

The Role of Curvature in Modifying Frontal Instabilities – Short Summary

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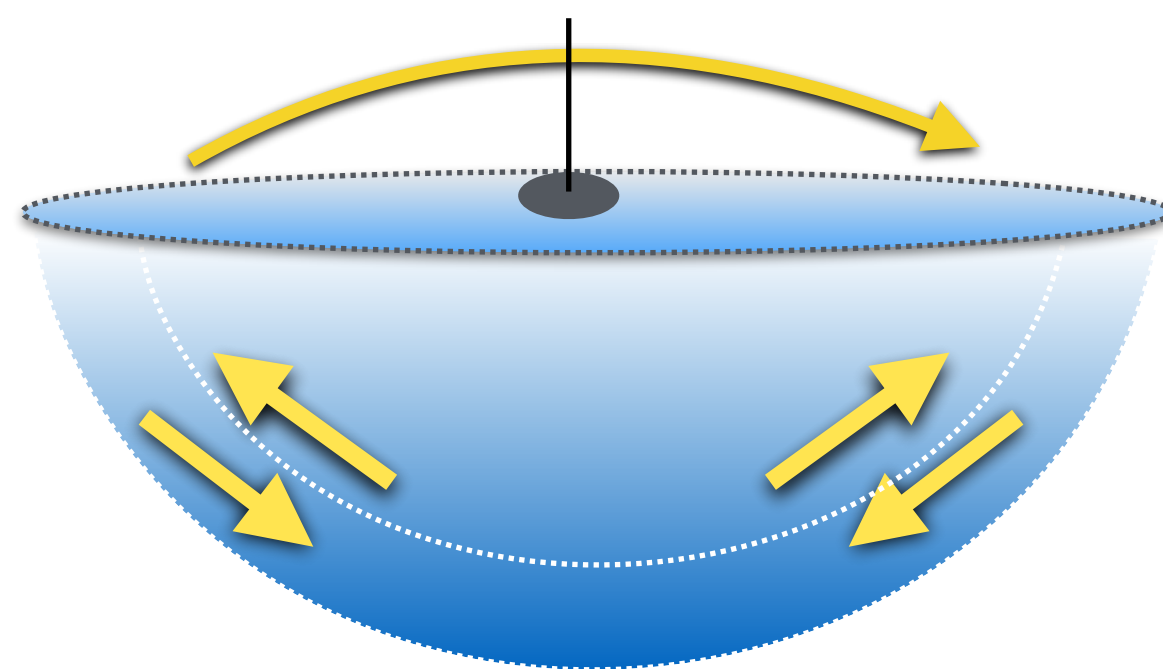
Abstract

In this study, we revisit the role of curvature in modifying frontal stability. We first consider the statement “ $f q < 0$ implies potential for instability”, where f is the Coriolis parameter and q is the Ertel potential vorticity (PV). This is true for any inviscid baroclinic flow. It is also evident in the transition of a governing equation for circulation within a front from elliptic to hyperbolic form as the discriminant changes sign. However, for curved fronts, an additional scale factor enters the discriminant owing to conservation of absolute angular momentum, L , leading to Solberg’s (1936) generalization of the Rayleigh criterion. In non-dimensional form, this expression also generalizes the classical instability criterion of Hoskins (1974) by accounting for centrifugal forces: modification of the front’s vertical shear and stratification owing to curvature tilts the absolute vorticity vector away from its thermal wind state and, in an effort to conserve the product of non-dimensional PV (q') and absolute angular momentum (L'), this alters Rossby and Richardson numbers permitted for stable flow. The criterion, $\Phi' = L' q' < 0$, is then investigated in non-dimensional parameter space representative of low-Richardson-number vortices. An interesting outcome is that, for Richardson numbers near one, anticyclonic flows increase in q' , while cyclonic flows decrease in q' . Though stabilization is muted for anticyclones (owing to multiplication by L'), the de-stabilization of cyclones is robust, and may help to explain an observed asymmetry in the distribution of submesoscale coherent vortices in the global ocean.

