# Data-driven simulation of rapid flux enhancement of energetic electrons with an upper-band whistler burst

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#### Abstract

The temporal variation of the energetic electron flux distribution caused by whistler mode chorus waves through the cyclotron resonant interaction provides crucial information on how electrons are accelerated in the Earth's inner magnetosphere. This study employing a data-driven test-particle simulation demonstrates that the rapid deformation of energetic electron distribution observed by the Arase satellite is not simply explained by a quasi-linear diffusion mechanism, but is essentially caused by nonlinear scattering: the phase trapping and the phase dislocation. In response to upper-band whistler chorus bursts, multiple nonlinear interactions finally achieve an efficient flux enhancement of electrons on a time scale of the chorus burst. A quasi-linear diffusion model tends to underestimate the flux enhancement of energetic electrons as compared with a model based on the realistic dynamic frequency spectrum of whistler waves. It is concluded that the nonlinear phase trapping plays an important role in the rapid flux enhancement of energetic electrons observed by Arase.

# Data-driven simulation of rapid flux enhancement of energetic electrons with an upper-band whistler burst

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# 15 Key Points:

16	• The data-driven simulation of rapid flux enhancement has been performed using
17	plasma/particle and wave data obtained by Arase.
18	• The simulation results reproduce the observed temporal variations of energetic elec-
19	tron flux distributions.
20	• The nonlinear phase trapping contributes to the flux enhancement of electrons above
21	20 keV.

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#### 22 Abstract

The temporal variation of the energetic electron flux distribution caused by whistler mode 23 chorus waves through the cyclotron resonant interaction provides crucial information on 24 how electrons are accelerated in the Earth's inner magnetosphere. This study employ-25 ing a data-driven test-particle simulation demonstrates that the rapid deformation of en-26 ergetic electron distribution observed by the Arase satellite is not simply explained by 27 a quasi-linear diffusion mechanism, but is essentially caused by nonlinear scattering: the 28 phase trapping and the phase dislocation. In response to upper-band whistler chorus bursts, 29 multiple nonlinear interactions finally achieve an efficient flux enhancement of electrons 30 on a time scale of the chorus burst. A quasi-linear diffusion model tends to underesti-31 mate the flux enhancement of energetic electrons as compared with a model based on 32 the realistic dynamic frequency spectrum of whistler waves. It is concluded that the non-33 linear phase trapping plays an important role in the rapid flux enhancement of energetic 34 electrons observed by Arase. 35

#### <sup>36</sup> Plain Language Summary

Energetic electrons could be a cause of satellite anomalies affected by electric dis-37 charge phenomena on its surface and interior materials. To minimize the anomalies through 38 satellite operation, it is important to forecast the temporal variation of the energetic elec-39 tron flux along the trajectories of a satellite. One of the causes of the variation of the 40 electron flux is whistler mode waves, which are right-handed, circularly polarized elec-41 tromagnetic waves that can resonate with energetic electrons. To understand how the 42 electrons are accelerated in realistic situations, we have performed a data-driven numer-43 ical simulation to demonstrate electron scattering, by importing the observation data of 44 the Arase satellite directly to the simulation. Results of the simulation reproduce the 45 temporal variations of energetic electron flux distributions in burst of whistler mode waves. 46 It is found that the nonlinear scattering contributes to the flux enhancement of energetic 47 electrons. It is confirmed that a quasi-linear diffusion model, which has been used in gen-48 eral so far, cannot explain such a rapid flux enhancement. We conclude that the non-49 linear scattering caused by the whistler burst plays an important role in the rapid flux 50 enhancement of energetic electrons observed by the Arase satellite. 51

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## 52 **1** Introduction

Whistler mode chorus waves are bursty electromagnetic emissions that are often 53 observed as the lower band mode below half of the electron cyclotron frequency  $f_{ce}$  and/or 54 the upper band mode between  $0.5f_{ce}$  and  $1.0f_{ce}$ . The Earth's magnetosphere naturally 55 generates the whistler chorus with the injection of several tens of keV electrons associ-56 ated with substorms (e.g., Tsurutani & Smith, 1977; Miyoshi et al., 2003, 2013). The 57 whistler chorus waves play an important role in accelerating energetic electrons over a 58 wide energy range on the keV to MeV order through Doppler-shifted cyclotron resonance 59 (Horne & Thorne, 2003). Cyclotron resonant interactions result in the pitch angle and 60 energy diffusion of electrons bouncing along a magnetic field line, and more energetic elec-61 trons can resonate with chorus waves at higher magnetic latitudes. A quasi-linear the-62 ory, which assumes the resonant interactions by incoherent, broadband, and small-amplitude 63 whistler waves, is commonly used to describe the evolution of the phase space density 64 of radiation belt electrons. Numerical simulations based on the theory reproduce the evo-65 lution of electrons trapped in the Earth's magnetosphere on a time scale range from an 66 hour to a day (Thorne et al., 2013; Glauert et al., 2014; Tu et al., 2014). 67

However, some previous observations suggest the rapid acceleration of energetic elec-68 trons on a time scale much shorter than the prediction based on a quasi-linear theory 69 (Fennell et al., 2014; Kurita et al., 2018). This indicates that some efficient acceleration 70 processes not described in this theory are involved. Many theoretical and simulation stud-71 ies have shown the importance of a nonlinear scattering process associated with the co-72 herent and bursty nature of the whistler chorus waves (e.g., Omura et al., 2007; Bort-73 nik et al., 2008; Lakhina et al., 2010; Saito et al., 2016). The contribution of the coher-74 ent nature of chorus waves, which is beyond the scope of the quasi-linear theory, plays 75 an important role in the efficient acceleration of energetic electrons. In particular, rel-76 ativistic turning acceleration (RTA) (Omura et al., 2007) requires the coherent nature 77 of whistler waves in order to efficiently accelerate electrons. Lakhina et al. (2010) also 78 discussed the importance of the coherent and bursty nature of whistler chorus waves. They 79 estimated the rate of the pitch angle change of electrons using coherent subelements with 80 durations of tens of milliseconds or longer. For typical parameters of whistler chorus el-81 ements, their study showed that the coherent chorus elements can realize a more rapid 82 pitch angle scattering of energetic electrons than a continuum of incoherent chorus waves 83 as assumed in the quasi-linear theory. Previous studies have thus revealed that quasi-84

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linear models may underestimate electron scattering in terms of pitch angle and energy
by whistler chorus bursts.

