Blending the Evaporation Precipitation Ratio with the Complementary Principle Function for the Prediction of Evaporation

Zhang Lu[1] and Wilfried Brutsaert[2]

[1]CSIRO Land and Water
[2]Cornell University

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Abstract

One class of descriptions of landscape evaporation is based on the principle that actual evaporation E and atmospheric evaporative demand exhibit complementary behavior. A feature of some recent implementations of this approach is the need for the estimation of a free parameter, usually by calibration. In a different class of representations of landscape evaporation, several functional forms have been proposed in the past for the dependency of the annual evaporation precipitation ratio (E/P) on the annual aridity index - the Schreiber-Oldekop hypothesis, also known as the Budyko framework. While there is no general agreement in the literature on the optimal formulation of the "maximum possible evaporation" , the functional forms of appear to be quite insensitive to its exact nature. This observation allows to be equated with the evaporative demand , and this immediately leads to a blending of the annual evaporation precipitation ratio (E/P) with the complementary evaporation principle, and the prediction of its unknown free parameter. As this free parameter is found to be relatively insensitive to time scale, the complementary functions become not only calibration-free at the annual time scale, but also applicable even at daily time scales. The results are shown to be applicable worldwide with experimental data from 516 catchment water balance set-ups and 152 high quality eddy covariance flux stations. The present approach offers a practical tool for the prediction of daily evaporation using only routine meteorological data such as air temperature, humidity, wind speed, net radiation, and long-term average precipitation.

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Lu Zhang

CSIRO Land and Water, Canberra, Australian Capital Territory, Australia

Wilfried Brutsaert

School of Civil and Environmental Engineering, Cornell University, Ithaca, New York, USA

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Abstract

One class of descriptions of landscape evaporation is based on the principle that actual evaporation E and atmospheric evaporative demand E_{pa} exhibit complementary behavior. A feature of some recent implementations of this approach is the need for the estimation of a free parameter, usually by calibration. In a different class of representations of landscape evaporation, several functional forms have been proposed in the past for the dependency of the annual evaporation precipitation ratio (E/P) on the annual aridity index $(E_{\max} / P)$ - the Schreiber-Oldekop hypothesis, also known as the Budyko framework. While there is no general agreement in the literature on the optimal formulation of the "maximum possible evaporation" $E_{\max}$, the functional forms of $(E / P) = f(E_{\max} / P)$ appear to be quite insensitive to its exact nature. This observation allows $E_{\max}$ to be equated with the atmospheric evaporative demand E_{pa}, and this immediately leads to a blending of the annual evaporation precipitation ratio (E/P) with the complementary evaporation principle, and the prediction of its unknown free parameter. As this free parameter is found to be relatively insensitive to time scale, the complementary functions become not only calibration-free at the annual time scale, but also applicable even at daily time scales. The results are shown to be applicable worldwide with experimental data from 524 catchment water balance set-ups and 156 high quality eddy covariance flux stations. The present approach offers a practical tool for the prediction of daily evaporation using only routine meteorological inputs including air temperature, specific humidity, wind speed, net radiation, and long-term average precipitation.

1. Introduction

As an important natural process, evaporation from the land surface has been studied extensively and various methods have been developed (Brutsaert, 1982, 2005; Wang and Dickinson, 2012; McMahon et al., 2013). One class of more practical methodologies for estimating evaporation is based on the well-known complementarity between actual evaporation E and atmospheric evaporative demand, also known as apparent potential evaporation E_{pa}. This complementary principle has been used in various ways to estimate evaporation at temporal scales ranging from hourly, to mean annual (Brutsaert and Stricker, 1979; Morton, 1983, Lemeur and Zhang, 1990; Parlange and Katul, 1992; Hobbins et al., 2001). Over the years the basic complementarity idea has been further generalized (e.g. Brutsaert and Parlange, 1998; Brutsaert, 2015) and this has enhanced an understanding of the underlying assumptions and has provided it a stronger base. The input data required by these more recent implementations of the complementary principle can be readily obtained from meteorological stations. However, their application still requires the estimation of at least one model parameter. Different ad-hoc methods have been used to deal with this issue, either by introducing additional rescaling assumptions with contrived variables (e.g. Crago et al., 2016; Crago and Qualls, 2018) or by calibrating the model parameters using measured evaporation data (e.g. Kahler and Brutsaert 2006; Liu et al. 2016, 2018; Brutsaert et al. 2017, Zhang et al. 2017; Brutsaert et al. 2020).

Another widely used methodology for estimating mean evaporation E, but only at the mean annual scale, is based on the Schreiber-Oldekop hypothesis, which makes use of maximum possible evaporation $E_{\max}$ and mean annual precipitation P. This hypothesis, which is also commonly known as Budyko's framework, states that the evaporation precipitation ratio (E/P) is a function of the aridity index $(E_{\max}/P)$ (e.g. Budyko 1974; Zhang et al., 2001, 2004; Andréassian et al., 2016; Wang et al., 2016; Sposito, 2017). Some comparable features of the Schreiber-Oldekop hypothesis and the complementary principle have been brought up and examined before (e.g. Zhang et al. 2004; Yang et al., 2006; Zhou et al., 2015; Lhomme and Moussa, 2016); but while these studies have greatly stimulated current understanding, so far this has not led to tangibly practical procedures.

In brief, the present study was motivated by the idea that the complementary principle and the evaporation precipitation ratio can be made to share a common variable, namely by equating the atmospheric demand of the former with the maximum possible evaporation of the latter. This way, the two methodologies become blended, and using their remaining input variables, namely the equilibrium evaporation and the precipitation, respectively, the unknown parameter can be estimated. Specifically, this study aims briefly to re-examine the assumptions under these two approaches and to develop a method for estimating the annual scale model parameter in the complementary principle. Because this parameter, coined β herein, is relatively insensitive to time scale it can then be used to calculate evaporation from routine meteorological data at any shorter time scale.

2. Theoretical Framework

2.1 The Evaporation Precipitation Ratio as a Function of Aridity Index

2.1.1 Background.

The search for general relationships among the main components of the annual water budget of river basins has a long history. Among the earliest attempts to relate mean annual runoff R with mean annual precipitation P in central Europe, Penck (1896) used a linear function, whereas Ule (1903) adopted a cubic polynomial. Further inspection of Ule's data and methodology by Schreiber (1904) "made him suppose" that an exponential function would be feasible, as this form had been used in many other problems in physics and meteorology; thus Schreiber proposed the following scaled relationship

$$R / P = \exp(-k_s / P) \tag{1}$$

Here k_s in the exponent is a constant for a given river basin. By a series expansion of (1), namely

$$R = P - k_s + \frac{k_s}{2}\left(\frac{k_s}{P}\right) - \frac{k_s}{6}\left(\frac{k_s}{P}\right)^2 + \frac{k_s}{24}\left(\frac{k_s}{P}\right)^3 - \ldots \tag{2}$$

Schreiber showed that k_s equals the value of the difference $(P - R)$ for very large *P*; he denoted this difference as the “remainder” or “leftover part”, which would start at zero for $P = 0$, and asymptotically approach its maximal value k_s for large increasing *P*. He did not further comment on the specific physical meaning of this remainder, nor on its maximal value. Obviously, Schreiber’s remainder, which is the part of the mean annual precipitation that does not run off, is in fact the mean annual evaporation from the basin. Schreiber was undoubtedly aware of this, as he was familiar with Penck’s (1896) equation $(P - R) = E$, whom he quotes in his paper. But it must not have occurred to him to call k_s the maximal evaporation rate, because his study and also Ule’s were solely focused on runoff prediction and not concerned with evaporation.