Revisiting symmetric instability in the oceans

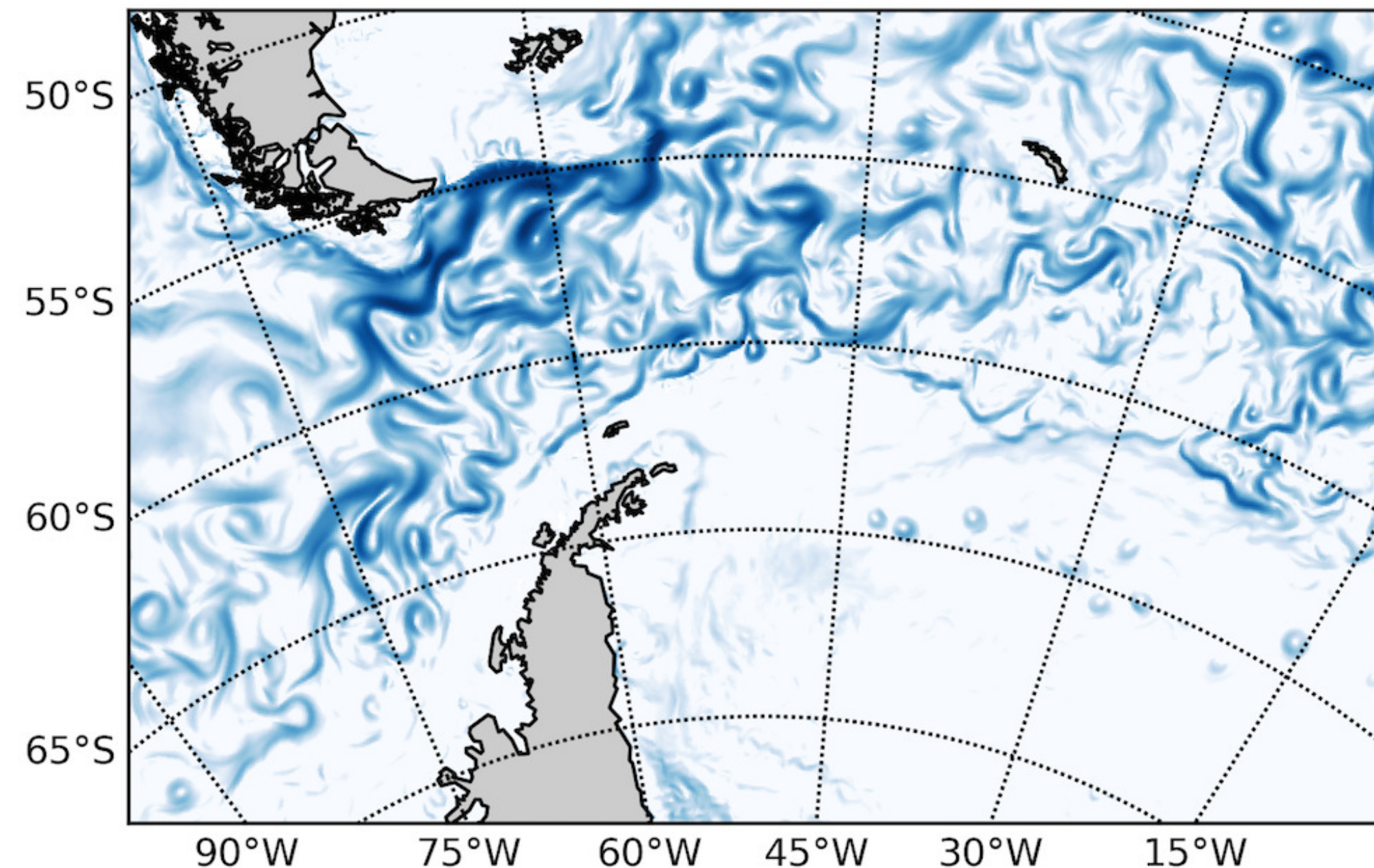