As studied by Kurita et al. (2018), the Arase satellite (Miyoshi, Shinohara, et al., 87 2018) observed the rapid flux enhancement of electrons with energies above 20 keV as-88 sociated with an intense upper-band whistler chorus burst. They found that the evolu-89 tion of electron pitch angle distributions in multi-energy channels occurs within 30 s. By 90 detailed data analysis of the cyclotron resonant condition between electrons and the cho-91 rus burst, they concluded that the evolution is a consequence of wave-particle interac-92 tions, which are faster than expected from a quasi-linear theory. Thus, it is of consid-93 erable interest to examine numerically how the electrons are accelerated rapidly through 94 the wave-particle interactions on such a short time scale, using in situ observational data 95 obtained by Arase. 96

In this paper, we demonstrate electron scattering by an upper-band whistler cho-97 rus burst with a duration of 32 s, using a test-particle simulation: Geospace Environ-98 ment Modeling System for Integrated Studies - Radiation Belt with Wave-particle in-99 teraction (GEMSIS-RBW) (Saito et al., 2012) with observational data obtained by the 100 Medium Energy Particle Experiment-electron analyzer (MEP-e; S. Kasahara et al. (2018)) 101 and Onboard Frequency Analyzer (OFA; Matsuda et al. (2018)) in Plasma Wave Exper-102 iment (PWE; Y. Kasahara et al. (2018)) onboard Arase. In Sec. 2, we describe the ob-103 servational data set applied to the test-particle simulation. In Sec. 3, we describe the 104 simulation model and its initial conditions. In Sec. 4, we compare the temporal varia-105 tions of electron flux distributions resulting from the simulation with the observational 106 data. Our test-particle simulation demonstrates that the upper-band whistler chorus burst 107 reproduces the Arase observations through the electron scattering by the nonlinear phase 108 trapping. In Sec. 5, we discuss the scattering processes in the whistler chorus burst, some 109 problems of our simulation, and potential future works. Finally, we conclude that the 110 nonlinear scattering, which should not be described by quasi-linear diffusion processes, 111 plays an important role in the deformation of electron flux distribution in a short du-112 ration, as observed by Arase. 113

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#### 114 **2** Arase observation

A rapid deformation of electron flux distribution at tens of keV during a period be-115 tween 19:20:13 UT and 19:20:46 UT on 8 April 2017 was reported by Kurita et al. (2018). 116 Arase was located near the magnetic equator (magnetic latitude of  $0.2^{\circ} - 1.5^{\circ}$ ), at the 117 magnetic local time of 4.3 - 4.5 h, and the radial distance of 5.5 Earth radii, correspond-118 ing to the McIlwain L  $(L_m)$  of about 5.4. Magnetic field data obtained by the Magnetic 119 Field Experiment (MGF; Matsuoka, Teramoto, Nomura, et al. (2018)) onboard Arase 120 show a background magnetic field intensity of 170 nT, indicating the local electron cy-121 clotron frequency  $f_{ce}$  of 4.7 kHz. The plasma density  $N = 3.4 \text{ cm}^{-3}$  was estimated by 122 Kurita et al. (2018) on the basis of HFA and MGF measurements, where HFA is the high-123 frequency analyzer onboard Arase (Kumamoto et al., 2018), indicating the local elec-124 tron plasma frequency  $f_{pe}$  of 16.5 kHz and thereby the frequency ratio  $f_{pe}/f_{ce}$  of about 125 3.5.126

Figure 1 is the summary plot of the event. The OFA magnetic spectrum (top panel) 127 shows an intense upper-band whistler chorus burst at frequencies over  $0.5f_{ce}$ . The wave 128 amplitude is highly variable in time and frequency. The maximum instantaneous am-129 plitude of the magnetic fluctuation exceeds 100 pT. Kurita et al. (2018) estimated the 130 wave normal angle of the upper-band whistlers by the singular value decomposition method 131 (Santolík et al., 2003). They found that the burst propagates in the quasi-parallel di-132 rection of the background magnetic field. Bottom panels of Figure 1 show electron flux 133 distributions obtained by MEP-e at four time intervals. Over 32 s represented by the pan-134 els, the flux of >20 keV increases at pitch angles of 70 – 80 degrees, while the flux of <20135 keV decreases at pitch angles of 40 - 50 degrees. Because the flux variation seems to oc-136 cur on the resonant ellipses of the upper-band whistlers, Kurita et al. (2018) concluded 137 that the low-energy electrons are accelerated to higher energies through the cyclotron 138 resonance with the upper-band whistler chorus burst. 139

<sup>140</sup> 3 Simulation models and initial conditions

We perform a test-particle simulation that demonstrates the electron scattering by the upper-band whistler chorus burst on a magnetic field line. The upper-band whistlers are assumed to be generated at the magnetic equator and propagate away from the equator along the field line. We set wave amplitudes and frequencies on the basis of the OFA

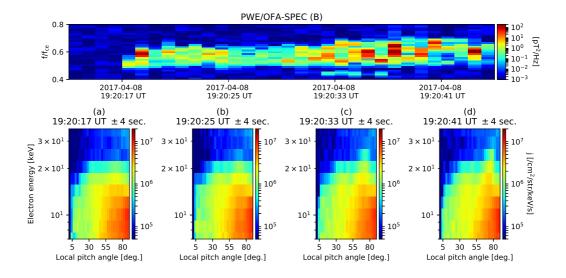


Figure 1. Observation by Arase located at the magnetic latitude of  $0.2^{\circ} - 1.5^{\circ}$ , the magnetic local time of 4.3 - 4.5 h, and the radial distance of 5.5 Earth radii (L<sub>m</sub> ~ 5.4). (Top) Dynamic frequency spectrum of magnetic fluctuations obtained from PWE/OFA. (Bottom) Electron flux distributions as functions of pitch angle and energy obtained from MEP-e at time intervals at around 19:20:17, 19:20:25, 19:20:33, and 19:20:41 UT. Electrons responsible for each flux distribution are detected in the time range of  $\pm 4$  s.

