	Section 2.2
$\theta_c$	IMF clock angle	degree	$\text{atan2}(B_y, B_z)$	Section 2.2
$V_{sw}, N_{sw}$	Solar wind flow speed and proton density, respectively	km/s, $\text{cm}^{-3}$	–	Section 2.2
$\varepsilon_t$	Coupling function	$(\text{km}^{4/3} \text{ s}^{-4/3} \text{ nT}^{2/3} \text{ cm}^{-1/2})$	$V_{sw}^{4/3} B_T^{2/3} N_{sw}^{1/6}$	Section 2.3
$J_E$	Differential energy flux	$\text{eV}/(\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ eV}^{-1})$	–	Section 3.1.1
$\phi$	Azimuthal angle	radian	$\frac{\text{MLT}}{12} \pi$	Section 3.1.1
$\omega$	IMF clock angle in radians	radian	$\frac{\theta_c}{180^\circ} \pi$	Section 3.1.1
$\Phi$	Electric potential	kV	–	Section 3.1.2
$\lambda_m$	MLAT in radians	radian	–	Section 3.1.2

$\theta$	Polar angle	radian	$\frac{\pi}{2} - \lambda_m$ $\frac{\pi}{4}$	Section 3.1.2
$E_{d1}, E_{d2}$	Electric field components in $\mathbf{d}_1$ and $\mathbf{d}_2$ directions (Richmond, 1995)	mV/m	–	Section 3.1.2
$E'_{d1}, E'_{d2}$	Residuals between the measured and modeled $E_{d1}$ and $E_{d2}$ , respectively	mV/m	–	Section 3.1.3
$\sigma_1, \sigma_2$	Standard deviation of $E'_{d1}$ and $E'_{d2}$ , respectively	mV/m	–	Section 3.1.3
$\Phi_{PC}$	Cross-polar-cap potential	kV	–	Section 3.2.1.2
$\varepsilon_{inf}$	Saturation level of $\varepsilon_t$	$(\text{km}^{4/3} \text{ s}^{-4/3}$ $\text{nT}^{2/3} \text{ cm}^{-1/2})$	–	Section 3.2.1.2
$\beta$	Saturation factor	–	$\frac{\varepsilon_t}{\sqrt{1 + \left(\frac{\varepsilon_t}{\varepsilon_{inf}}\right)^2}}$	Section 3.2.1.2
$r$	Averaged co-MLAT of the boundary identified on dawn and dusk sides	degree	–	Section 3.2.1.2
$Q_0, \bar{E}$	Total energy flux and average energy, respectively	mW/m <sup>2</sup> , eV	Eq. 2 in Robinson et al. (1987)	Section 5.1
$\bar{E}_{d1}, \bar{E}_{d2}$	$E_{d1}$ and $E_{d2}$ calculated from $\Phi$ , respectively	mV/m	Eqs. 4.8 and 4.9 in Richmond (1995)	Section 5.2
$E_1$	Mean electric field magnitude	mV/m	$\sqrt{\bar{E}_{d1}^2 + \bar{E}_{d2}^2}$	Section 5.2
$E_2$	Electric field variability magnitude	mV/m	$\sqrt{\sigma_1^2 + \sigma_2^2}$	Section 5.2

Table 2. Summary of variables defined in Sections 2-5.

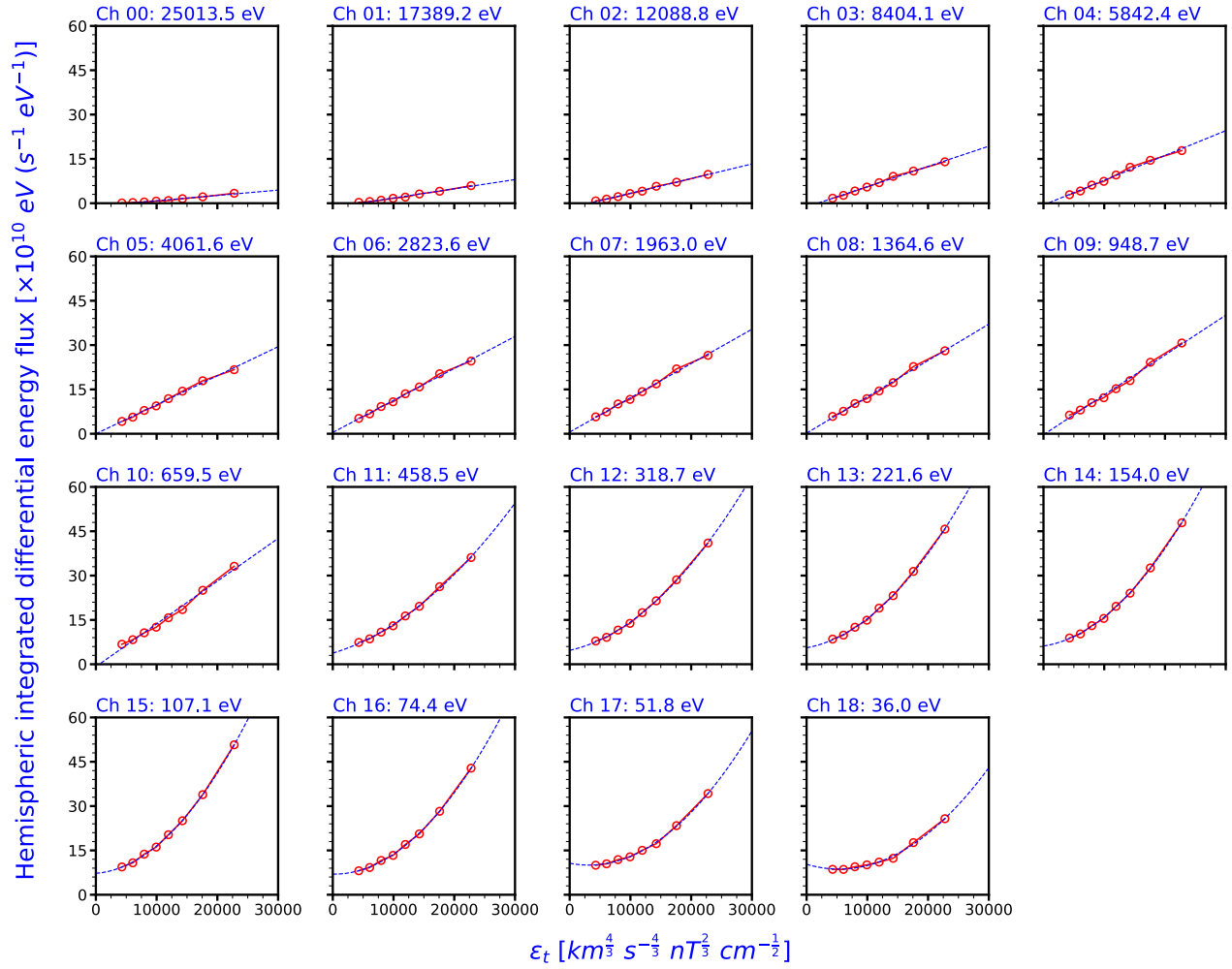


Figure 1. Hemispheric-integrated differential energy fluxes in the 19 DMSP energy channels from all 8  $\varepsilon_t$ - $\theta_c$  bins for the ASHLEY-A development where  $\theta_c$  is centered at  $225^\circ$ . The blue dashed lines represent the best-fit lines (parabolas) according to the red dots in the first 11 (last 8) plots.