What was already implicit in Schreiber’s analysis, was made explicit a few years later by Oldekop (1911) who formally identified $(P - R)$ as the mean annual evaporation *E* and, after Schreiber’s series expansion, k_s as the “possible maximum evaporation”, say $E_{\max}$. Thus with Oldekop’s stipulation, (1) would be reformulated for mean annual evaporation as:

$$E / P = 1 - \exp(-E_{\max} / P) \tag{3}$$

To describe his evaporation data Oldekop did not adopt (3), but expressed $(E / E_{\max})$ as a hyperbolic tangent function of $(P / E_{\max})$ instead. Subsequently Budyko (1958; 1974), using the dimensionless variables of Schreiber and Oldekop, felt that the geometric mean of Schreiber's exponential (3) and Oldekop's hyperbolic tangent would provide an even better fit to the available data. But this was only the beginning of a proliferation of other functions in terms of the same 3 variables, as intended improvements over the original functions of Schreiber, Oldekop and Budyko; all of these functions were in the general form

$$(E / P) = f(E_{\max} / P) \tag{4}$$

Note as an aside that the functional relationship (4) is often, as Oldekop did, also written in an equivalent, and mutually convertible form with $E_{\max}$ as repeating variable, instead of *P*, namely as

$$(E / E_{\max}) = F(P / E_{\max}) \tag{5}$$

Because Schreiber and Oldekop are clearly at the origin of (4) and (5), it would stand to reason that all these subsequent formulations would be referred to as resulting from the Schreiber-Oldekop hypothesis or as part of the Schreiber-Oldekop framework. Yet invariably these subsequent functions in the form of (4) or (5) are now referred to as belonging to the "Budyko framework". One can only guess why this discrepancy arose, because Budyko himself consistently gave credit to the two earlier authors; a major reason perhaps was that only Budyko's work can be read in English translation and served as the only source of (4), whereas the papers of Schreiber (1904) and Oldekop (1911) are only available in their original version. In any event, the attribution of the framework based on (4) or (5) to Budyko is another example of Stigler's (1980) law that "No scientific discovery is named after its original discoverer." While it may be too late to stop or even slow the current nomenclature advance of Budyko in the literature, it is never too late to try to correct mistaken practices of the past and give credit where credit is due. It is hoped that, if a name must be attached to it, henceforth (4) and (5) will also be acknowledged as the Schreiber-Oldekop hypothesis.

2.1.2 Present Implementation.

Reviews of the several implementations of (4) and (5), that have appeared in the literature after Budyko can be found in Andreassian et al. (2016), Andreassian and Sari (2019), Wang et al. (2016), and Sposito (2017), among others. Although these equations differ in their functional form, their numerical relationships are very similar. For the purpose of the present study it was decided to work with the function first derived by Tixeront (1964) and later by Fu (1981; Zhang et al., 2004) by a different analytical method; this function can be written in terms of the original variables of (4), as follows

$$\frac{E}{P} = 1 + \frac{E_{max}}{P} - \left[1 + \left(\frac{E_{max}}{P}\right)^{w}\right]^{1/w} \tag{6}$$

or, alternatively, in its equivalent form (5), as

$$\frac{E}{E_{max}} = 1 + \frac{P}{E_{max}} - \left[1 + \left(\frac{P}{E_{max}}\right)^{w}\right]^{1/w} \tag{7}$$

Here w is a model parameter, which is introduced for added flexibility; its estimation will be dealt with below in Section 4.1.1.

So far, the exact nature of this "possible maximum evaporation", E_{max}, has not been specified. In the literature on the Schreiber-Oldekop hypothesis various definitions have been implemented and there is still no consensus on its precise meaning, nor on the optimal method of its estimation. Budyko (1974) originally put it equal to the net radiation. Subsequently, it has been considered as the potential evaporation in some studies and as the atmospheric evaporative demand or the apparent potential evaporation in others. For example, in a survey of some 50 studies dealing with various implementations of (4) and (5), we found that $E_{\max}$ was implemented by net radiation in 14%, by the Priestley-Taylor equation in 26%, by the Penman and related equations in 41%, by pan evaporation (MOPEX) in 14%, and by temperature-based expressions in 5% of this admittedly limited sample. Thus put differently, $E_{\max}$ was represented by approximations of the true potential evaporation E_{po} in about 40%, and by estimates of the apparent potential evaporation, or the atmospheric evaporative demand E_{pa} in about 60% of the studies in this sample. As shown in what follows, for the present purpose it will be found suitable to assume that $E_{\max} = E_{pa}$ in (6) and (7).

2.2 The Complementary Evaporation Principle

This principle, introduced formally by Bouchet (1963), is based on the common observation that the actual evaporation E from a natural land surface under drying conditions and the evaporation E_{pa} from a small wet surface area, located in the same environment and surrounded by the drying surface, from which E is taking place, exhibit complementary trends; thus as drying proceeds and less moisture is available, E will decrease while E_{pa} will increase. The latter quantity is variously named the atmospheric evaporative demand or the apparent potential evaporation. When ample moisture is available at the surface, both E and E_{pa} assume the value of the potential evaporation, E_{po}, in accordance with Thornthwaite's (1948) definition; under such conditions one has $E = E_{po} = E_{pa}$. Bouchet (1963) originally assumed that as E becomes smaller than the potential evaporation E_{po} during drying, the energy not used for E becomes available and raises the apparent potential evaporation by the same amount; thus, one has $(E_{po} - E) = (E_{pa} - E_{po})$, and this yields immediately his proposed relationship

$$E = 2E_{po} - E_{pa} \tag{8}$$

This was later broadened in Brutsaert and Parlange (1998) by letting $(E_{po} - E)$ be proportional to $(E_{pa} - E_{po})$; this results in

$$E = [(b+1)E_{po} - E_{pa}]/b \tag{9}$$

where b is a constant. Both (8) and (9) satisfy the condition that $E = E_{po}$ whenever $E_{pa} = E_{po}$, as it should. In a further generalization, with the imposition of three additional conditions, in Brutsaert (2015) $(E_{po} - E)$ could be represented by a cubic polynomial of $(E_{pa} - E_{po})$. This relationship can be expressed as

$$E = \left(\frac{E_{po}}{E_{pa}}\right)^2 (2E_{pa} - E_{po}) \tag{10}$$

To apply (8), (9) or (10) it is necessary to estimate E_{pa} and E_{po}. As defined earlier, the atmospheric evaporative demand E_{pa} is the evaporation from a small moist surface area, located in the same environment and surrounded by the drying surface from which E is occurring. This definition indicates that it can be measured directly using a small pan (Kahler and Brutsaert, 2006; Brutsaert, 2006; 2013; Zhang et al., 2017); alternatively, as proposed in Brutsaert and Stricker (1979) with (8), and confirmed in subsequent studies, it can also be closely described by means of Penman's (1948, 1956) equation, with the variables measured under the ambient nonpotential conditions. This can be written as

$$E_{pa} = \frac{\Delta}{\Delta + \gamma} Q_{ne} + \frac{\gamma}{\gamma + \Delta} f_e(u_2)(e_1^* - e_1) \quad (11)$$

in which $\Delta \equiv de^* / dT$ is the slope of the saturation vapor pressure curve, γ is the psychrometric constant, and $Q_{ne} = (R_n - G) / L_e$ is the available energy expressed in evaporation units, with R_n the net radiation, G the heat flux into the ground (often neglected for daily averages), and L_e the latent heat of vaporization; the variable u_2 is the mean wind speed measured at a height z_2 above the ground, e_1 is the vapor pressure at a height z_1 above the ground, and the asterisk means saturation. Zhang et al.(2017) and Liu et al.(2018) showed that at the daily time scale the implementation of (11) is fairly insensitive to the selected wind function $f_e(u_2)$, in the context of the complementary approach; thus, the wind function can be represented by *Penman*'s[1948] simple empirical equation

$$f_e(u_2) = 0.26(1 + 0.54u_2) \quad (12)$$

The constants in this equation are for wind speeds (in $\mathrm{m\ s^{-1}}$) and vapor pressures (in hPa) measured at 2 m above the ground and they produce the resulting second term on the right of (11) in $\mathrm{mm\ d^{-1}}$.