- magnetic field spectrum shown in Fig. 1. The simulation solves the equations for adi-145 abatic and non-adiabatic momentum change of  $10^6$  electrons by the upper-band whistlers. 146 We set particle weights for all the electrons on the basis of the electron flux distribution 147 obtained by MEP-e before the upper-band whistlers are enhanced. By using the weights 148 for the electrons, we can calculate the time variation of the electron flux distribution at 149 any place along the field line throughout the simulation. The following subsections de-150 scribe the details of the test-particle model, the wave model, and the initial condition 151 used in the simulation. 152
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#### 3.1 Test-particle model

We use the test-particle simulation code GEMSIS-RBW (RBW) (Saito et al., 2012) to demonstrate the temporal variation of energetic electron flux distribution. Using the RBW simulation model, we calculate the adiabatic motion of electrons along a field line

using the equations of magnetic mirror motion of the guiding center, 157

$$\frac{dp_{\parallel}}{dt} = -\frac{\mu}{\gamma} \frac{\partial B}{\partial s},\tag{1}$$

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 $\frac{ds}{dt} = \frac{p_{\parallel}}{m_o \gamma},$ (2)160

where  $\mu = p_{\perp}^2 / (2m_o B)$  is the first adiabatic invariant. The invariant is assumed to be 161 constant as the mirror force is solved with the equation. Here, the relativistic Lorentz 162 gamma  $\gamma$  is  $\sqrt{1 + p^2/(m_o c)^2}$ , B is the magnetic field intensity at the electron position 163  $s,\,p^2=p_{\parallel}^2+p_{\perp}^2,\,p_{\parallel}$  and  $p_{\perp}$  are electron momenta parallel and perpendicular to the mag-164 netic field, respectively,  $m_o$  is electron rest mass, and c is the light speed. The equations 165 are solved by using the 4th-order Runge–Kutta method. In addition to the adiabatic mir-166 ror motion, the RBW simulation demonstrates the propagation of wave packets along 167 the field line with its own group velocity. The group velocity based on the cold plasma 168 dispersion relation is calculated at packet positions. Each wave packet has a wave am-169 plitude and frequency, which are constant over time. 170

By calculating multiple packets traveling along the field line in the RBW model, 171 we can estimate the wave frequency and amplitude acting on each of the electrons on 172 the field line. The wavenumber at the electron position  $k_s$  is calculated from the linear 173 dispersion relation of whistler waves with the the frequency  $f_s$ , and then right circularly 174 polarized electromagnetic fluctuations  $\delta \mathbf{E}$  and  $\delta \mathbf{B}$  at the electron position are constructed 175 using the RBW model. The temporal variation of the magnetic wave phase  $\phi$  at the elec-176 tron position is 177

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$$\phi = 2\pi f_s t - k_s v_{\parallel} t + \phi_o. \tag{3}$$

Here,  $\phi_o$  is the wave phase at which the wave-particle interaction starts. Note that  $\phi$  is 179 assigned to each of the electrons, which is the important core method for the RBW model. 180 By using the electromagnetic fluctuations, we can solve the following equation of mo-181 tion using the RBW model, 182

$$\frac{d\mathbf{p}}{dt} = q_e \left( \delta \mathbf{E} + \mathbf{v} \times \left( \mathbf{B} + \delta \mathbf{B} \right) \right), \tag{4}$$

where  $\mathbf{v}$  is the vector of electron velocity. Wave data applied to Eq. (4) are updated ev-184 ery  $\Delta t ~(\sim 0.035 \text{ ms})$  which is the time resolution of the solver of the guiding-center equa-185 tions (Eqs. (1) and (2)), whereas the time resolution for the equation of motion (Eq. 4) 186 is  $\delta t = \tau_{gyro}/64$ , which is quite shorter than  $\Delta t$ , where  $\tau_{gyro}$  is the in situ electron gy-187 ration period. The equation of motion is solved by the Buneman-Boris method (Buneman, 188

1993). The equation of motion is used to calculate  $\Delta \mathbf{p}$ , which is the change in  $\mathbf{p}$  in  $\Delta t$ ; 189 then  $\Delta \mathbf{p}$  is reflected in the guiding-center equations (Eqs. (1) and (2)). Then, the first 190 adiabatic invariant  $\mu$  is updated using the magnetic field intensity at the electron po-191 sition, corresponding to the break of  $\mu$  caused by the electron scattering. By using the 192 above sequence of calculations, we can solve the equations of magnetic mirror motion 193 coupled with the equation of motion in electromagnetic fluctuations of whistler mode 194 waves propagating along the field line. The RBW model has been successfully applied 195 to various wave-particle interaction phenomena in radiation belts, pulsating auroras, and 196 microbursts (Saito et al., 2012, 2016; Miyoshi, Oyama, et al., 2015; Miyoshi, Saito, et 197 al., 2015; Miyoshi et al., 2020). 198

#### 3.2 Wave model

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Wave packets are released every 1 ms from the magnetic equator according to the OFA magnetic spectrum shown in Fig. 1. The OFA resolves 15 frequencies between  $0.5f_{ce}$ and  $0.7f_{ce}$ . The frequency of each wave mode is defined as the discrete frequency  $f^i$ ,

$$f^i = f_o + \Delta f \times i, \tag{5}$$

where  $f_o$  is 2.368 kHz,  $\Delta f$  is 64 Hz, and *i* is an integer between 1 and 15. Since the time 204 resolution of the OFA is 1 s, the amplitude of wave packets released every  $\Delta t$  varies ev-205 ery second. When there are multiple modes at the electron position, the RBW model 206 is used to construct electromagnetic fluctuations acting on the electron by linearly su-207 perimposing these modes. On the basis of statistical studies of the upper band chorus 208 distribution (e.g., Bortnik et al., 2007), we assume that wave packet propagation is lim-209 ited to a magnetic latitude of 10 degrees. Note that there is no correlation of wave phase 210 among wave modes in the simulation. Moreover, we assume that the phase difference be-211 tween the wave phase and the electron gyrophase is randomly set between 0 and  $2\pi$  when 212 the electron passes through the magnetic latitudes of 10 degrees from higher latitudes. 213

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#### 3.3 Initial conditions

The magnetic field intensity along the field line in this simulation is assumed to be equal to the Earth's dipole field. On the basis of the MGF data of Arase, which was located close to the magnetic equator, the equatorial magnetic field intensity  $B_{eq}$  is estimated to be 170 nT. The background magnetic field is assumed to be the dipole field; thus, the magnetic field strength along the field line is

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 $B(\lambda) = 170 \times 10^{-9} \frac{\sqrt{1+3\sin^2 \lambda}}{\cos^6 \lambda},\tag{6}$ 

where  $\lambda$  is latitude. On the basis of the HFA and MGF measurements, as described in Sec. 2, the estimated plasma density  $N = 3.4 \text{ cm}^{-3}$  is applied in this simulation. Here, as in the previous simulation (Miyoshi, Oyama, et al., 2015), we assume that the plasma density is constant along the field line up to the magnetic latitude of 10 degrees.