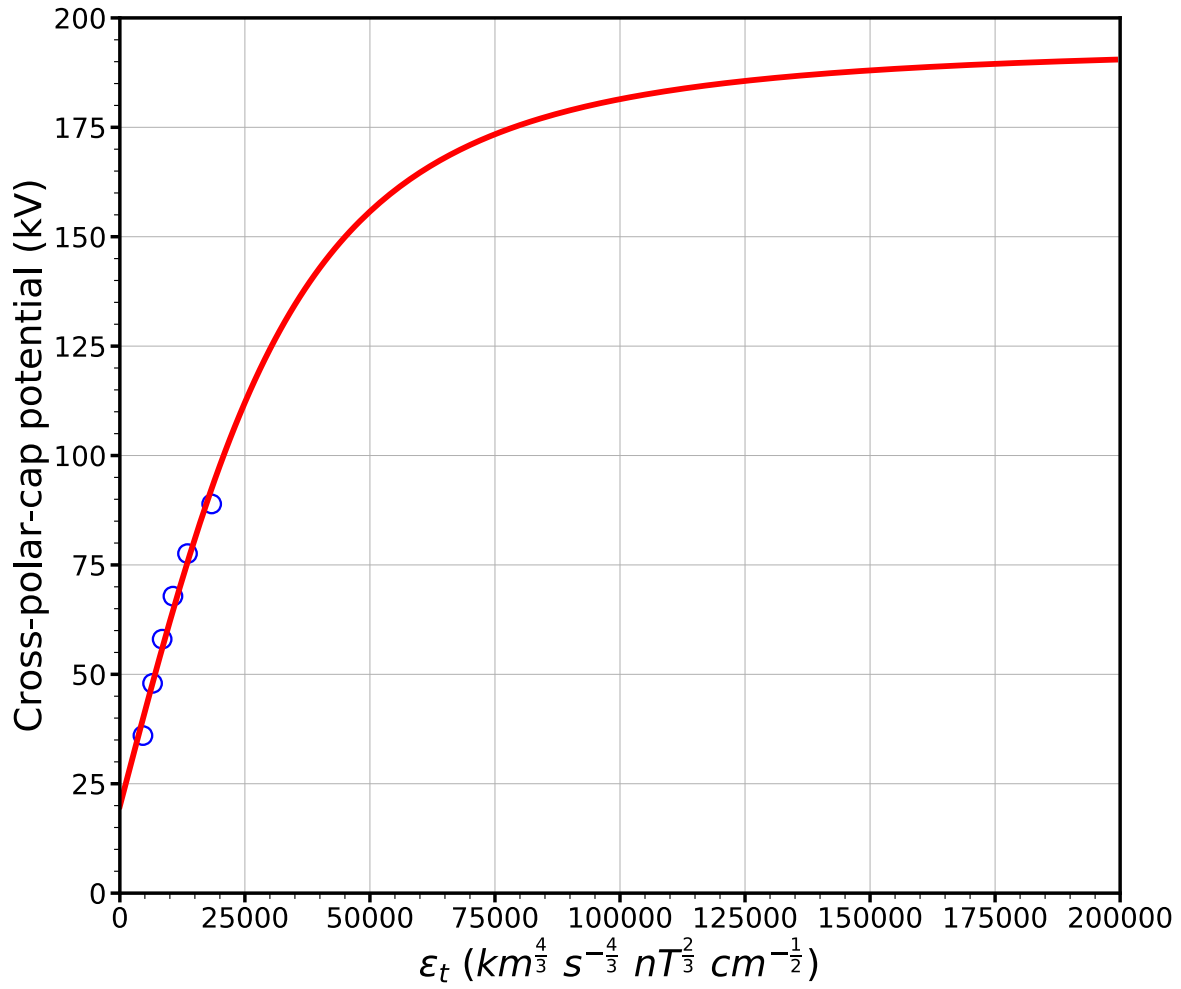


Figure 2. The cross-polar-cap potentials (CPCPs) from all 6  $\epsilon_t$ - $\theta_c$  bins for the ASHLEY-E development where  $\theta_c$  is centered at  $180^\circ$ . The red thick line represents the best-fit curve according to the blue circles.

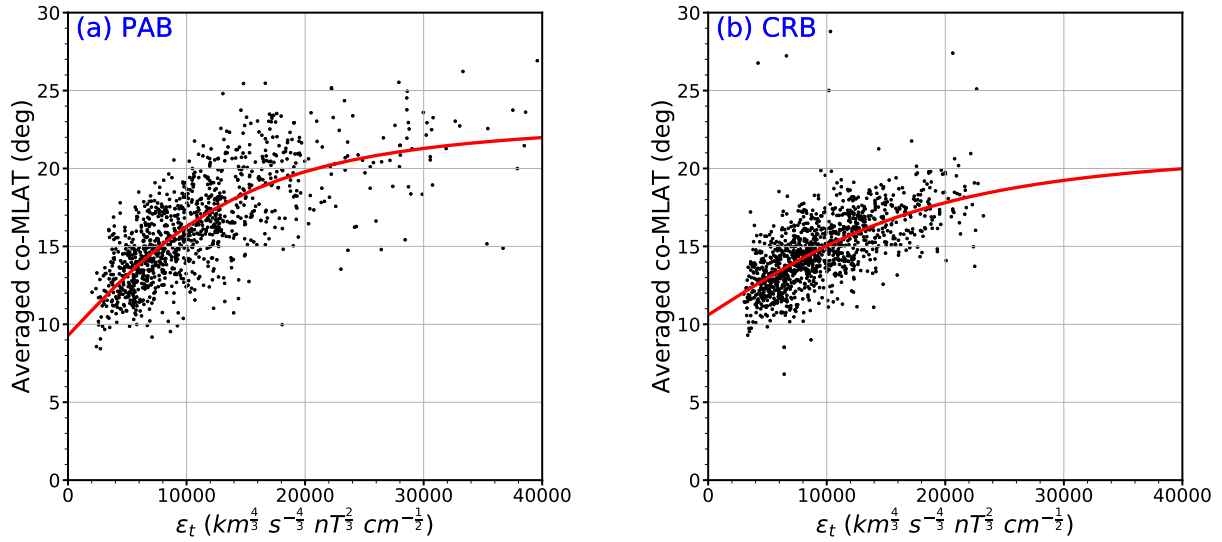
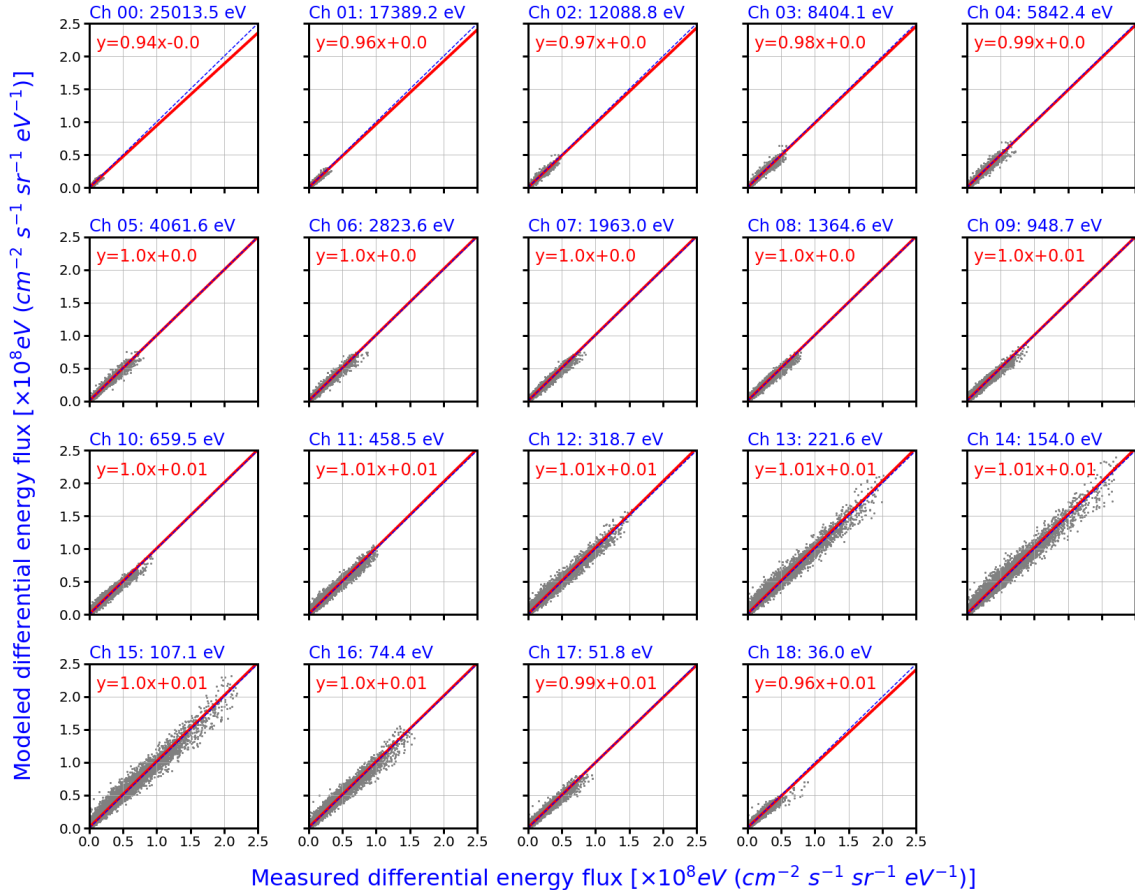