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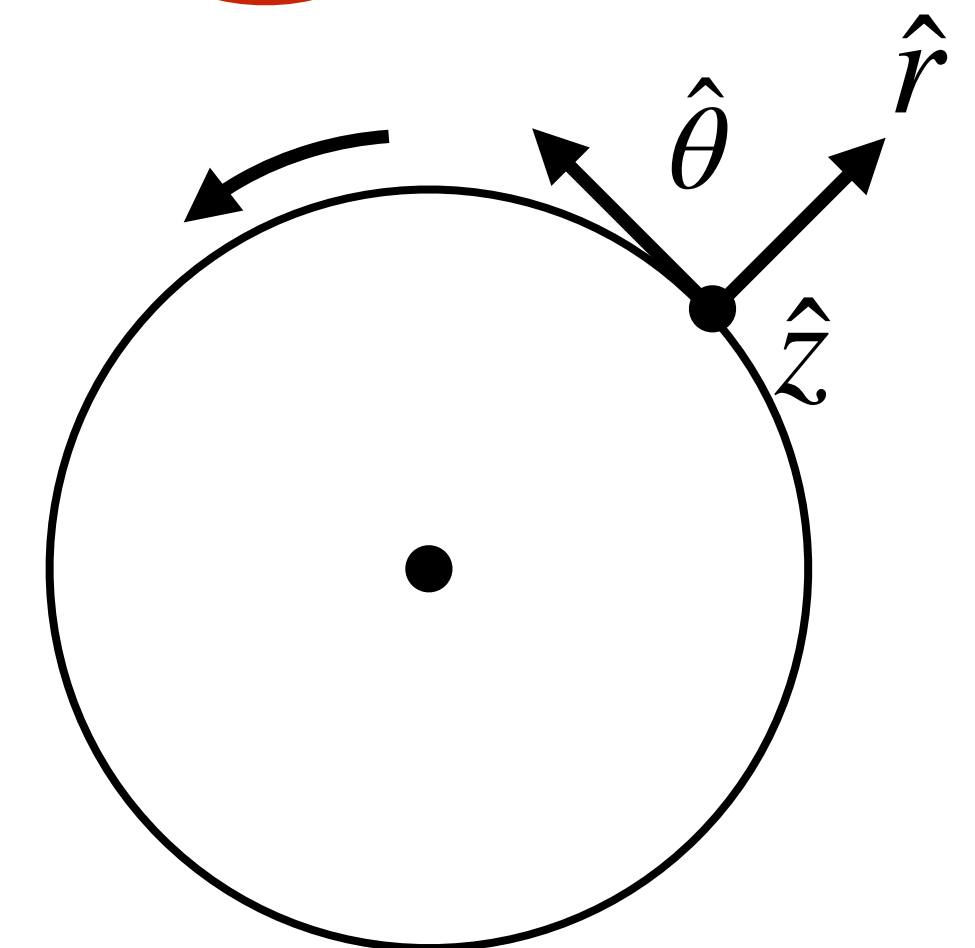
Symmetric instability