The number of electrons in the present simulation is  $10^6$ . These are distributed along the magnetic field line with random bounce phases. Their equatorial pitch angles range from the loss cone angle to slightly less than 90 degrees, and the kinetic energy ranges from 5 to 40 keV. The weight of electrons at energy E and the equatorial pitch angle  $\alpha_{eq}$ are derived from the weight table W,

$$W(E, \alpha_{eq}) = \frac{j(E, \alpha_{eq})}{j_u(E, \alpha_{eq})}.$$
(7)

Here,  $j(E, \alpha_{eq})$  is the flux distribution to be reproduced, and  $j_u(E, \alpha_{eq})$  is the flux dis-231 tribution calculated from the electrons with the unit weight.  $j(E, \alpha_{eq})$  can be reproduced 232 by setting the weight of the electrons. The weight of electrons is determined to fit the 233 distribution function obtained by MEP-e just before the chorus burst appears. The weight 234 is constant over time and there is no additional injection of electrons throughout the sim-235 ulation. The simulation starts from 2017-04-08 19:20:13 UT at which wave packets start 236 to inject from the equator following the OFA shown in Figure 1. The simulation time 237 t is the time that elapsed from the start. 238

#### **4** Simulation results

Figure 2 shows electron flux distributions calculated at the equator in the simulation. The time intervals labeled on the panels (a - d) correspond to the time intervals of observations for the flux distributions shown in the lower panels in Figure 1. The temporal evolution of electron fluxes shows that the electron flux increases at energies higher than 20 keV within 32 s. At energies lower than 20 keV, the electron flux at the pitch angles of 40 – 50 degrees decreases. These characteristics of flux deformation are similar to those of MEP-e shown in Figure 1.

To compare these distributions more directly, we plot the pitch angle distributions at 14.3, 17.1, 20.5, and 24.5 keV of the Arase observations (blue lines) with those of the

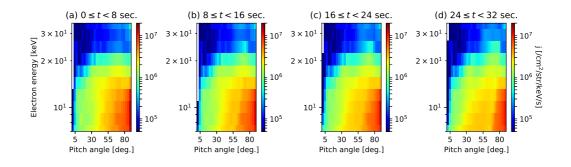


Figure 2. Equatorial electron flux distributions calculated from RBW simulation as a function of pitch angle and energy. The flux distributions are calculated during (a)  $0 \le t < 8$  s, (b)  $8 \le t < 16$  s, (c)  $16 \le t < 24$  s, and (d)  $24 \le t < 32$  s. These time ranges correspond to 19:20:17 UT  $\pm 4$  s, 19:20:25 UT  $\pm 4$  s, 19:20:33 UT  $\pm 4$  s, and 19:20:41 UT  $\pm 4$  s, respectively.

simulation (red lines) in Figure 3. The Arase observations over 8 s are averaged, whereas 249 black error bars show the standard deviations of the Arase observations at each pitch 250 angle and energy bins during the indicated time interval. In both the simulation and ob-251 servation, electron fluxes at around the pitch angle of 50 degrees decrease over time at 252 energies of 14.3 and 17.1 keV, whereas those at around the pitch angle of 75 degrees at 253 energies of 20.5 and 24.5 keV increase. A butterfly distribution is formed at energies higher 254 than 20 keV. It is shown that the simulation reproduces the characteristics of the ob-255 served flux enhancement. 256

Figure 4 shows the initial flux distributions of electrons that contribute to the fluxes 257 within the energy range between 23.5 and 25.5 keV and the pitch angle range between 258 70 and 80 degrees in (a)  $0 \le t < 8$ , (b)  $8 \le t < 16$ , (c)  $16 \le t < 24$ , and (d)  $24 \le 16$ 259 t < 32 s. Here, we define the pitch angle and energy ranges as  $\Lambda$ . As shown in panel 260 (a), the initial flux distribution of electrons has a peak in the  $\Lambda$  at the time of less than 261 8 s. Only a small fraction of the electrons originate from outside of  $\Lambda$  at this moment. 262 At later times, as shown in panels (b-d), the distribution spreads in both directions of 263 energy and pitch angle, and the peak of the flux distribution moves toward lower ener-264 gies and smaller pitch angles. At the time interval shown in panel (d), there is a peak 265 of the distribution at the pitch angle between 40 and 50 degrees and the energy between 266 15 and 16 keV, indicating that the electrons that were initially distributed there dom-267 inate  $\Lambda$ . 268

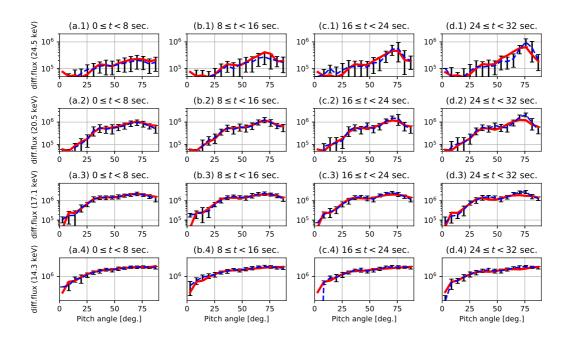


Figure 3. Pitch angle distributions at 14.3, 17.1, 20.5, and 24.5 keV. Blue lines are electron fluxes taken from the MEP-e onboard Arase averaged over 8 s, black error bars are the standard deviations of the fluxes obtained during the time interval, and red lines are calculated from the RBW simulation.

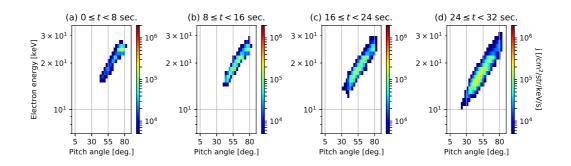


Figure 4. Initial flux distributions of electrons contributing to flux within the energy range between 23.5 keV and 25.5 keV and the pitch angle range between 70 and 80 degrees in the time ranges of (a)  $0 \le t < 8$ , (b)  $8 \le t < 16$ , (c)  $16 \le t < 24$ , and (d)  $24 \le t < 32$  s.