The E_{po} term is more difficult to estimate. One reason is that it was originally conceived as potential evaporation. Thus, as defined by Thornthwaite (1948) the measurements needed in its estimation must be made under truly potential conditions, that is, with sufficient water present at the evaporating surface; these are never available when *E* is to be estimated under non-potential conditions. In the advection-aridity approach with (8) in Brutsaert and Stricker (1979), and other early implementations of the complementary approach, the E_{po} term was assumed to be proportional to the equilibrium evaporation introduced by Slatyer and McIlroy (1961) as

$$E_e = \frac{\Delta}{\Delta + \gamma} Q_{ne} \tag{13}$$

The proportionality constant was taken to be $\alpha_e = 1.26$, the standard value for potential conditions given by Priestley and Taylor (1972). In Brutsaert and Stricker (1979) it was assumed (and hoped) that $\alpha_e E_e$ would be sufficiently robust to provide a stable estimate of E_{po} under any conditions. However, subsequent calibration studies with better experimental data revealed that this is not the case and that the α_e parameter must be allowed to vary; this variation was shown to depend mostly on the aridity index $\mathrm{AI} = E_{pa} / P$ (Liu et al. 2016; 2018; Brutsaert et al., 2020). To avoid further confusion with the α_e parameter of Priestley and Taylor, henceforth the variable proportionality will be denoted β. Accordingly, after replacing E_{po} by βE_e, Equation (10) becomes finally

$$\frac{E}{E_{pa}} = 2\left(\frac{\beta E_e}{E_{pa}}\right)^2 - \left(\frac{\beta E_e}{E_{pa}}\right)^3 \tag{14}$$

in which E_e is defined in (13), and E_{pa} in (11). For further analysis (14) can be written more concisely as

$$y = 2x^2 - x^3 \tag{15}$$

in which $y = (E / E_{pa})$ and $x = (\beta E_e / E_{pa})$. The estimation of the variable parameter β and of its functional form is the main objective of this paper; this is treated next.

2.3 Blending of Evaporation/Precipitation Ratio with Complementary Function

Elimination of (E/E_{pa}) between (14) (or (15)) and (7) in which $E_{\max} = E_{pa}$, yields their combination as a cubic equation, to wit

$$x^3 - 2x^2 + z = 0 \tag{16}$$

in which $z = F(P/E_{pa}) = 1 + (P/E_{pa}) - \left[1 + (P/E_{pa})^w\right]^{1/w}$. The appropriate solution of (16) can be written as (Oldham, Myland and Spanier, 2009)

$$x = \frac{2}{\sqrt{3}}\sqrt{-p}\,\sin\left(\frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}}{2(\sqrt{-p})^3}q\right)\right) + \frac{2}{3} \tag{17}$$

where $p = -4/3$ and $q = (-16/27 + z)$. This yields the parameter β in terms of E_e, E_{pa} and P, as follows

$$\beta = \left(\frac{E_{pa}}{E_e}\right)\left[\frac{4}{3}\sin\left(\frac{1}{3}\sin^{-1}\left(\frac{27}{16}z - 1\right)\right) + \frac{2}{3}\right] \tag{18}$$

This can also be written concisely as

$$\beta = \Psi^{-1}\left[\frac{4}{3}\sin\left(\frac{1}{3}\sin^{-1}\left(\frac{27}{16}F(\Phi) - 1.0\right)\right) + \frac{2}{3}\right] \tag{19}$$

where $\Psi = E_e / E_{pa}$, $\Phi = P / E_{pa}$, and $F(\Phi) = 1 + \Phi - [1 + \Phi^w]^{1/w}$. The optimal value of the parameter w will be determined in Section 4.1.1. The analogous relationship for the linear complementary relationship (9) is shown in Appendix 1.

3. Data Description

This study used the data obtained from global catchment water balance data and eddy covariance flux measurements. The global catchment water balance data were used for method development, while the eddy covariance flux data were used for method application and validation.

3.1. Data for method development

The global catchment water balance data include reliable mean annual streamflow and precipitation data from 524 catchments located in different geographic regions of the world (Figure 1).

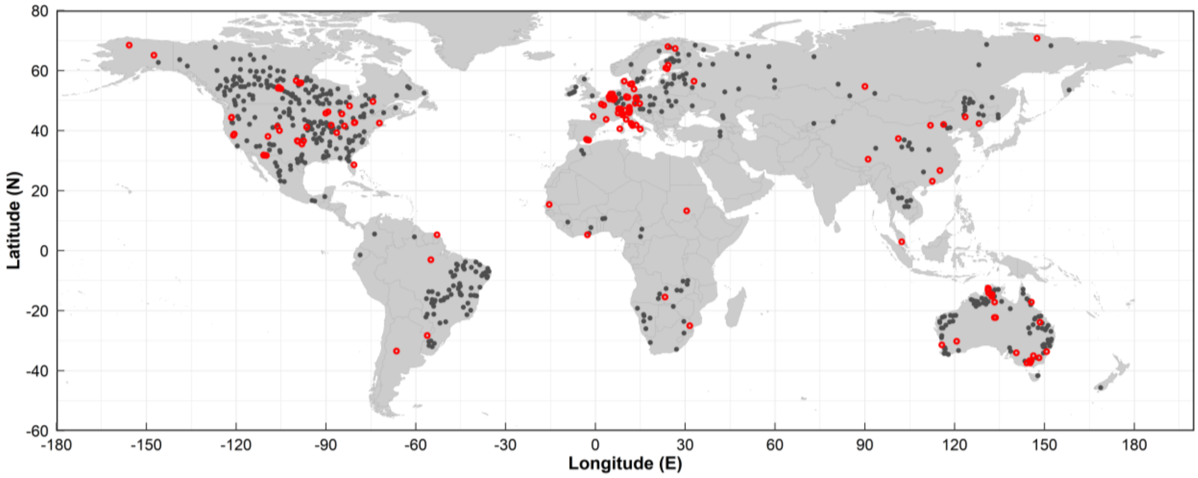

Figure 1. Location map of the global catchments (black dots) (n=524) where the annual evaporation was measured with the water budget equation; these values were used to develop the proposed method. Also shown is the spatial distribution of the global flux stations (red circles) (n=156) used in the validation process.

All the 524 selected catchments are identified as unregulated with minimal effect of dams or reservoirs. The streamflow data from these catchments for the period of 2001-2013 were obtained from (i) the Global Runoff Data Centre (http://www.bafg.de/GRDC/EN/Home/homepage_node); (ii) unregulated Australian catchments (Zhang et al., 2013); (iii) the Model Parameter Estimation Experiment (MOPEX) across the United States (http://www.nws.noaa.gov/oh/mopex/index.html); and (iv) runoff data gathered by the Chinese Academy of Sciences. Precipitation data were obtained from the global cover MSWEP rainfall dataset (Beck et al. 2017; Sun et al. 2018). For each of these catchments, the mean annual precipitation for 2001-2013 was calculated with an area weighted averaging technique. The mean annual values of evaporation from these catchments were calculated based on the water balance equation by neglecting changes in catchment water storage.

Daily values of meteorological variables such as air temperature, humidity, and wind speed were obtained from the CRU-NECP dataset (version 7, New et al., 1999). This dataset was produced by merging observed mean monthly data of the Climate Research Unit (CRU) at the University of East Anglia, with NOAA's high temporal resolution NCEP reanalysis data at a spatial resolution of 0.5°. The mean annual pressure was used to represent the atmospheric pressure and the wind speed at 10 m above the ground was converted to a height of 2 m by multiplying it by $(2/10)^{1/7}$ (Brutsaert, 2005). Daily net radiation values were obtained from the Clouds and the Earth's Radiant Energy System (CERES) SYN1deg-Day dataset with spatial resolution of 1.0° (Wielicki et al., 1996). A local averaging method was used to resample the net radiation data to 0.5° to achieve the resolution of the other atmospheric inputs. The ground heat flux was considered negligible on a daily basis.

The above daily meteorological data were then used to calculate E_e and E_{pa} by means of (13) and (11) over the period of 2001-13 at a spatial resolution of 0.5°. The catchment mean annual values of E_e and E_{pa} were calculated with an area weighted averaging method, in the same way as mean annual precipitation.