Antarctic Circumpolar Current (ACC)



When does the curvature term matter?

$$(f + 2\bar{v}/r)\partial_z \bar{v} = \partial_r \bar{b}$$



Most theory assumes thermal wind balance (Hoskins 1974)
Wouldn't **gradient wind balance** be a better approximation?
Particularly for small scales (**i.e. submesoscales**) and **strong current systems**?

Changes in stability introduced by curvature ...

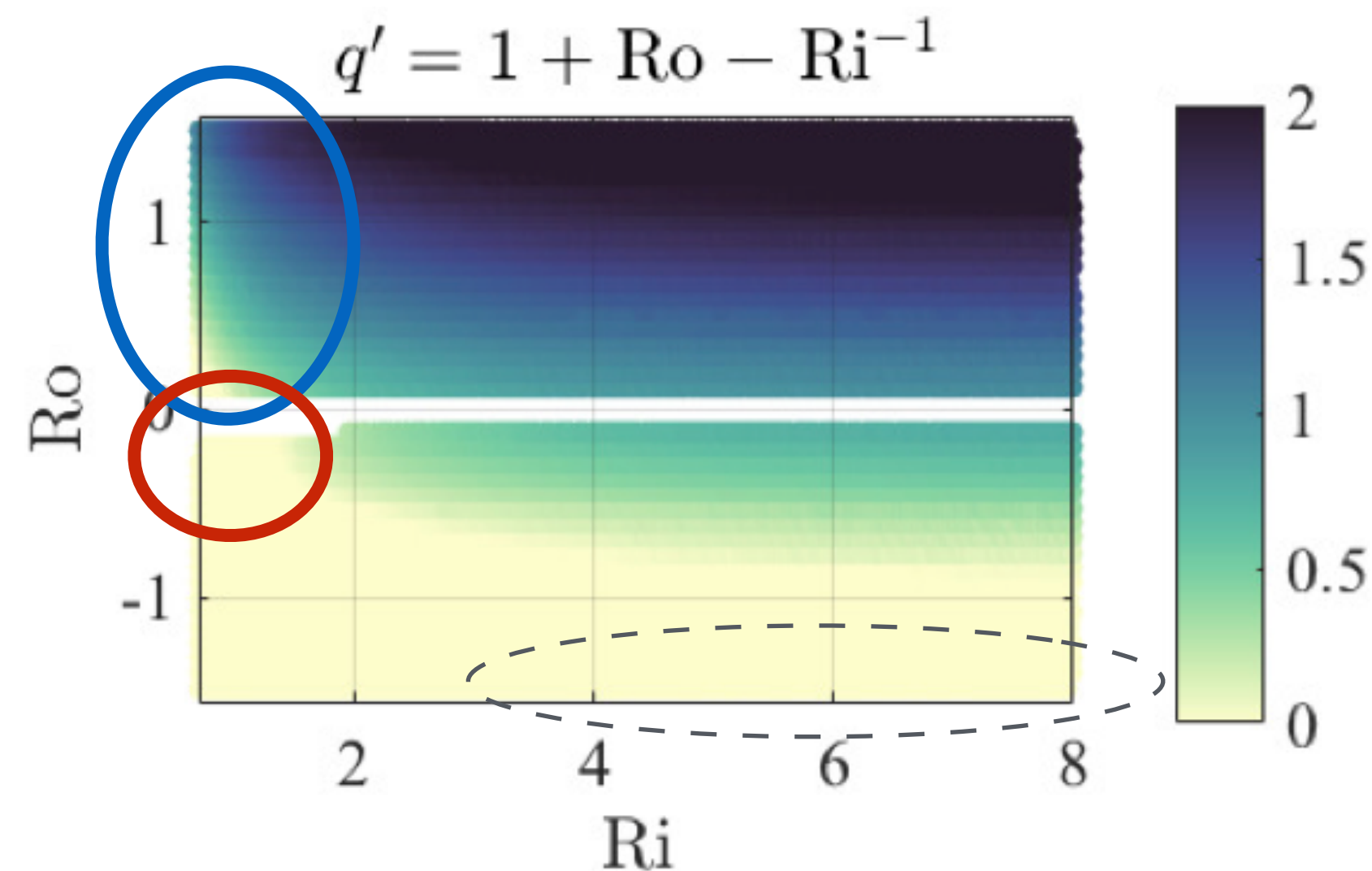
Thermal wind balance (TWB)

$$f\partial_z \bar{v} = \partial_r \bar{b}$$

Relevant instability criteria

$$fq < 0 \text{ (dim.)} \implies q' < 0 \text{ (non . dim.)}$$

No curvature



Occurs at low Ri
(i.e. symmetric instability)