Figure 5 confirms whether the pitch angle change can be realized with a quasi-linear 269 diffusion model. The black-dashed line in the top panel shows the magnetic wave power 270 in frequency averaged in time during the burst event, calculated from the OFA magnetic 271 spectrum. The dashed red line is a Gaussian fitting curve for the time-averaged frequency 272 spectrum. The Gaussian fitting gives the maximum amplitude of 33.6 pT, the center of 273 frequency of  $0.59 f_{ce}$ , and the frequency band width of  $0.027 f_{ce}$ . With the derived pa-274 rameters of the magnetic wave power distribution, we calculate the pitch angle diffusion 275 coefficients of the quasi-linear diffusion model (Albert, 1999) at energies of 10, 20, and 276 30 keV. Here, the coefficients are averaged in the bounce motion. As electrons move along 277 the distribution shown in Figure 4(d) before reaching  $\Lambda$ , the averaged pitch angle coef-278 ficient of the electrons would be less than  $2 \times 10^{-3}$  [/sec.]. We found in Figure 4 (d) that 279 the main flux source contributing to the formation of the butterfly distribution is at a 280 pitch angle between 40 and 50 degrees and an energy between 15 and 16 keV. From a 281 simple estimation, a value corresponding to a pitch angle diffusion coefficient is about 282  $4 \times 10^{-3}$  [/sec.] as electrons at the source region ( $\alpha_{eq} = 45$  degrees) move to the flux 283 peak at 24.5 keV ( $\alpha_{eq} = 75$  degrees) within 32 s. This value seems to be slightly larger 284 than the diffusion coefficients estimated using the quasi-linear diffusion model, but roughly 285 of the same order. It suggests that some electrons initially at 15 keV can contribute to 286 the butterfly distribution formation through pitch angle scattering with comparable timescales 287 predicted by the quasi-linear process. 288

Figure 6 shows the probability of nonlinear scattering of electrons with energies be-289 tween 24 and 25 keV at t = 32 s as a function of equatorial pitch angle. The probabil-290 ity is defined as  $N_{rapid}/N_{total}$  with the energy range in a pitch angle bin, where  $N_{rapid}$ 291 is the number of electrons that have experienced a rapid change in the pitch angle and 292 the energy at least once during 32 s and  $N_{total}$  is the total number of electrons. Here, 293 the number of electrons in each bin is calculated considering particle weights defined by 294 the initial flux distribution. The rapid change for an electron means that a value of  $D_{TP} =$ 295  $\Delta \alpha_{eq}^2/(2\Delta t)$  corresponding to an instantaneous pitch angle diffusion coefficient exceeds 296 the threshold coefficients  $D_{th} = 0.05, 0.1, \text{ and } 0.2, \text{ where } \Delta \alpha_{eq}$  is an equatorial pitch 297 angle change calculated every  $\Delta t = 0.2$  s. The threshold coefficients are 20 - 100 times 298 larger than the pitch angle coefficients in Figure 5. As seen in Figure 6, there is a peak 299 in pitch angles of 70 and 80 degrees at each  $D_{th}$ . Around a pitch angle of 75 degrees, al-300 most all electrons experience  $D_{TP} > 0.05$  at least once during the burst. The value is 301

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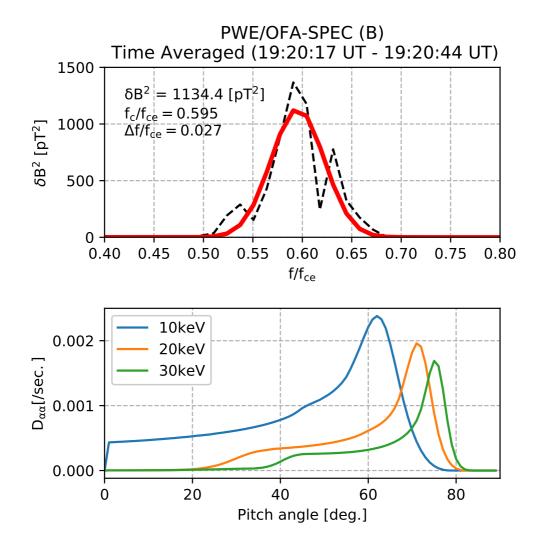


Figure 5. (Top) Magnetic frequency spectrum averaged in time during the burst event obtained from PWE/OFA (Black-dashed) and a Gaussian fitting curve for the time-averaged frequency spectrum (Red). (Bottom) Bounce-averaged, pitch angle diffusion coefficients as a function of pitch angle with energies of 10, 20, and 30 keV. The quasi-linear model uses the Gaussian distribution of magnetic wave power as shown in the top panel, which has the maximum amplitude of 33.6 pT, the center of frequency of  $0.59f_{ce}$ , and the frequency band width of  $0.027f_{ce}$ . We assume a Gaussian distribution of the wave propagation angle with the half width of 45 degrees, centered at the zero degree along the magnetic field line.

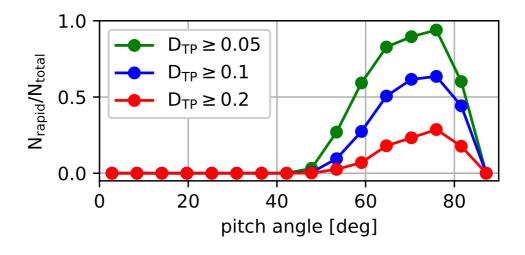


Figure 6. Probability of nonlinear scattering for electrons with energies between 24 and 25 keV at t = 32 s as a function of equatorial pitch angle.

about 20 times larger than the diffusion coefficients. About 30% of electrons at the peak 302 experience a rapid scattering with  $D_{TP} > 0.2$ , which is 100 times larger than the es-303 timation with the quasi-linear model. The peaks of  $N_{rapid}/N_{total}$  in pitch angle are within 304 a range of electron flux enhancement shown in the top panels (E = 24.5 keV) in Figure 305 3. From the OFA spectrum (Figure 1), the maximum instantaneous amplitude is esti-306 mated to be about 120 pT, which is four times higher than the average amplitude. If we 307 assume a 16  $(= 4^2)$  times higher magnetic power distribution, pitch angle diffusion co-308 efficients would be estimated to be up to 0.032 [/sec.]. The diffusion coefficients estimated 309 using the quasi-linear model cannot exceed 0.05. 310

We examine linear/nonlinear scattering processes in more detail by analyzing the 311 motion of typical electrons in the simulation. Figure 7 shows the time histories of en-312 ergy and equatorial pitch angle of five electrons labeled as A to E, which are sampled 313 from 100 electrons with energies between 15 and 16 keV and equatorial pitch angles be-314 tween 40 and 50 degrees at t=0. Because the whistlers propagate away from the equa-315 tor up to the magnetic latitude of 10 degrees, electrons have opportunities to be scat-316 tered through cyclotron resonance when they travel toward the equator at magnetic lat-317 itudes less than 10 degrees. Thus, electrons can be scattered every half of the bounce 318 period ( $\sim 0.5$  s), as seen in all the time histories. Almost all scattering times are on the 319 order of 100 ms or shorter. In many cases, electrons experience energy changes of less 320

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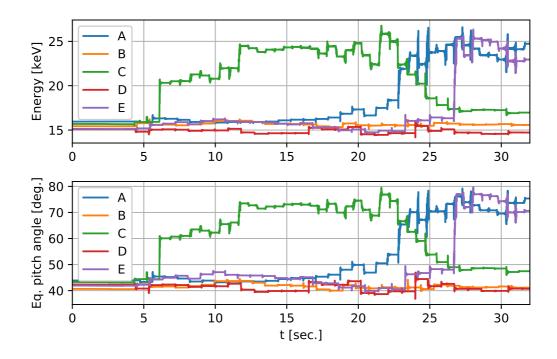


Figure 7. Time histories of the energy (top) and equatorial pitch angle (bottom) of five electrons labeled as A to E. The initial pitch angle and energy are between 40 and 50 degrees and between 15 and 16 keV, respectively.