3.2 Data for method application and validation

For the purpose of more detailed application and independent validation of the proposed method, a selection was made of 156 eddy covariance flux stations from the global FLUXNET2015 dataset (http://fluxnet.fluxdata.org/), with the criterion that each station had at least one full year of complete and continuous daily data. The data from these 156 FLUXNET stations comprise evaporation measurements using the eddy covariance technique, beside measurements of the standard near surface atmospheric variables. At all stations energy budget closure was achieved by adjusting the eddy covariance flux measurements as suggested by Twine et al. (2000) and others. The β value of each station was obtained by solving equation (14) at the mean annual time scale, with annual averages of daily values. Among the 156 flux stations, 28 stations were selected for estimating daily evaporation rates and details of these stations are listed in Table 1. These 28 stations were selected so they would represent as wide a geographic and climatic distribution as possible.

Table 1. Details of flux stations used for estimating daily evaporation rates in the study.

Station ID	Lat.	Lon.	IGBP	Period of Record
AU-DaS	-14.1593	131.3881	GRA	2008-2014
AU-Gin	-31.3764	115.7138	WSA	2011-2014
AU-GWW	-30.1913	120.6541	SAV	2013-2014
AU-How	-12.4943	131.53	WSA	2003-2014
AU-Stp	17.1507	133.3502	GRA	2008-2014
AU-Tum	-35.6566	148.1517	EBF	2001-2014
BE-Vie	50.30493	5.99812	MF	1996-2014
BR-Sa3	-3.01803	-54.97144	EBF	2000-2004
CA-Gro	48.2167	-82.1556	MF	2003-2014
CN-Du3	42.0551	116.2809	GRA	2009-2010
DE-Geb	51.09973	10.91463	CRO	2001-2014
DE-Kli	50.89306	13.52238	CRO	2004-2014
DE-SfN	47.80639	11.3275	WET	2012-2014
DK-Eng	55.69053	12.19175	GRA	2005-2008
ES-Amo	36.83361	-2.25232	OSH	2007-2012
ES-LJu	36.92659	-2.75212	OSH	2004-2013
FI-Let	60.64183	23.95952	ENF	2009-2012
FI-Lom	67.99724	24.20918	WET	2007-2009
FR-LBr	44.71711	-0.7693	ENF	1996-2008
GH-Ank	5.26854	-2.69421	EBF	2011-2014
IT-BCi	40.52375	14.95744	CRO	2009-2011
IT-Noe	40.60618	8.11529	CSH	2004-2014

IT-Tor	45.84444	7.57806	GRA	2008-2014
MY-PSO	2.9730	102.3062	EBF	2003-2009
RU-Fyo	56.46153	32.92208	ENF	1998-2014
US-Blo	38.8953	-120.6328	ENF	1997-2007
US-MMS	39.3232	-86.4131	DBF	1999-2014
US-Syv	46.2420	-89.3477	MF	2001-2008

4. Results and Discussion

4.1. Method development

4.1.1. Estimation of Tixeront-Fu parameter *w*

The Tixeront-Fu parameter *w* was determined by calibrating (7) against the water balance estimates of evaporation from the 524 selected catchments with $E_{\max} = E_{pa}$. The calibration was done by trial and error, adjusting *w* until the slope through the origin was exactly 1.0 between observed and estimated mean annual evaporation (Figure 2). The Nash-Sutcliffe efficiency (NSE) is 0.93, the correlation coefficient is 0.96 and the bias 2.12%, indicating a good model calibration. The value of the optimized *w* is 2.41; this is close to the reported value of 2.53 by Zhang et al. (2004) and 2.50 by Xu et al. (2013). Also shown in Figure 2 are the mean annual values of evaporation estimated using (7) with the optimized $w = 2.41$ for the 156 global flux sites. The NSE is 0.71 and the correlation coefficient is 0.83 with a bias of 2.15%. These results indicate that equation (7) with the optimized $w = 2.41$ can accurately predict mean annual evaporation when compared with global catchment water balance data and flux measurements.

It can be noted in Figure 2 that equation (7) provided more accurate estimates of mean annual evaporation for the selected catchments than the flux stations. This may be due to the fact that the catchment water balance estimates of evaporation are averaged over the catchments for a period of 12 years, while the flux stations represent point measurements and the record lengths are also shorter. This difference in performance for the two sets of data will also be seen again in the other figures which follow in this paper.

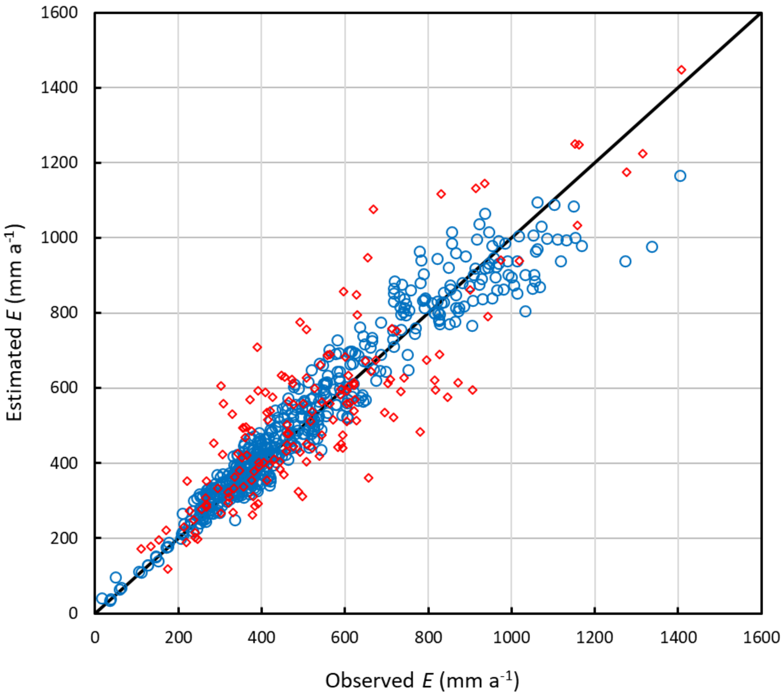

Figure 2. Comparison between the mean annual evaporation estimated using (7) with optimal parameter $w = 2.41$ against the water balance estimates for 524 selected catchments (blue open circles). Also shown are the mean annual evaporation estimates using (7) with optimal parameter $w = 2.41$ against observed mean annual evaporation from 156 global flux stations (red diamonds).

4.1.2. Estimation of the model parameter β

The main part of this study is to develop a prediction method for the model parameter β from climatic variables, i.e. E_e and E_{pa} in the absence of information on E. For each catchment, the parameter β was predicted from (19) with the optimized $w = 2.41$. These predicted parameter β values can be compared with "observed" β values for the 524 selected catchments in Figure 3. The "observed " parameter β values were obtained by inverting equation (14) numerically with the value of mean annual evaporation calculated from the water balance method and E_e and E_{pa} determined with (13) and (11). The Nash-Sutcliffe efficiency (NSE) is 0.85, the correlation coefficient is 0.92 and the bias 2.11%. The performance of (19) in predicting the parameter β was also assessed by comparing the predicted and "observed" β values for the flux stations (Figure 3). Similarly, the observed β values for the flux stations were obtained with (14) from the measured mean annual evaporation values using the eddy covariance technique. The Nash-Sutcliffe efficiency (NSE) is 0.49, the correlation coefficient is 0.67 and the bias 1.45%, indicating a reasonably accurate prediction of the parameter β for the flux stations.