Figure 3. (a) Evolution of the averaged co-MLATs of the poleward auroral boundary (PAB) identified on the dawn (4-8 MLT) and dusk (16-20 MLT) sides along the same polar crossing as a function of  $\varepsilon_t$  when  $157.5^\circ < \theta_c < 202.5^\circ$ . (b) Evolution of the averaged co-MLATs of the convection reversal boundary (CRB) identified on the dawn (4-8 MLT) and dusk (16-20 MLT) sides along the same polar crossing as a function of  $\varepsilon_t$  when  $157.5^\circ < \theta_c < 202.5^\circ$ . The red-thick line in each plot indicates the best-fit curve according to the black dots. (MLAT=magnetic latitude)



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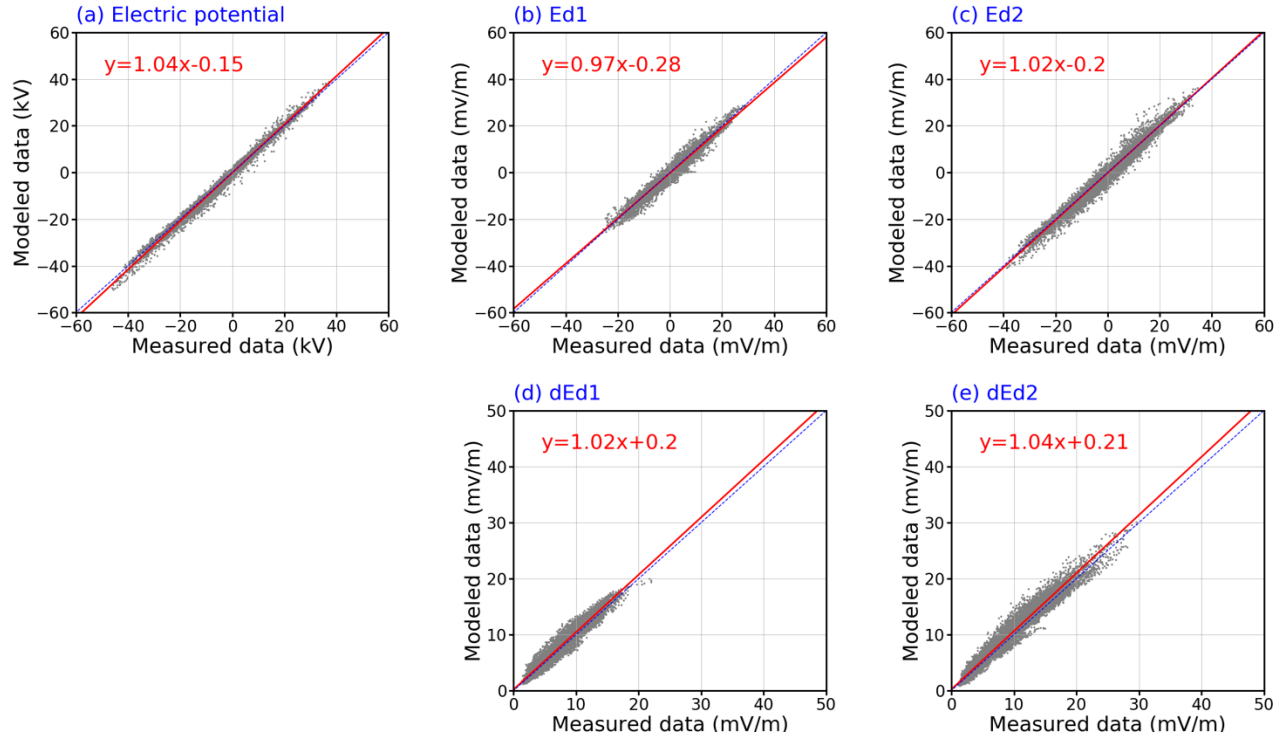
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Figure 4. Comparisons of the averages of measured and modeled differential energy fluxes from all MLT-MLAT and  $\varepsilon_t$ - $\theta_c$  bins in the 19 DMSP energy channels. The numbers of the MLT, MLAT and  $\varepsilon_t$ - $\theta_c$  bin are 24, 40 (50°-90° MLAT) and 65, respectively. The blue dashed line in each plot denotes the  $y = x$  line, and the red thick line indicates the best-fit line according to the grey dots. The equation of the best-fit line is given in each plot.

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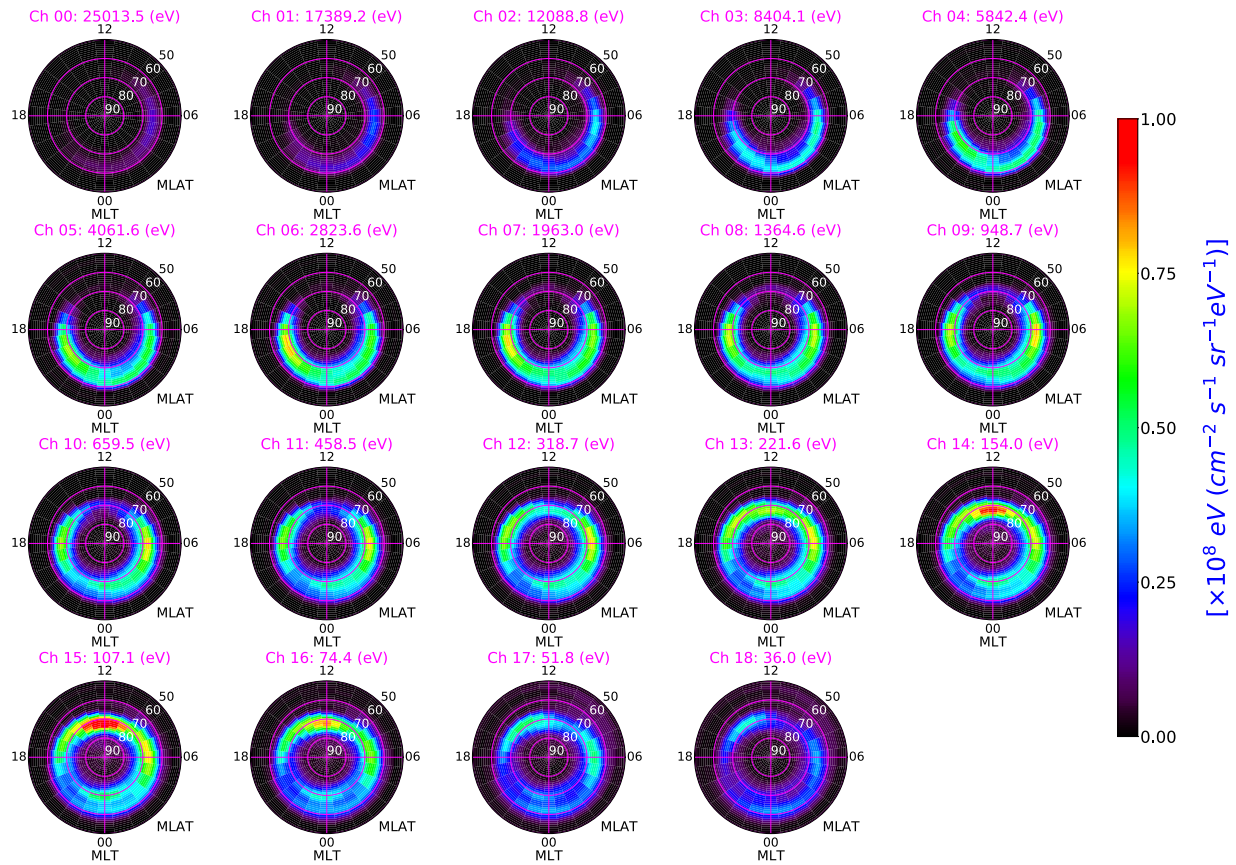
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1219 Figure 5. (Top) Comparisons of the averages of measured and modeled (a) electric potential, (b)  
 1220  $E_{d1}$  and (c)  $E_{d2}$  from all MLT-MLAT and  $\varepsilon_t$ - $\theta_c$  bins. (Bottom) Comparisons of (d) the standard  
 1221 deviation of measured  $E_{d1}$  and the root mean square (RMS) of modeled  $E_{d1}$  variability along with  
 1222 (e) the standard deviation of measured  $E_{d2}$  and the RMS of modeled  $E_{d2}$  variability from all  
 1223 MLT-MLAT and  $\varepsilon_t$ - $\theta_c$  bins. The numbers of the MLT, MLAT and  $\varepsilon_t$ - $\theta_c$  bin are 24, 20 ( $50^\circ$ - $90^\circ$   
 1224 MLAT) and 49, respectively. The blue dashed line in each plot denotes the  $y = x$  line, and the red  
 1225 thick line represents indicate the best-fit line according to the grey dots. The equation of the best-  
 1226 fit line is given in each plot.