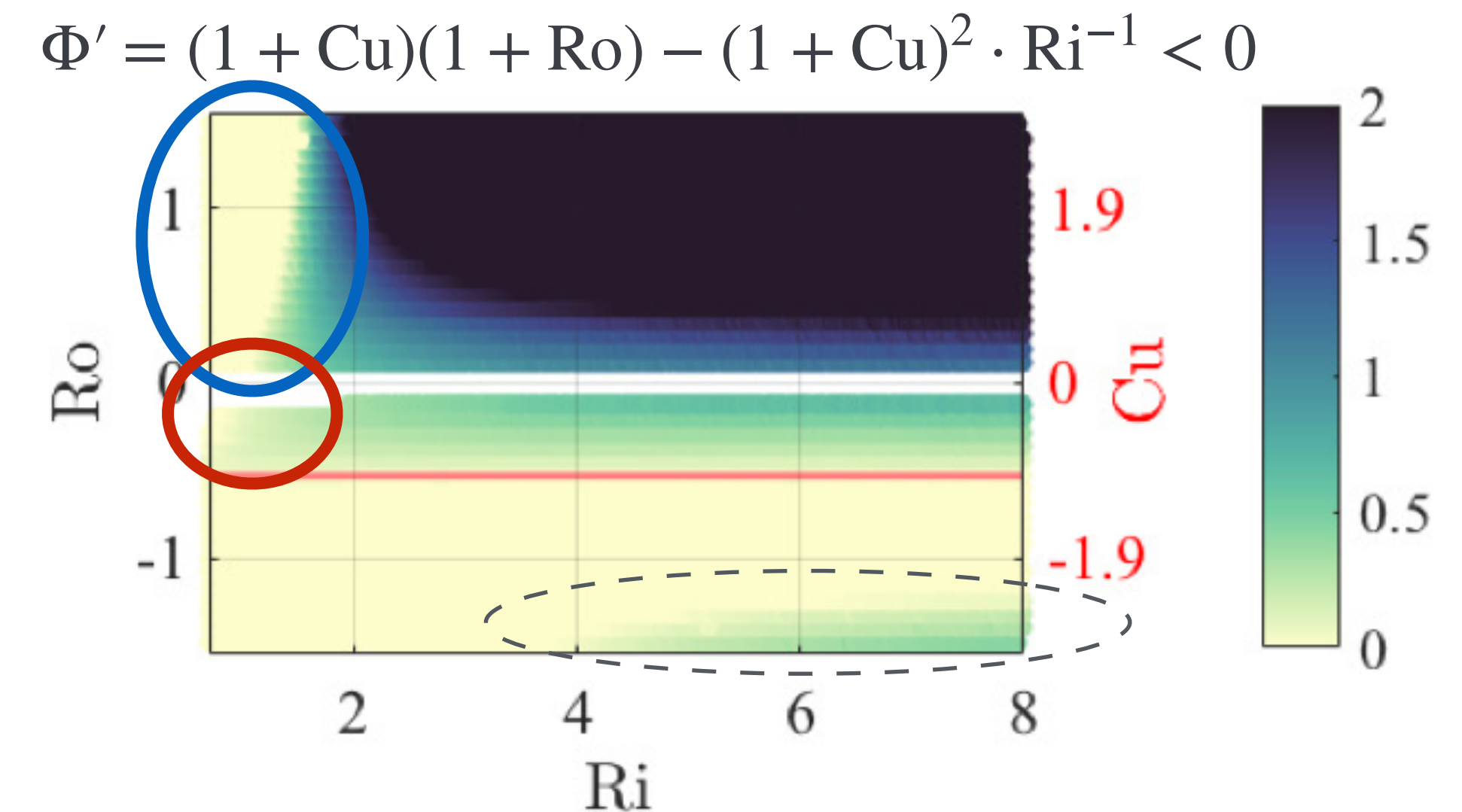
Gradient wind balance (GWB)

$$f(1 + Cu)\partial_z \bar{v} = \partial_r \bar{b}, \text{ where } Cu = 2\bar{v}/(fr)$$