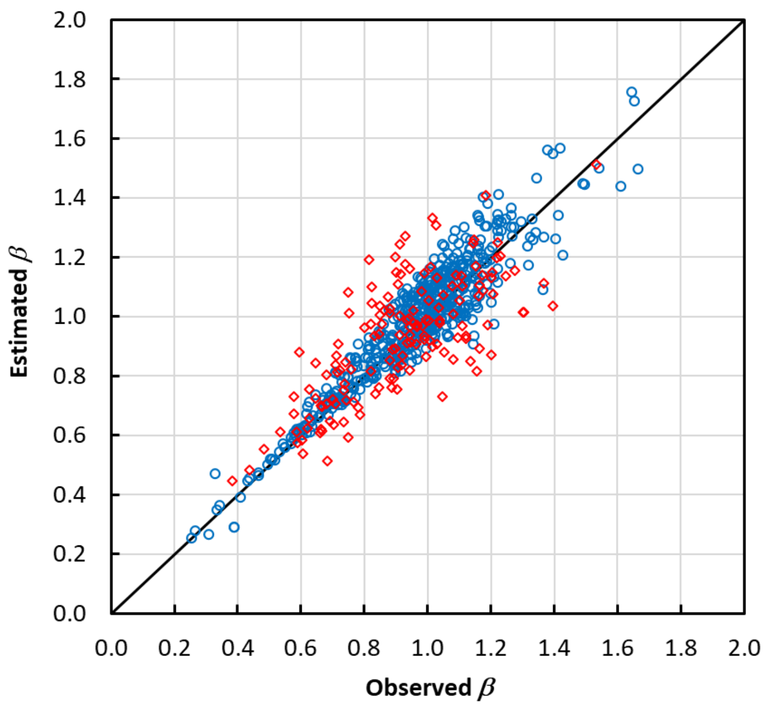

Figure 3. Comparison of the calculated β values by equation (19) against "observed" values by inverting (14) for the 524 catchments shown in Figure 1 (blue circles). Also shown are the corresponding β values from the data at the 156 flux stations (red diamonds).

4.1.3. Estimation of mean annual evaporation

The values of mean annual evaporation estimated using (14) with the model parameter β predicted using (19) are compared against the estimates made by the water balance of the 524 selected catchments in Figure 4. The Nash-Sutcliffe efficiency (NSE) is 0.93, the correlation coefficient is 0.96 and the bias is -2.08%. These results indicate a good fit between the estimated and observed values. Also shown in Figure 4 is a comparison of the mean annual values of evaporation values for the 156 global flux sites estimated the same way. The NSE is 0.71 and the correlation coefficient is 0.83 with a bias of 1.88%. Both the water balance estimates of mean annual evaporation and flux measurements support the validity of the complementary relationship in (14) (Figure 5) and show that actual evaporation and apparent potential evaporation exhibit a strong asymmetrical complementary relationship (Figure 6). These results indicate that the complementary principle-based approach with the parameter β determined by (19) can accurately predict mean annual evaporation. As illustrated in Figures 2 and 4, both equations (7) and (14) prove to be accurate for estimating mean annual evaporation. However, it should be noted that equation (7) needs only one constant parameter *w*, while the complementary principle-based approach with (14) requires a variable parameter β which is a function of two dimensionless quantities as indicated in (19). But the disadvantage of (7) is that it can only be applied at annual or larger time scales.

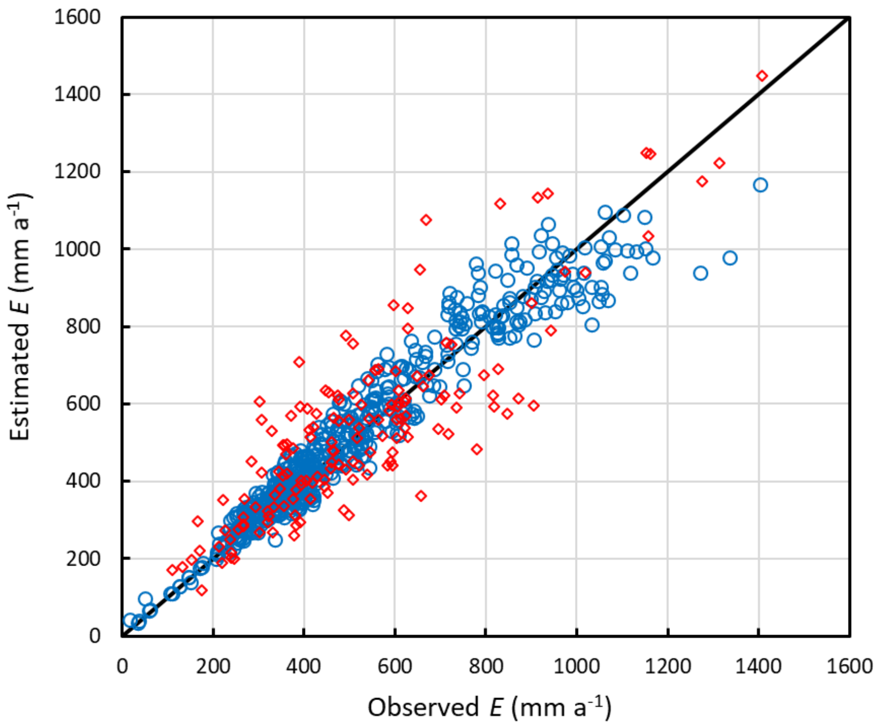

Figure 4. Comparison between mean annual evaporation estimated using (14) with parameter β determined by (19) against water balance estimates for 524 selected catchments (blue open circles). Also shown are the mean annual evaporation values estimated the same way, against the observed mean annual evaporation for the 156 global flux stations (red diamonds).

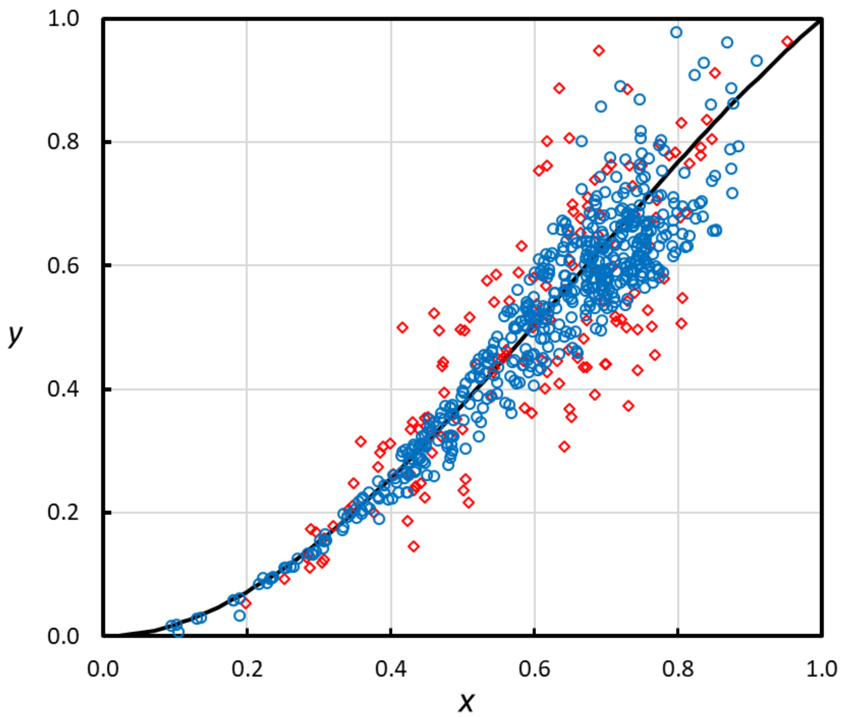

Figure 5. Relationship between scaled evaporation $y = (E / E_{pa})$ and scaled reference evaporation $x = (\beta E_e / E_{pa})$. The catchment water balance data are shown as blue circles (n = 524) and the global flux measurements as red diamonds (n = 156). The theoretical complementary relationship (14) or (15) is represented by the curve.