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Figure 6. Distributions of the differential energy flux in the 19 DMSP energy channels as a function of MLT and MLAT (IMF  $B_y = 0$ , IMF  $B_z = -8$  nT,  $V_{sw} = 450$  km/s and  $N_{sw} = 5$  cm $^{-3}$ ). All plots are presented in geomagnetic coordinates. (DMSP: Defense Meteorological Satellite Program; MLT=magnetic local time; MLAT=magnetic latitude; IMF=interplanetary magnetic field)

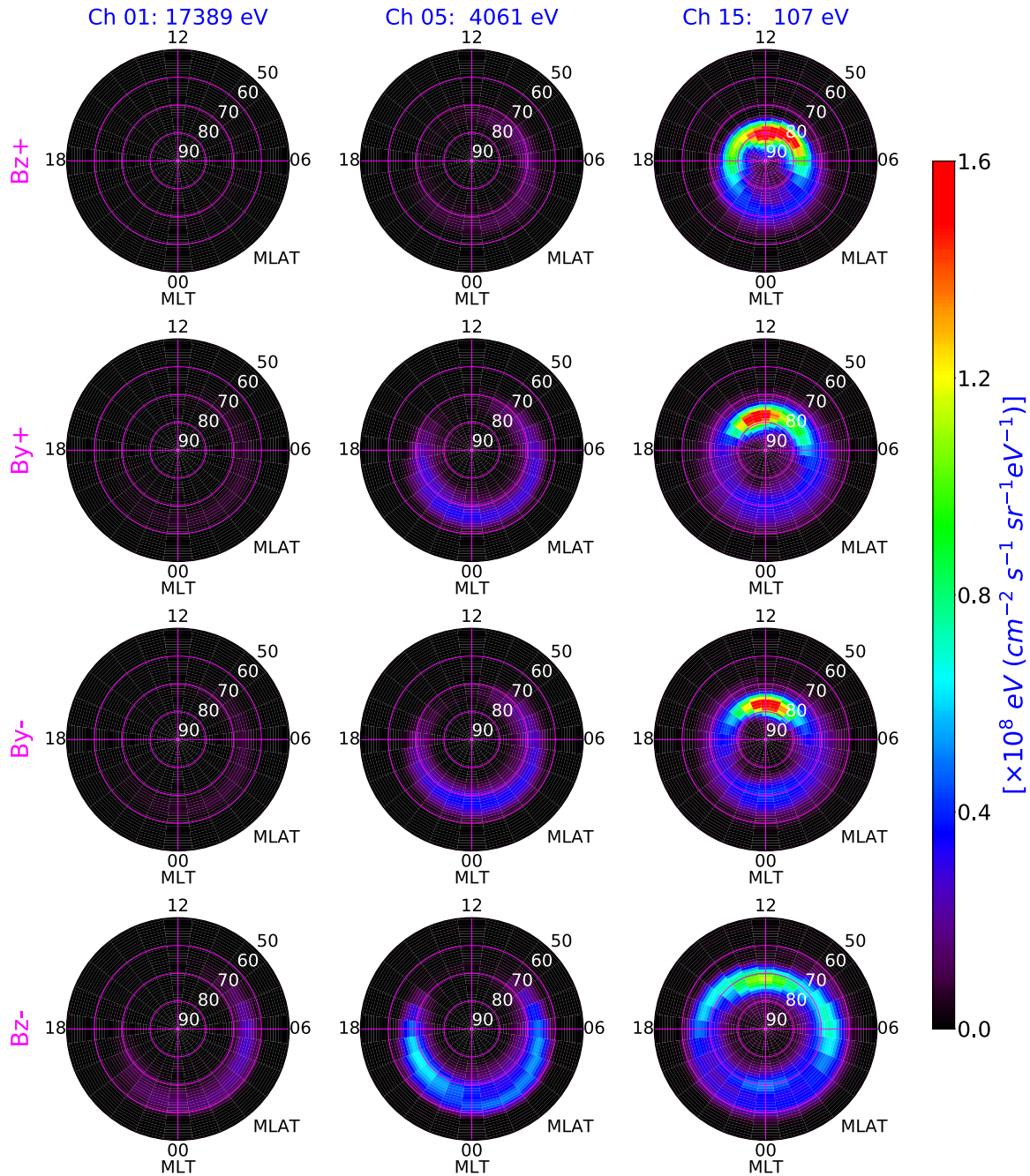
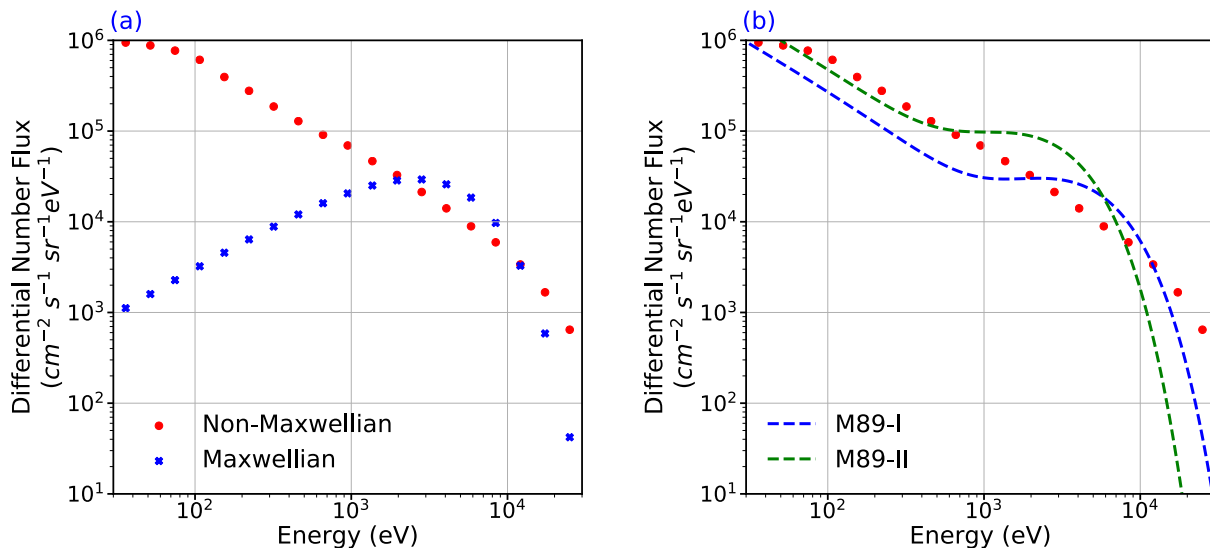