# than 1 keV, but in some cases, the interaction increases or decreases their energy higher than 1 keV at a single scattering.

Figure 8 shows the results of a detailed analysis of electron E. The top panel shows 323 the energy time history of electron E, which is the same as that shown as the purple line 324 in Figure 7. For the simulation time of less than 26 s, the electron undergoes energy changes 325 of 1 keV or less in each of the scattering events. We label one of the events as (I). In the 326 time range between 26 and 27 s, the electron gains an energy of about 8 keV, which is 327 labeled as (II). Soon after the efficient energy gain, the electron loses an energy of about 328 2 keV, which is labeled as (III). For the three events labeled here, we show the distance 329 of the electron from the magnetic equator (2nd row), the kinetic energy (3rd row), the 330 inhomogeneity ratio S (4th row), and the phase differences  $\zeta$  between the wave phase 331 and the electron gyrophase (5th row). As described by Omura et al. (2007), the inho-332 mogeneity ratio is 333

$$S = \frac{1}{2kv_{\perp}\Omega_w\delta^2} \left[ \left( 2 + \delta^2 \frac{\Omega_e - \gamma\omega}{\Omega_e - \omega} \right) V_R - \frac{k\gamma v_{\perp}^2}{\Omega_e} \right] \frac{\partial\Omega_e}{\partial s},\tag{8}$$

where  $\delta^2 = 1 - (\omega/ck)^2$ ,  $\Omega_w = -q_e \delta B/m_o$ ,  $V_R = (\omega - \Omega_e/\gamma)/k$ ,  $v_\perp$  is the speed of an electron in the direction perpendicular to the background magnetic field, and  $\Omega_e$  is the angular electron cyclotron frequency. The inhomogeneity ratio and the phase difference are calculated for 15 wave modes described in Sec. 3.2.

To estimate the energy change and the interaction time of each of the events, we calculate the fitting curve using the function based on the hyperbolic tangent curve,

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$$F(t) = \frac{\Delta E}{2} \left( 1 + \tanh \frac{t - t_c}{t_w} \right) + E_o, \tag{9}$$

by solving a nonlinear least-squares problem by an algorithm (Branch et al., 1999) that 342 is used in the function scipy.optimize.curve\_fit() in SciPy library (Virtanen et al., 343 2020). Here,  $\Delta E$  is the amount of energy change,  $t_c$  is the center time of the scattering 344 event,  $t_w$  is the half width of the time interval of the energy change, and  $E_o$  is the ini-345 tial value of the function. The curve obtained using the estimated parameters is shown 346 as the blue dashed lines in the third row in Figure 8, and the parameters are shown in 347 each of the panels. Event (I) has the smallest energy change ( $\Delta E = 130 \text{ eV}$ ) with the 348 shortest duration of 4.8 ms  $(=2t_w)$  among the three events. On the other hand, event 349 (II) has the longest duration of 100.4 ms with the largest energy change (7.98 keV). Event 350 (III) shows the energy reduction (-1.93 keV) with a moderate duration of 27.4 ms. Note 351 that the pitch angle immediately before the rapid energy loss is relatively large ( $\alpha_{eq} >$ 352 75 degrees), as is shown in Figure 7. 353

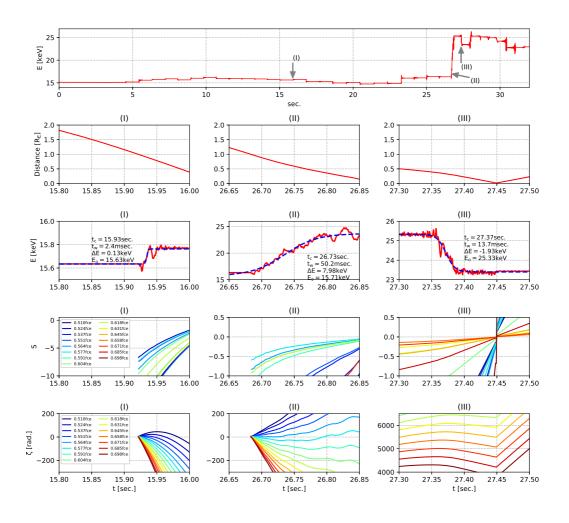
We calculate the inhomogeneity ratio S (Omura et al., 2007) for 15 wave modes, 354 which is the key parameter for the resonant phase trapping of electrons. The necessary 355 condition for the trapping is  $|S| \leq 1$ , which satisfies the pendulum equation for the phase 356 difference  $\zeta$ . When the pendulum equation is satisfied, the electrons are trapped in the 357 wave phase and then gain energy efficiently. Bortnik et al. (2008) defined the inhomo-358 geneity forcing term  $\rho$ , which is used to categorize three scattering types: the linear scat-359 tering, the phase trapping, and the phase dislocation. The condition of  $\rho < 1$  leads to 360 the diffusive behavior of electrons, whereas the condition of  $\rho \gg 1$  leads to the phase 361 dislocation, which generally reduces the electron energy. When  $\rho \sim 1$ , some electrons 362 are trapped by a wave, leading to the efficient energy gain of electrons. Saito et al. (2016) 363 showed that  $S^2 \rho^2 = 1$ , so the three types of scattering are also categorized on the ba-364 sis of S, namely,  $|S| \gg 1$  for linear scattering,  $|S| \sim 1$  or slightly less for the phase 365 trapping, and  $|S| \ll 1$  for the phase dislocation. The inhomogeneity ratio for 15 wave 366

modes in event (I) shows that all |S| values are much larger than 1 within the duration 367 of the small energy change, indicating the linear scattering. The time histories of  $\zeta$  for 368 15 wave modes do not show any signature of the phase trapping. Several high-frequency 369 modes have  $\zeta$  decreasing monotonically, whereas others have convex profiles indicating 370 that the resonance conditions  $d\zeta/dt \sim 0$  are satisfied at the peak of  $\zeta$ . In event (II), 371 several wave modes have  $|S| \sim 1$  or slightly less during the efficient energy gain. The 372 phase differences  $\zeta$  of several modes remain roughly constant. It means that the elec-373 tron is phase-trapped by some of these modes. Thus, the electron undergoes efficient en-374 ergy gain over a duration longer than that of event (I). In the case of event (III), sev-375 eral wave modes have S values close to zero, and the electron loses energy, whereas  $d\zeta/dt$ 376 of the wave modes is close to zero. The signature of event (III) is consistent with that 377 of the phase dislocation. 378