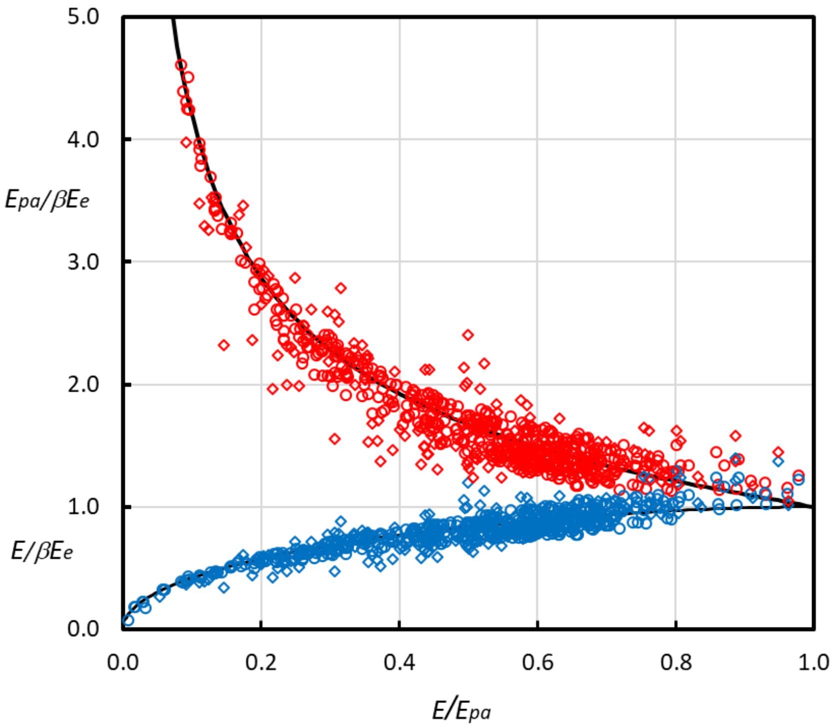

Figure 6. Illustration of the complementary relationship between the actual evaporation E and the atmospheric evaporative demand (or apparent potential evaporation) E_{pa}, relative to βE_e, for varying conditions of moisture availability, as expressed by their ratio E / E_{pa}. The open circles represent the water balance data (n = 524) and the diamonds represent global flux measurements (n = 156). The curves show the theoretical relationship (14) or (15), as (y / x) and $(1 / x)$ against y.

4.2. Prediction of daily evaporation

4.2.1. Proposed procedure

Equation (19) is the result of a fusion of the evaporation/precipitation ratio with the complementary principle function and it allows the direct prediction of the parameter β without the need of evaporation measurements for its calibration. The results shown in Figure 4 indicate that the complementary principle-based approach can provide accurate estimates of mean annual evaporation from routine meteorological data with the predicted parameter $\beta;$ from equation (19). Here we now make the assumption that the mean annual β relationship (i.e. equation (19)) can also be used at shorter time scales, such as a day. To test this assumption, the complementary principle equation (14) can now be used to estimate daily values of evaporation from the 28 selected flux stations under different climatic conditions listed in Table 1. The model parameter β for each flux station was estimated with equation (19) and assumed constant through the year. The estimated daily evaporation rates using (14) were compared against the flux measurements, and the resulting statistics are listed in Table 2. The Nash-Sutcliffe efficiency (NSE) is larger than 0.50 in 66% of the flux stations, the correlation coefficient is larger than 0.83 in 27 out of the 28 flux stations. The slope of the daily evaporation plots through the origin is close to 1.0 for most stations. As expected, flux stations under dry climatic conditions (e.g. aridity index larger than 3) showed less accurate results as compared to the more humid flux stations. However, this may not necessarily be a reflection of the performance of the model itself. As a further illustration, some results of the daily evaporation estimates are also shown in Figure 7 for a subset of the flux stations (n=12); these graphs reinforce the good agreement shown in Table 2. Figure 7 also illustrates why a low correlation may not necessarily reflect poorly on the performance of the model. For instance, graph (a) (AU-Gin in Table 2) and graph (k) (RU-Fyo in Table 2) display a similar spread of the data, yet in Table 2 the former has a correlation coefficient r of 0.89 and a slope of 0.82, whereas for the latter these values are 0.95 and 1.03. The main reason for this difference is that the drier Australian site has a narrower range of *E* values than the more humid Russian site.

Table 2. Statistics of daily evaporation estimated with predicted parameter β from Eq. (19) against eddy covariance measurements from selected global flux stations.

Station ID	AI	β	Slope	r	NSE	Bias (%)	RMSE (mm/d)
AU-DaS	1.40	0.95	1.02	0.98	0.63	-6.7	0.7
AU-Gin	3.10	0.71	0.82	0.89	-0.44	12.1	0.86
AU-GWW	6.18	0.61	0.74	0.88	-0.35	19.5	0.53
AU-How	1.05	0.98	0.95	0.97	0.34	-0.10	0.88
AU-Stp	3.01	0.91	0.91	0.91	0.03	-18.4	0.88
AU-Tum	1.20	1.07	0.95	0.98	0.83	7.5	0.68
BE-Vie	1.13	0.97	1.01	0.97	0.74	-3.3	0.61
BR-Sa3	0.83	1.02	0.91	0.99	0.70	11.60	0.47
CA-Gro	1.37	1.11	1.05	0.94	0.65	-11.7	0.89
CN-Du3	2.91	1.14	0.95	0.95	0.78	-1.9	0.45
DE-Geb	1.53	1.14	0.91	0.94	0.57	-0.4	0.8
DE-Kli	0.92	1.29	1.08	0.95	0.59	-13.1	0.89
DE-SfN	1.06	0.91	0.94	0.99	0.88	7.8	0.49
DK-Eng	1.21	1.16	0.99	0.96	0.71	3.7	0.6
ES-Amo	6.08	0.57	0.80	0.63	-1.30	-27	0.61
ES-LJu	3.14	0.78	1.13	0.89	0.14	-23.9	0.71
FI-Let	1.57	0.90	0.97	0.96	0.75	0.4	0.52
FI-Lom	1.14	1.14	1.04	0.99	0.94	-2.1	0.26
FR-LBr	1.23	1.00	0.97	0.95	0.53	0.9	0.89
GH-Ank	0.76	0.93	0.96	0.98	0.48	2.2	0.56
IT-BCi	0.87	1.13	0.98	0.93	0.32	-3.4	1.26
IT-Noe	2.60	0.67	1.06	0.83	-0.40	-10.6	0.79
IT-Tor	0.73	1.19	0.97	0.99	0.87	18.4	0.79
MY-PSO	0.90	0.92	0.94	0.99	0.67	5.2	1.06
RU-Fyo	1.21	0.94	1.03	0.95	0.68	-7.9	0.69
US-Blo	1.09	1.03	1.06	0.98	0.78	-17.5	1.24
US-MMS	1.16	1.09	0.96	0.97	0.75	0.1	0.72
US-Syv	1.23	1.06	0.92	0.95	0.69	0.1	0.72