Figure 7. Distributions of the differential energy flux in 3 selected DMSP energy channels as a function of MLAT and MLT when the IMF is purely northward, downward, duskward and southward (from top to bottom). For these four cases, the IMF and solar wind conditions are: IMF  $B_T = 8$  nT,  $V_{SW} = 450$  km/s and  $N_{SW} = 5$  cm<sup>-3</sup>. All plots are presented in geomagnetic coordinates. (DMSP: Defense Meteorological Satellite Program; MLT=magnetic local time; MLAT=magnetic latitude; IMF=interplanetary magnetic field)

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Figure 8 (a) Comparisons of the differential number fluxes between the modeled spectrum (Red dots) and a Maxwellian spectrum (Blue crosses) derived from the total energy flux and average energy of the  $>500$  eV portion of the modeled spectrum. (b) Comparisons of the differential number fluxes between the modeled spectrum (Red dots) and two spectra calculated by using the Meier 1989 formula (blue and green dashed lines). The blue and green dashed lines indicate the spectra calculated by using the total energy flux and average energy of the  $>500$  eV portion of the modeled spectrum and the whole modeled spectrum, respectively. The location is at MLT=4.5 h and MLAT=64.5°. The IMF and solar wind conditions are: IMF  $B_y = 0$ , IMF  $B_z = -8$  nT,  $V_{\text{SW}} = 450$  km/s and  $N_{\text{SW}} = 5 \text{ cm}^{-3}$ . (MLT=magnetic local time; MLAT=magnetic latitude; IMF=interplanetary magnetic field)

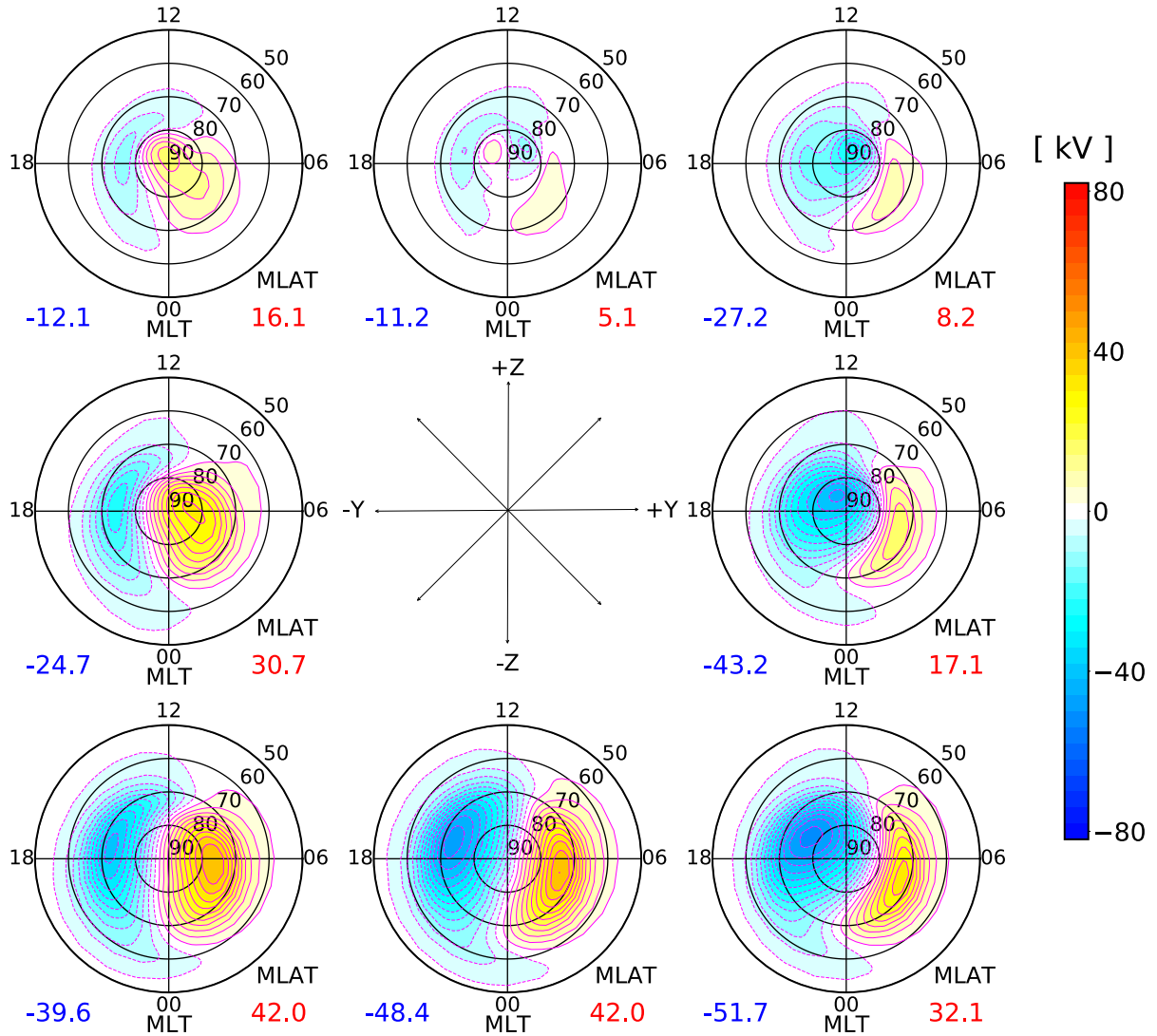


Figure 9. High-latitude electric potential outputs of ASHLEY-E at eight different IMF clock angles as a function of MLAT and MLT. For the cases shown in this figure, the solar wind and IMF conditions are: IMF  $B_T = 8$  nT,  $V_{SW} = 450$  km/s and  $N_{SW} = 5$  cm<sup>-3</sup>. The maximum and minimum electric potential of each case are indicated on the bottom left and right sides of each plot, respectively, and the contour interval is 4 kV. All plots are presented in geomagnetic coordinates. (IMF=interplanetary magnetic field)