We calculate the energy time histories of 100 electrons that have had the initial en-379 ergy between 15 and 16 keV and the initial equatorial pitch angle between 40 and 50 de-380 grees. From the entire dataset of the time histories, we estimate  $t_c$  and  $\Delta E$  in each of 381 the scattering events by using the fitting method as used in Figure 8. In each of the scat-382 tering events, we calculate the inhomogeneity ratio for 15 wave modes at  $t_c$ . A total of 383 3,413 scattering events are identified from the dataset. Here, we choose scattering events 384 whose standard deviation of  $\Delta E$  is smaller than the estimated  $\Delta E$ . The qualified event 385 list is further grouped into two classes: weak and intense, according to the magnitude 386 of  $\Delta E$ , which are defined as  $10 \text{eV} \leq |\Delta E| < 100 \text{eV}$  and  $1 \text{keV} \leq |\Delta E| < 10 \text{keV}$ , re-387 spectively. We further classify each of the two classes of the events into two types ac-388 cording to the sign of  $\Delta E$ , that is, positive and negative. 389

Figure 9 shows the probability density functions (PDFs) of the inhomogeneity ra-390 tio S calculated at  $t_c$  for the two types in two classes. Note that the integral of the PDFs 391 over S becomes unity. The number of identified events for each of the four cases is shown 392 as N. The PDFs for intense events (left panels) show a clear peak at |S| slightly lower 393 than 1. According to the classification of the scattering processes discussed by Bortnik 394 et al. (2008), the intense events are expected to be accompanied by phase trapping or 395 phase dislocation. Considering the characteristics of the scattering processes, it is shown 396 that the phase trapping and phase dislocation contribute to the intense-positive and intense-397 negative events, respectively. Note that the PDFs of the intense-negative events show 398 the distribution confined slightly closer to |S| = 0 than that of the intense-positive events. 399

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**Figure 8.** Results of a detailed analysis of electron E. (Top panel) Time history of the electron energy. (2nd row) Distance from magnetic equator in events (I), (II), and (III) labeled in top panel. (3rd row) The red line shows an enlarged view of the time history of the electron energy in each of the events. The blue dashed line is the curve fitting of the red line through the hyperbolic tangent function. The fitting parameters are shown in each of the panels. (4th row) Inhomogeneity ratio of the electron E associated with 15 wave modes. (Bottom row) Phase differences between the electron gyrophase and the wave phases of 15 wave modes.

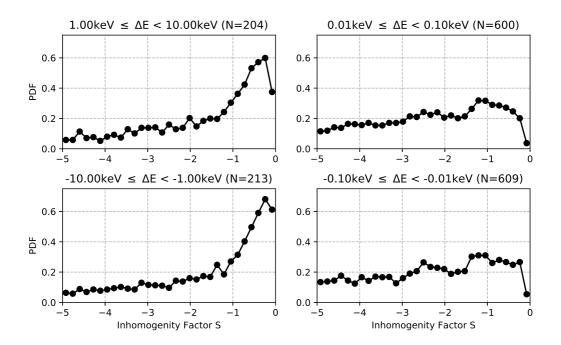


Figure 9. Probability distribution functions of the inhomogeneity ratio S for (upper left) intense-positive, (lower left) intense-negative, (upper right) weak-positive, and (lower right) weak-negative scattering events. The scattering events are identified from the energy-time histories of 100 electrons by the fitting method of the hyperbolic tangent fitting function for estimating  $t_c$  and  $\Delta E$ . The inhomogeneity ratios for 15 wave modes for each of the electrons are calculated at  $t_c$ .

The slight difference in the distributions may reflect the differences in the conditions between the phase dislocation and the phase trapping. In weak events, there is no significant peak on the PDFs. Moreover, unlike the intense events, the PDFs tend to become smaller with decreasing |S| when |S| < 1. On the other hand, at |S| > 1, the PDFs tend to be higher and relatively flatter than those in intense events. It is indicated that the linear scattering, which has  $|S| \gg 1$ , dominates both the weak-positive and weaknegative events.

Table 1 shows the probabilities calculated from the integral of PDFs in |S| < 1for the four categorized events. The integral of the PDF for the intense-positive events shows that roughly half of the wave modes at  $t_c$  have |S| < 1. The intense-negative events also show the high probabilities of wave modes with |S| < 1. It is clear that the wave modes that lead to nonlinear scatterings definitely contribute to the intense events. On

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**Table 1.** Integral of PDFs in |S| < 1 for each of the scattering events.

	Intense	Weak
Positive	46.1~%	21.4~%
Negative	49.9~%	22.1~%

the other hand, in weak events, about 80% of the wave modes are found to be beyond the range (|S| > 1), indicating the dominance of the linear scattering.

### 414 5 Discussion

Our simulation results reproduce the temporal variation of the electron flux dis-415 tribution observed by Arase (Figures 1-3). A pitch angle distribution at 24.5 keV pro-416 duces a butterfly distribution within 32 s because of an upper-band chorus burst. It has 417 a flux peak of the pitch angle between 70 and 80 degrees. The main flux source contribut-418 ing to the formation of the butterfly distribution is at a pitch angle between 40 and 50 419 degrees and an energy between 15 and 16 keV at t=0 (Figure 4). From a simple estima-420 tion, we confirmed that some electrons initially at 15.5 keV may possibly contribute to 421 the formation of the butterfly distribution through a quasi-linear process (Figure 5). How-422 ever, we found that the instantaneous pitch angle changes  $(D_{TP})$  of some electrons are 423 much larger than those in the quasi-linear process. Electrons that experience  $D_{TP} \geq$ 424 0.1 within 32 s are dominant in the peak of the butterfly distribution (Figure 6). It sug-425 gests the importance of individual scattering processes not exactly described in quasi-426 linear processes. Furthermore, a timescale ( $\Delta t$ ) for a pitch angle change of an electron 427 is different in each scattering and tends to be shorter than 200 ms (See timescales of  $t_w$ 428 shown in Figure 8). Thus, the instantaneous pitch angle changes  $(D_{TP})$  with  $\Delta t = 200$ 429 ms tend to be underestimated here. We suggest that the contribution of the rapid scat-430 tering processes to the butterfly distribution is more important than our estimations. Lakhina 431 et al. (2010) reported that instantaneously coherent chorus waves can realize a more rapid 432 pitch angle scattering of electrons than that expected in quasi-linear models. The OFA 433 spectrum (Figure 1) frequently shows relatively narrow frequency spectra, so the scat-434 tering process proposed by Lakhina et al. (2010) could work effectively. Moreover, when 435 a wave amplitude of a narrow frequency spectra exceeds a certain threshold, phase trap-436