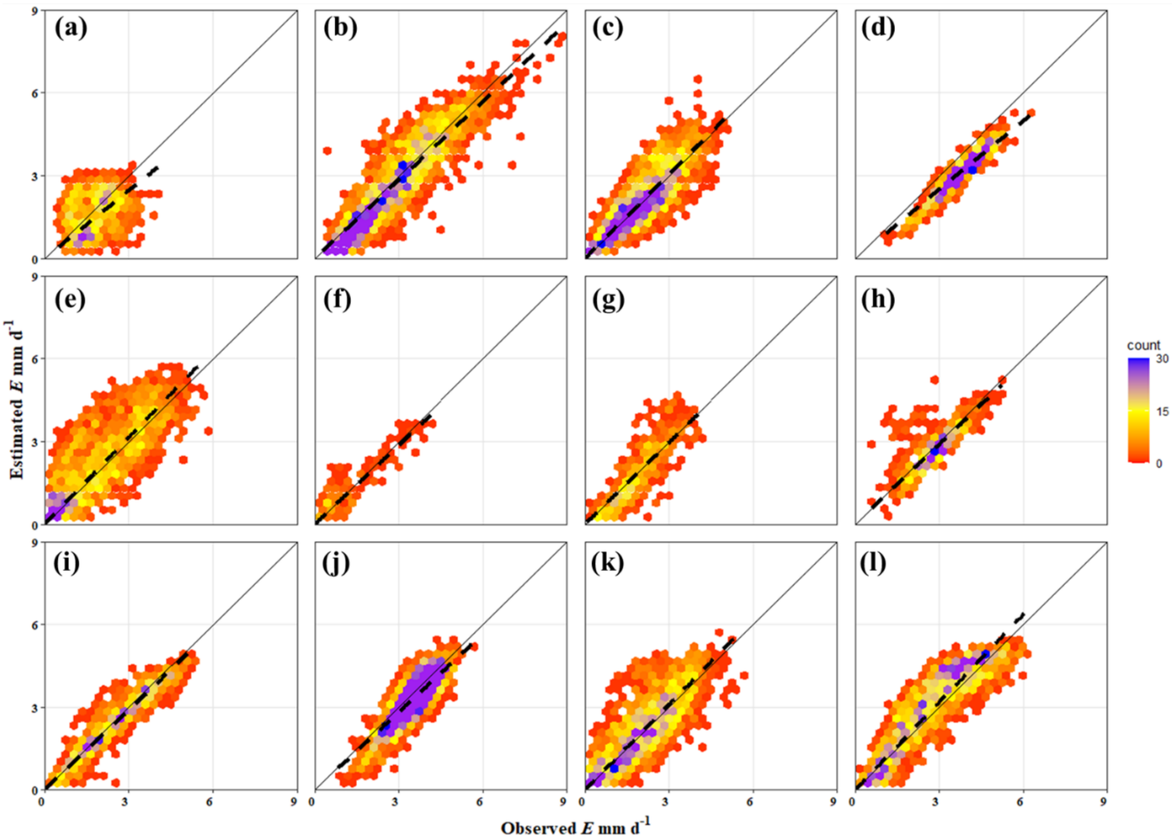

Figure 7. Comparison of daily evaporation estimated using equation (14) with model parameter β obtained from equation (19) against the eddy covariance measurements at (a) Gingin (Australia), (b)Tumbarumba (Australia), (c)Vielsalm (Belgium), (d) Santarem (Brazil), (e) Ontario (Canada), (f) Duolun (China), (g) Enghave (Denmark), (h) Ankasa (Ghana), (i) Torgnon (Italy), (j) Pasoh Forest Reserve (Malaysia), (k) Fyodorovskoye (Russia), (l) Blodgett Forest (USA). Filling color of the bins depends on the number of points (i.e., counts) in each bin.

5. Conclusions

The method for estimating evaporation based on the Schreiber-Oldekop hypothesis, also commonly known as the Budyko framework, relates the evaporation precipitation ratio with aridity index. In this study, an equivalent and mutually convertible form of the relationship (i.e. the Tixeront-Fu equation) with $E_{\max}$ estimated using the Penman equation as repeating variable was tested against observed evaporation from global datasets of catchment water balance and eddy covariance flux measurements and was found to yield excellent agreement at the mean annual time scale, with an optimized parameter $w = 2.41$. This value is close to w values obtained in previous studies. By selecting the atmospheric evaporative demand, E_{pa} to represent the maximum possible evaporation $E_{\max}$, the evaporation precipitation ratio and the complementary relationship could be blended into an analytical equation to predict the parameter β of the latter; this made the generalized complementary approach essentially calibration-free. With predicted parameter values from the analytical equation, the complementary principle approach provided accurate estimates of evaporation not only at the mean annual time scale, but also at shorter such as daily time scales. One of the main strengths of this approach is that it allows the prediction of daily evaporation using only routine meteorological inputs including air temperature, humidity, wind speed, net radiation, and long-term average precipitation.

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References

Andréassian, V., Mander, Ü., Pae, T. (2016). The Budyko hypothesis before Budyko: The hydrological legacy of Evald Oldekop. J. Hydrol., 535, 386–391. http://dx.doi.org/10.1016/j.jhydrol.2016.02.002

Andréassian, V and Sari, T. (2019). Technical Note: On the puzzling similarity of two water balance formulas – Turc–Mezentsev vs. Tixeront–Fu. Hydrol. Earth Syst. Sci., 23, 2339–2350, https://doi.org/10.5194/hess-23-2339-2019

Beck, H. E., van Dijk, A. I. J. M., Levizzani, V., Schellekens, J., Miralles, D. G., Martens, B. and de Roo, A. (2017). MSWEP: 3-hourly 0.258 global gridded precipitation (1979–2015) by merging gauge, satellite, and reanalysis data. Hydrol. Earth Syst. Sci., 21, 589–615, https://doi.org/10.5194/hess-21-589-2017.

Bouchet, R. (1963). Evapotranspiration réelle et potentielle, signification climatique, IAHS Publ., 62, 134–142 (in French).

Brutsaert, W. (1982). Evaporation Into the Atmosphere: Theory, History, and Applications, pp. 299, Kluwer Acad., Dordrecht, Netherlands。

Brutsaert, W. (2005). Hydrology: An Introduction, 605 pp., Cambridge Univ. Press, N. Y.

Brutsaert, W. (2006). Indications of increasing land surface evaporation during the second half of the 20th century, Geophys. Res. Lett., 33, L20403, doi:10.1029/2006GL027532.

Brutsaert, W. (2013). Use of pan evaporation to estimate terrestrial evaporation trends: The case of the Tibetan Plateau. Water Resour. Res., 49, 3054–3058, https://doi.org/10.1002/wrcr.20247.

Brutsaert, W. (2015). A generalized complementary principle with physical constraints for land-surface evaporation, Water Resour. Res., 51, 8087–8093, doi:10.1002/2015WR017720.

Brutsaert, W., Li, W., Takahashi, A., Hiyama, T., Zhang, L., and Liu, W. Z. (2017). Nonlinear advection-aridity method for landscape evaporation and its application during the growing season in the southern Loess Plateau of the Yellow River basin, Water Resour. Res, 53, doi:10.1002/2016WR019472.

Brutsaert, W., and Stricker, H. (1979). An advection-aridity approach to estimate actual regional evapotranspiration, Water Resour. Res., 15, 443–450, doi:10.1029/WR015i002p00443.

Brutsaert, W., and Parlange, M. B. (1998). Hydrologic cycle explains the evaporation paradox, Nature, 396(5), 300.

Brutsaert, W., Cheng, L., and Zhang, L. (2020). Spatial Distribution of Global Landscape Evaporation in the Early Twenty First Century by Means of a Generalized Complementary Approach. Journal of Hydrometeorology, https://doi.org/10.1175/JHM-D-19-0208.1

Budyko, M. I. (1958). The heat balance of the Earth’s surface, Natl. Weather Serv., U.S. Dep. of Commer., Washington, D. C.

Budyko, M. I. (1974). Climate and Life, 508 pp., Academic, San Diego, Calif.

Crago, R. D., and Qualls, R. J. (2018). Evaluation of the generalized and rescaled complementary evaporation relationships. Water Resources Research, 54, 8086–8102. https://doi.org/10.1029/2018WR023401

Crago, R., Szilagyi, J., Qualls, R. and Huntington, J. (2016). Rescaling the complementary relationship for land surface evaporation, Water Resour. Res., 52, 8461–8471. doi:10.1002/2016WR019753.

Fu, B. P. (1981). On the calculation of the evaporation from land surface (in Chinese), Sci. Atmos. Sin., 5, 23– 31.

Hobbins, M. T., J. A. Ramírez, and Brown, T. C. (2001). The complementary relationship in estimation of regional evapotranspiration: An enhanced Advection-Aridity Model, Water Resour. Res., 37(5), 1389–1403, doi:10.1029/2000WR900359.

Kahler, D. M., and Brutsaert, W. (2006). Complementary relationship between daily evaporation in the environment and pan evaporation. Water Resour. Res., 42, W05413, https://doi.org/10.1029/2005WR004541.

Lemeur, R., and Zhang, L. (1990). Evaluation of three evapotranspiration models in terms of their applicability for an arid region, J. Hydrol., 114(3), 395–411, doi:10.1016/0022-1694(90)90067-8.

Lhomme, J-P and Moussa, R. (2016). Matching the Budyko functions with the complementary evaporation relationship: consequences for the drying power of the air and the Priestley-Taylor coefficient. Hydrol. Earth Syst. Sci., 20, 4857–4865, doi:10.5194/hess-20-4857-2016.