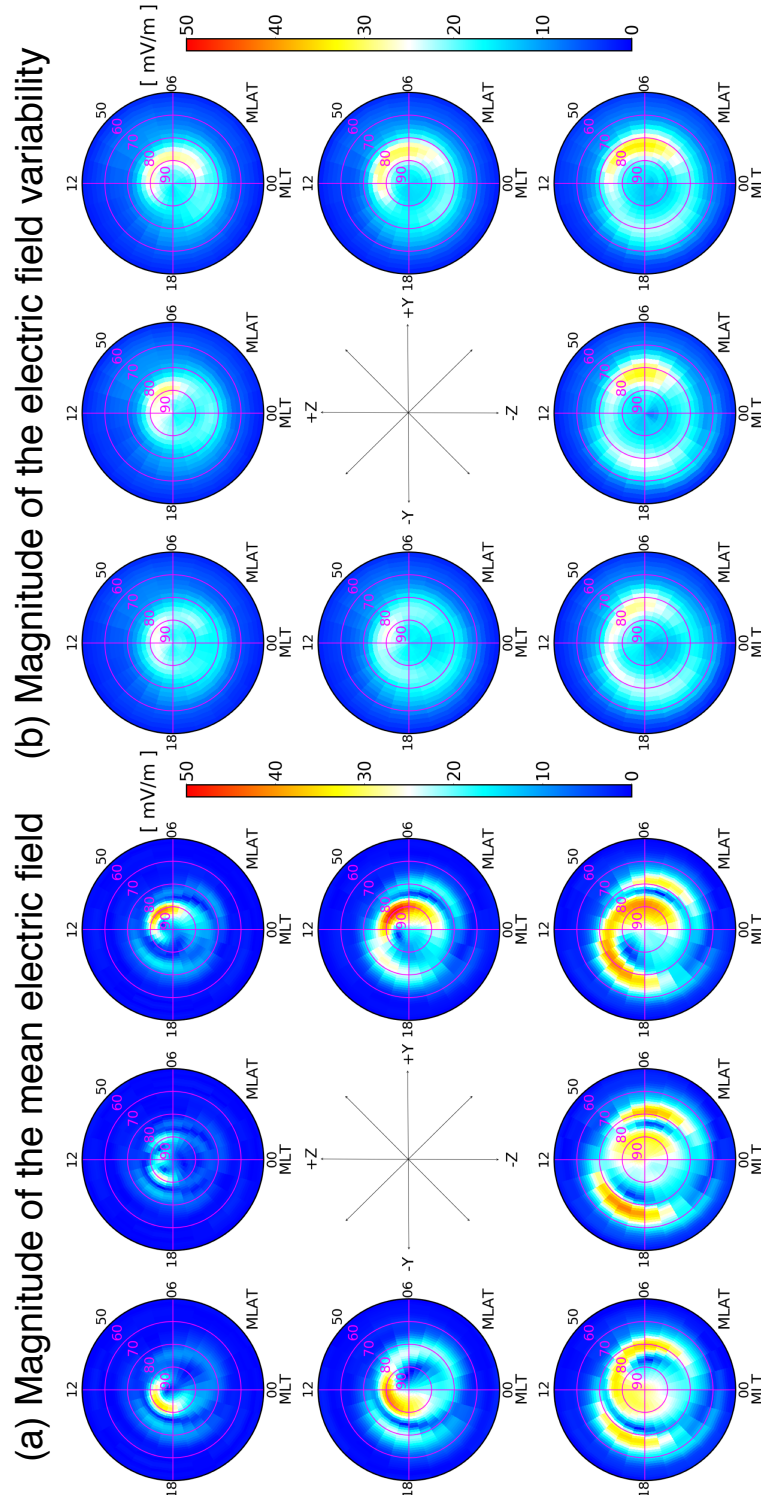
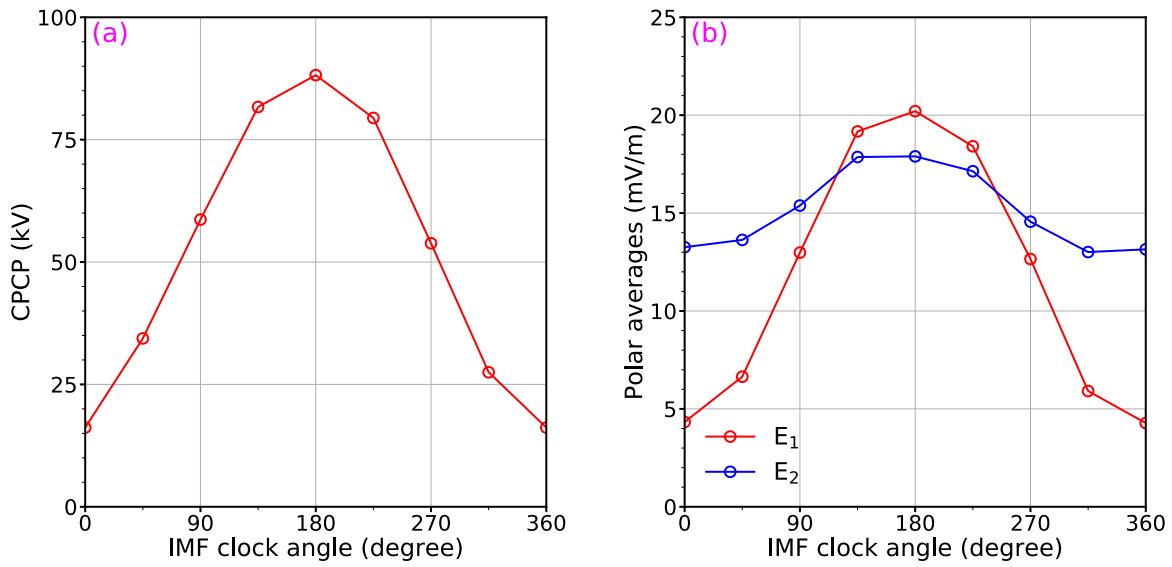


Figure 10. Distributions of the (a) mean electric field and (b) electric field variability magnitudes as a function of MLAT and MLT for eight different IMF clock angles. For the cases shown here, the IMF and solar wind conditions are: IMF  $B_T = 8$  nT,  $V_{SW} = 450$  km/s and  $N_{SW} = 5$  cm<sup>-3</sup>. All plots are presented in geomagnetic coordinates. (MLT=magnetic local time; MLAT=magnetic latitude; IMF=interplanetary magnetic field)