Relevant instability criteria

$$(1 + Cu)fq < 0 \text{ (dim.)} \implies \Phi' = L'q' < 0 \text{ (non . dim.)}$$

With curvature

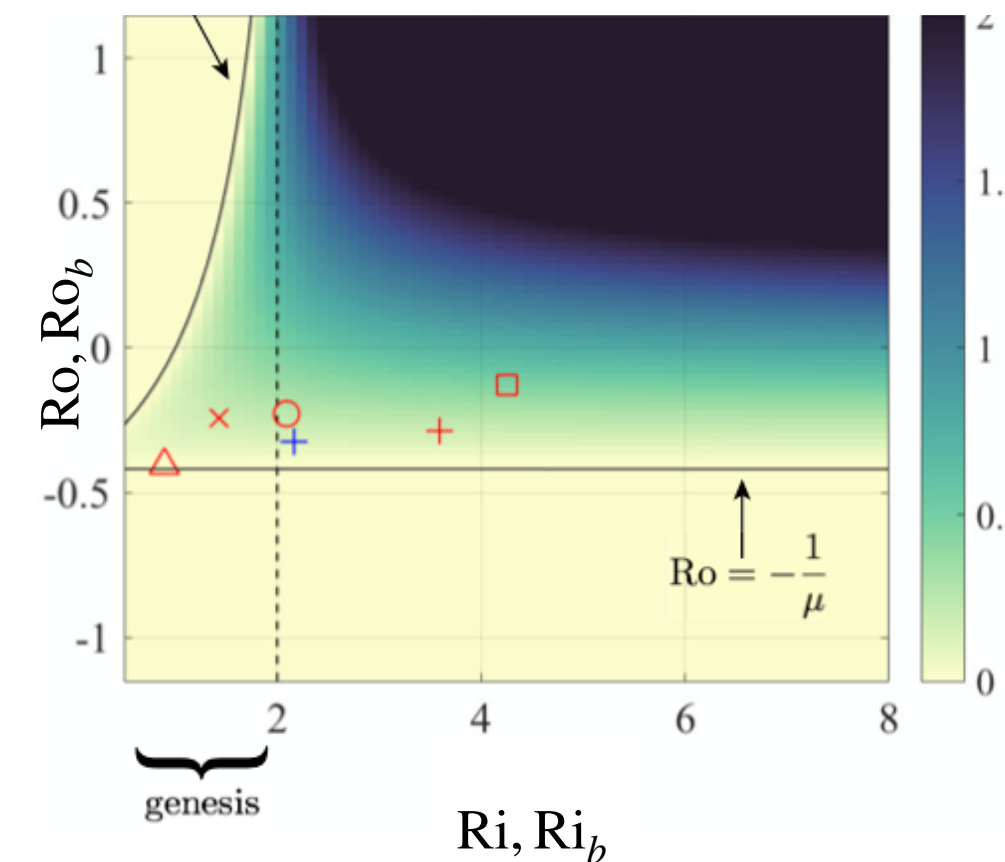


1. **Pronounced decrease in stability for curved cyclonic fronts ($Ro > 0$).**
2. **Marginal increase in stability for curved, anticyclonic fronts ($Ro < 0$).**
3. Stability for really strong anticyclones ($Ro < -1$). Occurs for negative PV!

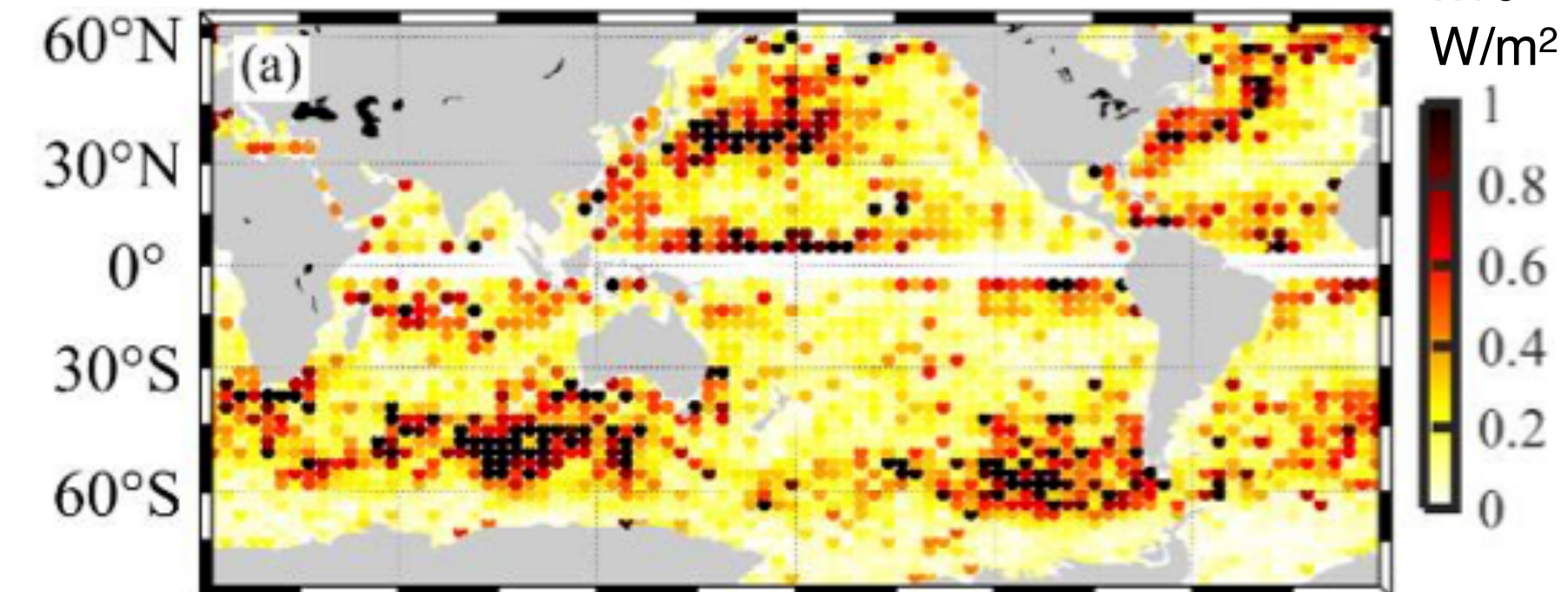
Potential Implications

- Ocean circulation
- Ocean-atmosphere exchange
- Tracer flux between boundaries and interior ocean
- Transport of water masses (**think biogeochemistry!**)
- Representation of these within Earth System models