Li, D., Cong, ZT, Zhang, L, and Wood, E (2013). Vegetation control on water and energy balance within the Budyko framework. Water Resour. Res. 49, 1–8, doi:10.1002/wrcr.20107

Liu, X., C. Liu, and Brutsaert, W. (2016). Regional evaporation estimates in the eastern monsoon region of China: Assessment of a nonlinear formulation of the complementary principle. Water Resour. Res., 52, 9511–9521, https://doi.org/10.1002/2016WR019340.

Liu, X., C. Liu, and Brutsaert, W. (2018). Investigation of a generalized nonlinear form of the complementary principle for evaporation estimation. J. Geophys. Res. Atmos., 123, 3933–3942, https://doi.org/10.1002/2017JD028035.

McMahon, T. A., Peel, M. C., Lowe, L., Srikanthan, R and McVicar, T.R. (2013). Estimating actual, potential, reference crop and pan evaporation using standard meteorological data: A pragmatic synthesis, Hydrol. Earth Syst. Sci., 17(4), 1331–1363, doi:10.5194/hess-17-1331-2013.

Morton, F. I. (1983). Operational estimates of areal evapotranspiration and their significance to the science and practice of hydrology, J. Hydrol., 66, 1–76.

New, M., Hulme, M, and Jones, P. (1999). Representing twentieth century space–time climate variability. Part I: Development of a 1961–90 mean monthly terrestrial climatology. J. Climate, 12, 829–856, https://doi.org/10.1175/1520-0442(1999)012,0829: RTCSTC.2.0.CO;2.

Oldham, K, Myland, J, and Spanier, j. (2009). An Atlas of Functions with Equator, the Atlas Unction Calculator, Second Edition. Springer Science+Business Media, LLC, 748pp.

Oldekop, E. (1911). On evaporation from the surface of river basins. (In Russian: Ob Isparenii s Poverkhnosti Rechnykh Basseinov), (With abstract in German, p. 201-209). Collection of the Works of Students of the Meteorological Observatory. University of Tartu-Jurjew-Dorpat, Tartu, Estonia, 209pp.

Parlange, M.B. and Katul, G.G. (1992). An advection-aridity evaporation model, Water Resour. Res., *28*, 127-132, 1992.

Penck, A. (1896). Untersuchungen über Verdunstung und Abfluss von grösseren Landflächen. Geogr. Abh. Wien, 5 (5), 10-29.

Penman, H. L. (1948). Natural evaporation from open water, bare soil, and grass, Proc. R. Soc. A, 193, 120–146.

Penman, H.L. (1956). Evaporation an Introductory Survey. Netherlands Journal of Agricultural Science, 4, 9-29.

Priestley, C. H. B., and Taylor, R. J. (1972). On the assessment of the surface heat flux and evaporation using large-scale parameters, Mon. Weather Rev., 100, 81–92.

Schreiber, P. (1904). Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der Flüsse in Mitteleuropa. Meteorol. Zeitsch., 21, 441-452.

Slatyer, R. O. and McIlroy, I. C. (1961). Practical Microclimatology, CSIRO, Melbourne, Victoria, Australia, 310 pp.

Sposito, G. (2017). Understanding the Budyko equation. Water, 9 (4), 236, doi:10.3390/w9040236

Stigler, S. M. (1980). Stigler's law of eponymy. Trans. New York Acad. Sci., 39, 147–157. doi: 10.1111/j.2164-0947.1980.tb02775.

Sun, Q., Miao, C., Duan, Q., Ashouri, H., Sorooshian, S. and Hsu, K.-L. (2018). A review of global precipitation data sets: Data sources, estimation, and intercomparisons. Rev. Geophys., 56, 79–107, https://doi.org/10.1002/2017RG000574.

Thornthwaite, C. W. (1948). An approach toward a rational classification of climate. Geogr. Rev., 38, 55–94. https://doi.org/ 10.2307/210739.

Tixeront, J. (1964). Prévision des apports des cours d'eau (Prediction of streamflow), IAHS Publication 63: General Assembly of Berkeley. IAHS, Gentbrugge, pp. 118–126.

Twine, T. E., and Coauthors, (2000). Correcting eddy-covariance flux underestimates over a grassland. Agric. For. Meteor., 103, 279–300, https://doi.org/10.1016/S0168-1923(00)00123-4.

Ule, W. (1903). Niederschlag und Abfluss in Mitteleuropa. Forsch. Deutsche Volks- u. Landesk., 14 (5), 24-39.

Wang, C., Wang, S., Fu, B.J., and Zhang, L. (2016). Advances in hydrological modelling with the Budyko framework: A review. Progress in Physical Geography, 1-22. doi:10.1177/0309133315620997.

Wang, K., and Dickinson, R. E. (2012). A review of global terrestrial evapotranspiration: Observation, modeling, climatology, and climatic variability. Rev. Geophys., 50, RG2005, https://doi.org/10.1029/2011RG000373.

Wielicki, B. A., Barkstrom, B. R., Harrison, E. F., Lee, R. B., Smith, G. L. and Cooper, J. E. (1996). Clouds and the Earth's Radiant Energy System (CERES): An Earth observing system experiment. Bull. Amer. Meteor. Soc., 77, 853–868, https://doi.org/10.1175/1520-0477(1996)077,0853:CATERE.2.0.CO;2.

Xu, X.L., Liu, W., Scanlon, B.R., Zhang, L., and Pan, M. (2013). Local and global factors controlling water-energy balances within the Budyko framework. Geophysical Research Letters. Vol, 40, 6123-6129, doi: 10.1002/2013GL058324.

Yang, D., Sun, F., Liu, Z., Cong, Z., and Lei, Z. (2006). Interpreting the complementary relationship in non-humid environments based on the Budyko and Penman hypotheses, Geophys. Res. Lett., 33, L18402, doi:10.1029/2006GL027657.

Zhang, L. Hickel, K., Dawes, W.R., Chiew, F., and Western, A. (2004). A rational function approach for estimating mean annual evapotranspiration. Water Resources Research, 40, W02502, doi:10.1029/2003WR002710.

Zhang, L., Cheng, L. and Brutsaert, W. (2017). Estimation of land surface evaporation using a generalized nonlinear complementary relationship. Journal of Geophysical Research – Atmospheres, 122, doi:10.1002/2016JD025936.

Zhang, L., Dawes, W. R. and Walker, G. R. (2001). The response of mean annual evapotranspiration to vegetation changes at catchment scale, Water Resour. Res., 37, 701–708.

Zhou, S., Yu, B., Huang, Y., and Wang, G. (2015). The complementary relationship and generation of the Budyko functions, Geophys. Res. Lett., 42, 1781–1790, doi:10.1002/2015GL063511.

Appendix 1. Estimation of Parameter β for the Linear Complementary Function

The generalized linear complementary function (9), with E_{po} replaced by βE_e, relating the actual with the apparent potential evaporation can be written as

$$\frac{E}{E_{pa}} = \left[(1+b)\beta \frac{E_e}{E_{pa}} - 1 \right] / b \tag{20}$$

Substitution of (7) with $E_{\max} = E_{pa}$ into equation (20) leads to

$$\beta = \frac{E_{pa}}{E_e} \left\{ 1 + \frac{b}{1+b} \left[\frac{P}{E_{pa}} - \left[1 + \left(\frac{P}{E_{pa}} \right)^w \right]^{1/w} \right] \right\} \tag{21}$$

or concisely

$$\beta = \left\{ 1 + c\left[\Phi - (1+\Phi^w)^{1/w} \right] \right\} / \Psi \tag{22}$$

where $\Psi = E_e / E_{pa}$, $\Phi = P / E_{pa} (= \mathrm{AI}^{-1})$, and $c = b/(b+1)$ in which b = 4.5, following Brutsaert (2015, Fig. 1). Interestingly, a few trial computations with (21) or (22) have shown that the results are quite close to those obtained herein with (19).