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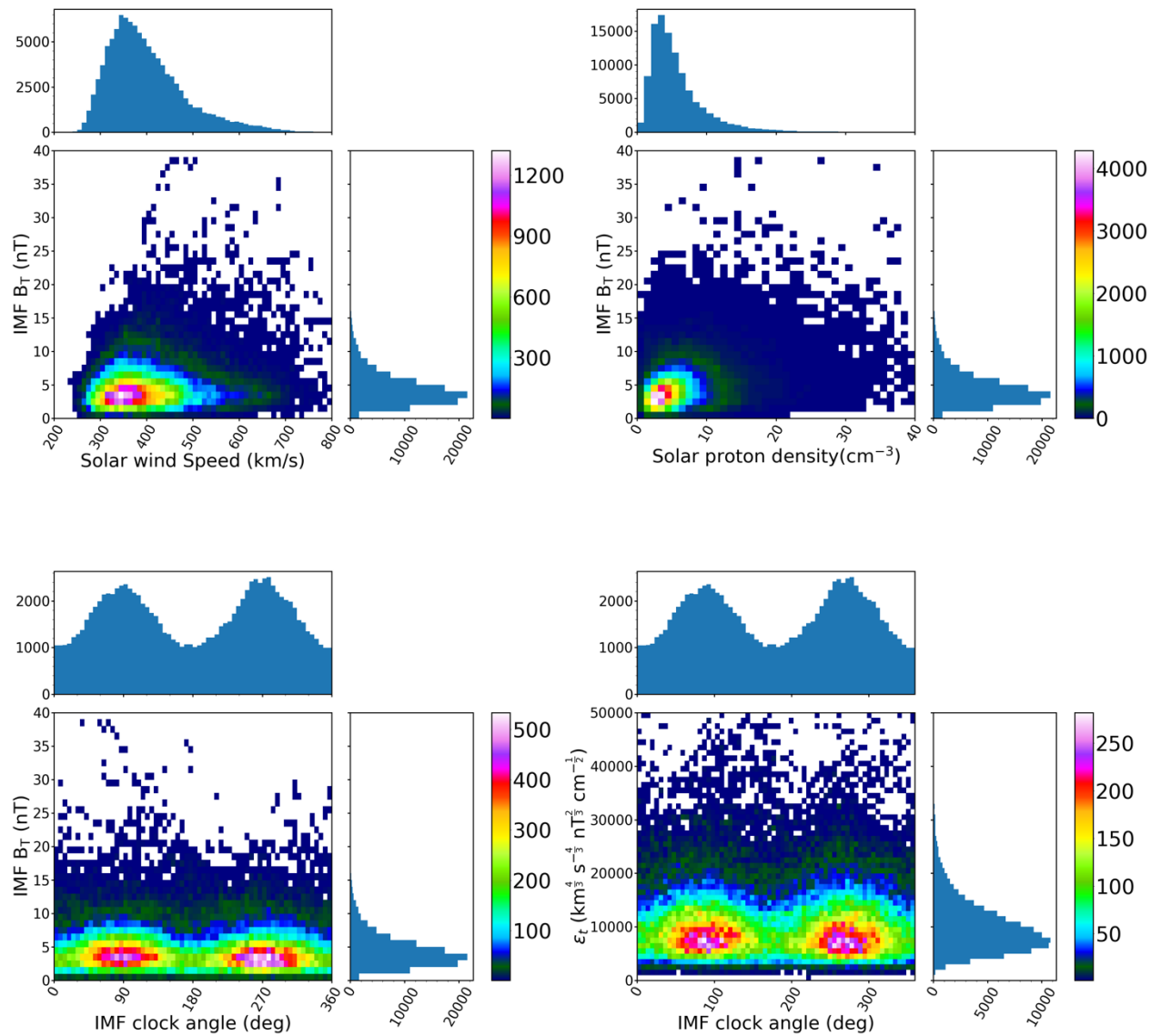
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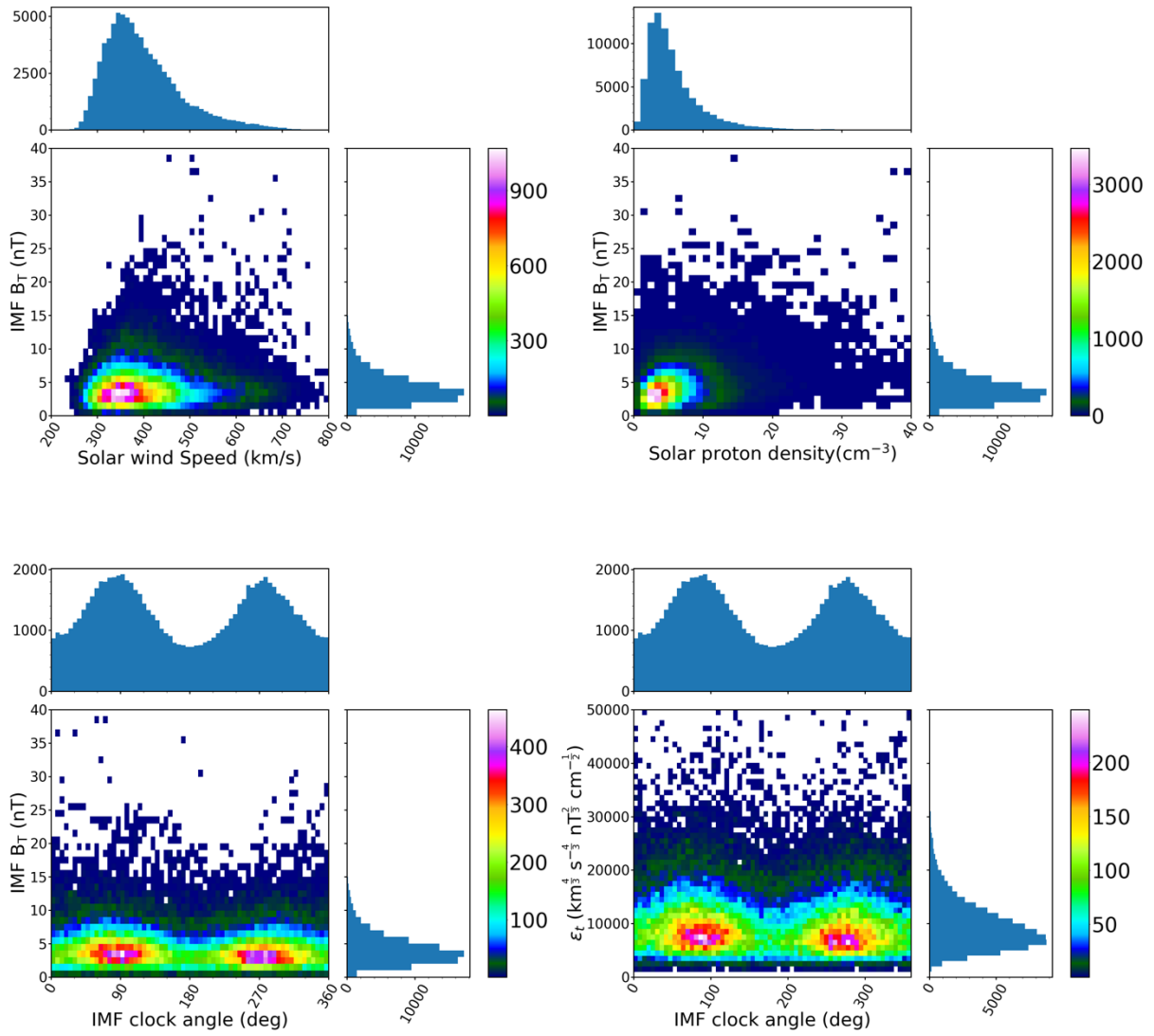
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Figure 11. (a) Variation of the cross-polar-cap potential (CPCP) with respect to the IMF clock angle; (b) Variations of the averages of the mean electric field magnitude ( $E_1$ , red) and electric field variability magnitude ( $E_2$ , blue) over the region where  $|\text{MLAT}| > 60^\circ$  as a function of the IMF clock angle. For the cases shown here, the IMF and solar wind conditions are: IMF  $B_T = 8$  nT,  $V_{\text{SW}} = 450$  km/s and  $N_{\text{SW}} = 5 \text{ cm}^{-3}$ . (MLAT=magnetic latitude; IMF=interplanetary magnetic field)