Changes in symm. instability

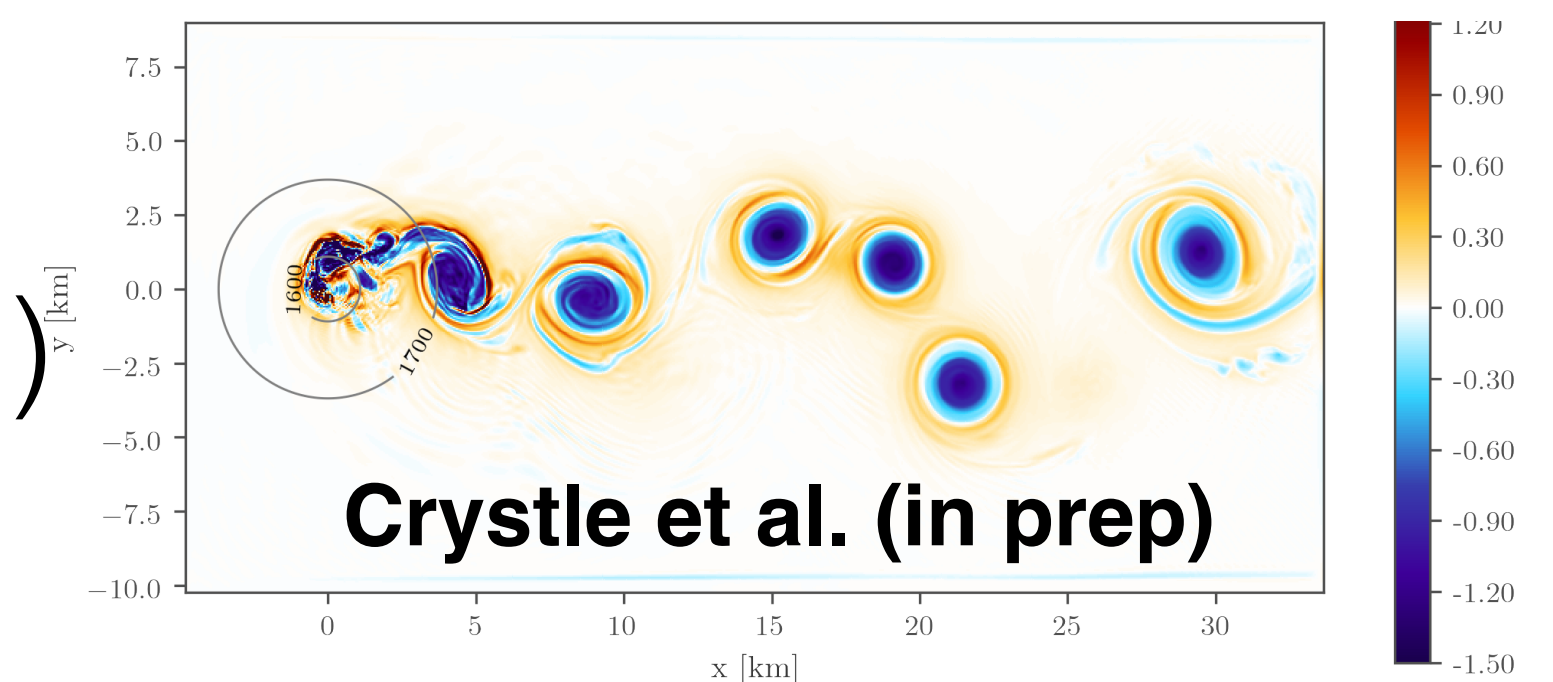


Energy loss from ocean circulation in winter $\times 10^{-2}$ W/m²



Dong et al. 2020, *in review*

Vortex Generation by Hydrothermal Vents



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