ping/dislocation largely increases/decreases the pitch angle and energy of electrons, as 437 shown in Figures 7 and 8. The pitch angle and energy transports by the phase trapping/dislocation 438 show large  $D_{TP}$  of electrons, which contribute to the formation of the butterfly distri-439 bution. Recently, Gan, Li, Ma, Artemyev, and Albert (2020) have also reported the im-440 portance of nonlinear scattering in rapid electron acceleration at energies of tens of keV 441 in terms of the formation of a butterfly distribution due to whistler chorus bursts. The 442 scattering in instantaneously coherent chorus waves and the scattering by phase trap-443 ping/dislocation are not described as quasi-linear models. We expect that non-quasi-linear 444 processes play an important role in forming a butterfly distribution of energetic electrons. 445 Furthermore, the phase trapping process plays a more important role than the disloca-446 tion process, because the phase dislocation reduces the energy of electrons. The phase 447 trapping would have a dominant contribution to the rapid acceleration of electrons that 448 form the butterfly distribution at 24.5 keV. 449

In our simulation, there is no phase correlation among the wave modes, as described in Sec. 3.2. Usually, electrons are not easily trapped by broadband fluctuations with random wave phases. However, if there is an amplitude modulation in time and frequency, a particular wave mode can possibly be dominant. As a situation in which the wave mode that has  $|S| \sim 1$  continues for a finite time, efficient acceleration associated with the phase trapping can occur even if there is no phase coherency with other wave modes with lower amplitudes.

The upper-band chorus burst used in the simulation is highly modulated in am-457 plitude (Figure 1). The wave amplitude in the burst intermittently exceeds 100 pT in 458 a narrow frequency range of about 100 - 200 Hz. Thus, some electrons are phase-trapped 459 and gain energy in a short duration owing to intermittently enhanced wave modes (Fig-460 ure 8). It is expected that the energy gain by the phase trapping is more efficient than 461 the quasi-linear diffusion process even without a long-duration trapping ( $\sim 1$  sec.), as 462 described by Omura et al. (2007). Recently, the important role of the phase trapping pro-463 cess in an amplitude-modulated whistler chorus wave has also been studied numerically 464 by Hiraga and Omura (2020) and Gan, Li, Ma, Albert, et al. (2020). Our simulation and 465 other numerical studies suggest the importance of considering realistic wave modes that 466 contribute to electron accelerations. In quasi-linear diffusion models, statistical wave mod-467 els are utilized, which eliminate the contribution of the amplitude modulation by aver-468 aging in time. The model that takes averaged signatures of waves may underestimate 469

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the acceleration of electrons by each of the whistler chorus bursts or elements. In theoretical and numerical studies, appropriate wave models should be chosen to reproduce transient energetic electron dynamics that are actually observed in the magnetosphere.

As seen in Figures 7 and 8, the electron scattering shows not only small energy changes 473 but also large energy changes induced by phase trapping and dislocation. A scattering 474 process can be evaluated using an instantaneous inhomogeneity ratio S defined in Equa-475 tion (8). The contribution of wave modes with  $|S| \gg 1$  to electrons becomes dominant 476 when the energy change is small, whereas that with |S| < 1 becomes dominant when 477 the energy change is large (Figure 9). Larger wave amplitudes reduce |S| because of the 478  $\Omega_m^{-1}$  term, and the electron position also contributes to the reduction in |S| because  $|\partial \Omega_e/\partial s|$ 479 tends to be smaller at lower latitudes. Note that  $\partial \Omega_e / \partial s$  at the electron position is pro-480 portional to the spatial gradient of the background magnetic field along its field line. An 481 instantaneous change of an inhomogeneity factor leads to a variety of scattering processes 482 for an electron in the upper-band chorus burst. 483

The wave model used in the RBW simulation shown in this paper has a frequency 484 gap of 64 Hz among wave modes, and thus, it does not perfectly construct the incoher-485 ent wave burst as defined in a quasi-linear diffusion model. Moreover, the time resolu-486 tion of the wave model is limited to 1 s; thus, amplitude modulations shorter than 1 s 487 are not reproduced. However, the simulation has demonstrated the Arase observations 488 relatively well, implying that the observed upper-band whistler chorus burst may be close 489 to the condition assumed in the wave model. That is, the observed burst might be co-490 herent with a finite frequency (of about 64 Hz) and the amplitude modulation might not 491 be much shorter than 1 s. It is necessary to consider the actual wave form data of whistler 492 chorus waves covering a longer time scale in order to reproduce the actual electron scat-493 tering processes in future simulations. 494

495

## 6 Summary and conclusions

We have performed a data-driven RBW simulation using Arase observations to study the rapid flux enhancement of energetic electrons with the upper-band whistler chorus burst in the duration of about 30 s. The simulation reproduces the temporal variation of the electron flux distribution observed by Arase. As a result of detailed analysis of the simulation data, it is found that 15 – 16 keV electrons with the equatorial pitch an-

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gle of 40 – 50 degrees contribute to the flux enhancement at energies higher than 20 keV at large pitch angles. We have found that scattering processes not described by the quasilinear diffusion model contribute to the electron acceleration that forms the butterfly distribution at 24.5 keV. Our simulation suggests that a time-averaged statistical wave model as used in quasi-linear models underestimates the acceleration efficiency of radiation belt electrons in each of the whistler chorus bursts and elements.

We conclude that the rapid flux enhancement of energetic electrons observed by Arase is caused by the phase trapping of electrons associated with a highly amplitudemodulated upper-band whistler chorus burst. It is also suggested that the contribution of the amplitude modulation, which leads to the intermittent enhancement of the wave amplitude, should be properly taken into account in wave models for theoretical and numerical studies.

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- <sup>517</sup> u.ac.jp/index.shtml.en, Miyoshi, Hori, et al. (2018)). Part of the work of SK, YM, TH,
- MS, SN, and SI was carried out at ERG-SC. In this study, we analyzed the MEP-e level
- <sup>519</sup> 3 v1.01, MGF level 2 v1.02 and the PWE/OFA-SPEC level 2 v2.01. Arase MGF Level-
- <sup>520</sup> 2 dataset used for this research is available in Matsuoka, Teramoto, Imajo, et al. (2018).

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