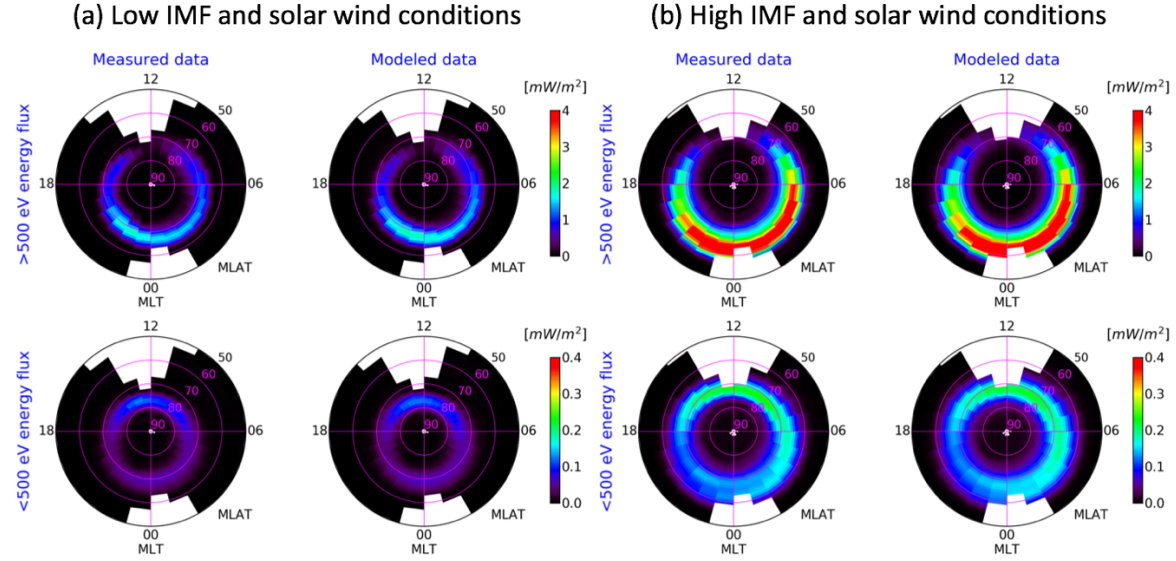




**Figure S1.** Distributions of the IMF and solar wind data used for the ASHLEY-A development.

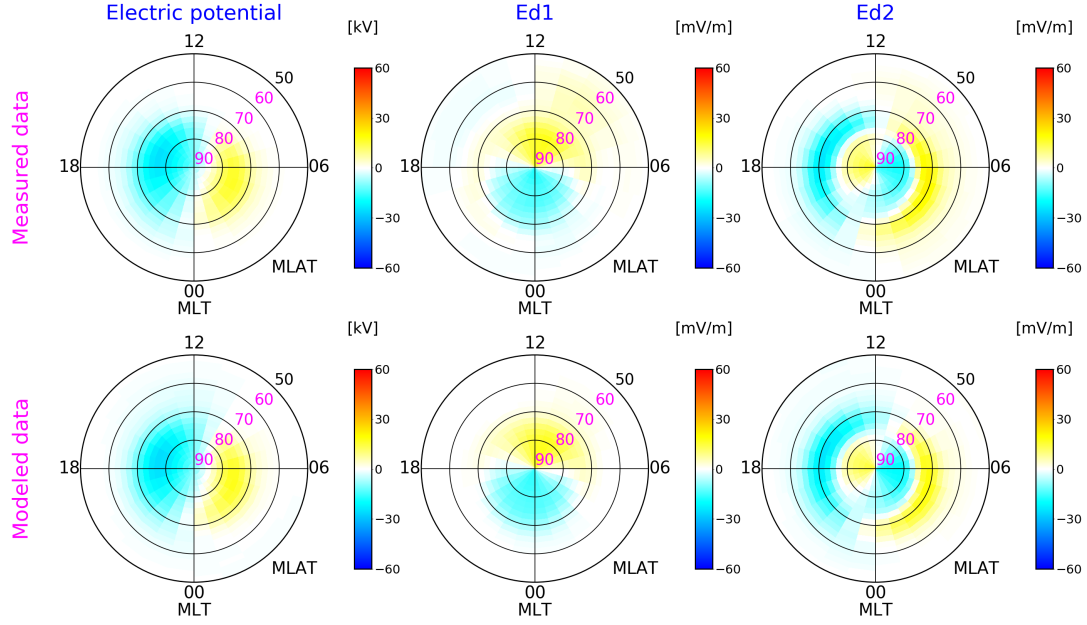


**Figure S2.** Distributions of the IMF and solar wind data used for the ASHLEY-E and ASHLEY-Evar developments.

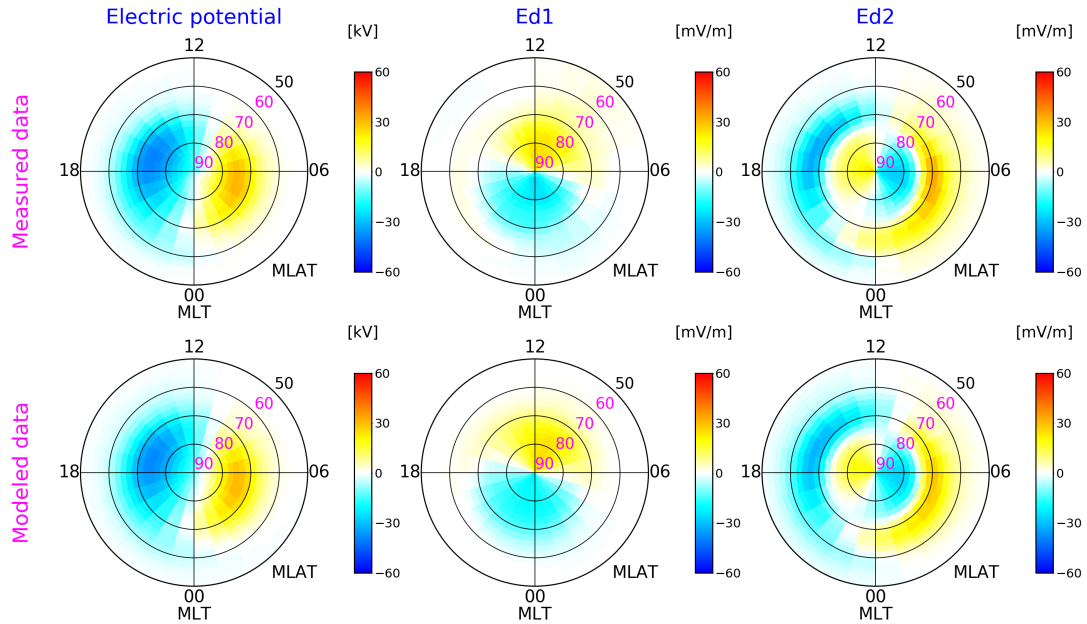


**Figure S3.** Comparisons of averages of the measured and modeled integrated differential energy fluxes in different MLAT-MLT bins under (a) low IMF and solar wind conditions ( $5000 \leq \varepsilon_t < 7000$ ,  $157.5^\circ < \theta_c < 202.5^\circ$ ) and (b) high IMF and solar wind conditions ( $20000 \leq \varepsilon_t < 30000$ ,  $112.5^\circ < \theta_c < 157.5^\circ$ ). The parameters shown in the top and bottom row of each plot are the integrated differential energy fluxes of  $>500$  eV and  $<500$  eV electrons, respectively. All plots are presented in geomagnetic coordinates.

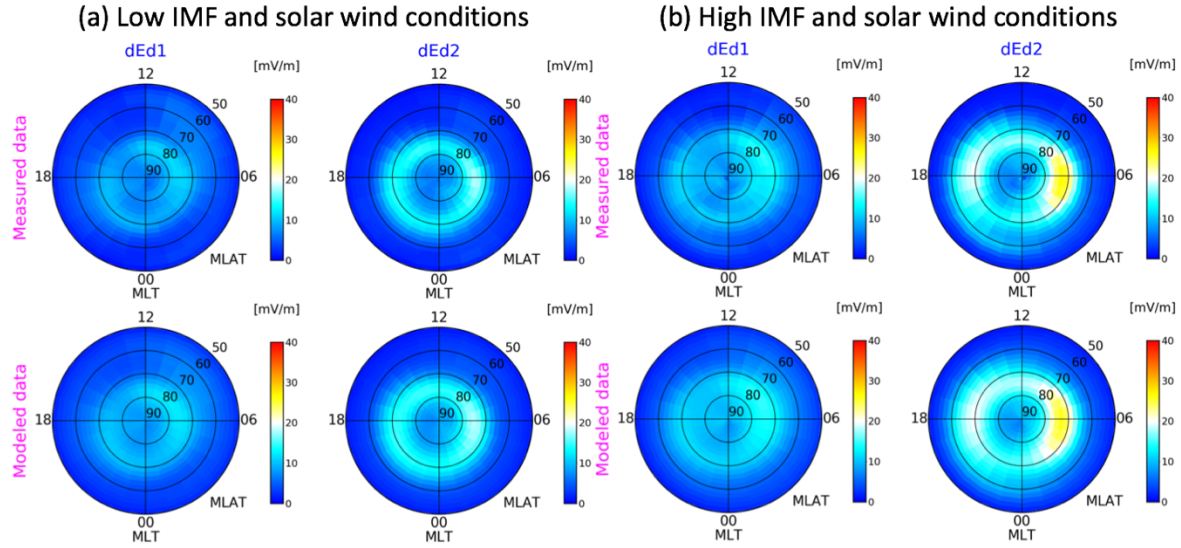
(a) Low IMF and solar wind conditions



(b) High IMF and solar wind conditions



**Figure S4.** Comparisons of averages of the measured and modeled electric potential,  $E_{d1}$  and  $E_{d2}$  in different MLAT-MLT bins under (a) low IMF and solar wind conditions ( $5500 \leq \epsilon_t < 7500$ ,  $157.5^\circ < \theta_c < 202.5^\circ$ ) and (b) high IMF and solar wind conditions ( $12000 \leq \epsilon_t < 16000$ ,  $157.5^\circ < \theta_c < 202.5^\circ$ ). Here,  $E_{d1}$  and  $E_{d2}$  are magnetic eastward and northward components of the electric field, respectively. All plots are presented in geomagnetic coordinates.



**Figure S5.** Comparisons of the standard deviations of measured  $E_{d1}$  and  $E_{d2}$  and the root mean squares (RMSs) of modeled  $E_{d1}$  and  $E_{d2}$  variabilities in different MLAT-MLT bins under (a) low IMF and solar wind conditions ( $5500 \leq \varepsilon_t < 7500$ ,  $157.5^\circ < \theta_c < 202.5^\circ$ ) and (b) high IMF and solar wind conditions ( $12000 \leq \varepsilon_t < 16000$ ,  $157.5^\circ < \theta_c < 202.5^\circ$ ). Here,  $E_{d1}$  and  $E_{d2}$  are magnetic eastward and northward components of the electric field, respectively. All plots are presented in geomagnetic coordinates.