A Fourier-series modeling approach to develop corrections to atmospheric drag in orbit

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Abstract

Atmospheric drag is one of the primary sources of error in the orbit determination and prediction of satellites in the low altitude LEO regime. Accurate modeling of the drag force is limited by uncertainties in the atmospheric density model used in the filter and the assumption of a constant drag coefficient, the so-called 'cannonball' model. Over the last two decades, various advances in density and drag-coefficient modeling have been made possible through the development of empirical and physics-based dynamical calibration techniques and machine-learning methods respectively. But even with high-fidelity models for density and drag coefficient, systematic uncertainties can remain in both due to the lack of temporal and spatial resolution of data and insufficient knowledge of parameters that feed into these models. In this work, we develop an estimation-based Fourier expansion model that can provide corrections to the nominal values of density and drag coefficient during the orbit determination process. In an earlier work (Ray et al., 2018), we demonstrated improved orbit prediction performance over the standard cannonball model with Fourier series expansions of the drag coefficient in body frame and orbit frame of a satellite. Whereas a body-fixed Fourier model captures the dependence of the drag coefficient on satellite attitude, the orbit-fixed model corrects for periodic changes in the gas-surface interaction in orbit. Since changes in the gas-surface interaction parameters in orbit are highly correlated with atmospheric density, any existing errors in the density are absorbed in the estimated orbit-fixed coefficients. Here, we derive a body-orbit Fourier model such that the orbit-fixed terms provide corrections for combined error variations of density and drag coefficient in orbit while the body-fixed terms account for the drag coefficient attitude dependence. We analyze the performance of the proposed approach with various atmospheric models such as NRLMSISE-00 (Picone et al., 2002), JB08 (Bowman et al., 2008), HASDM (Storz et al., 2002) and densities derived by Mehta et al. (2017) for varying geomagnetic conditions for the GRACE satellite.

A Fourier-series modeling approach to develop corrections to drag in orbit

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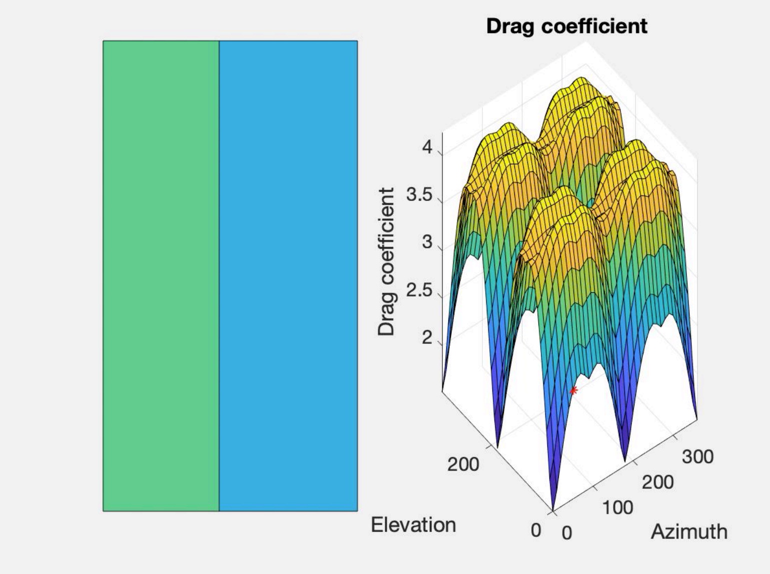
Once upon a drag-coefficient

- Parameter governing atmosphere-satellite interaction
- Models used in orbit-determination

$$a_{drag} = -\frac{1}{2}\rho C_d \frac{A_{ref}}{m} v_r^2 \hat{\boldsymbol{u}}$$

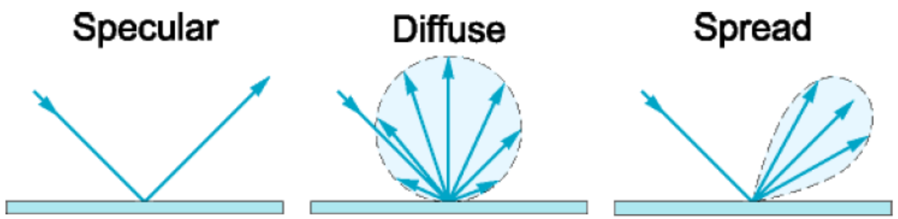


- Analytical approach
- Numerical approach
- Machine-learning

} Physics-based

- Cannonball model

} Estimation-based



Specular, diffuse, and spread reflection from a surface.

A Fourier-expansion based approach

Physics-based models

- Capture time-variation based on actual physics
- Uncertainties in input parameters. Do not use real-time orbit tracking data

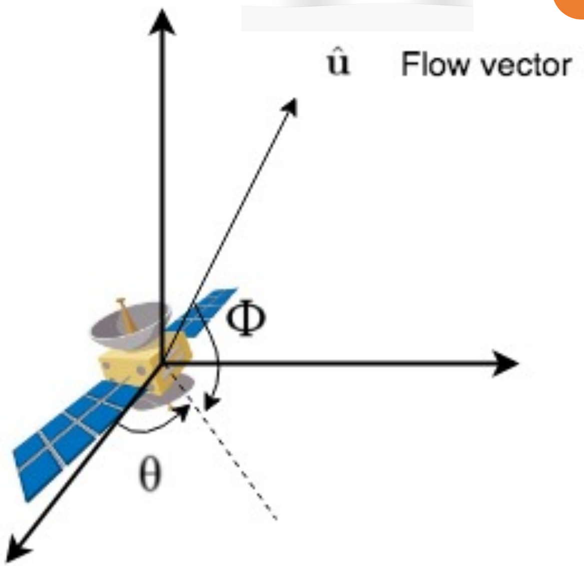
Proposal
Estimate time-varying corrections to nominal physics-based models using Fourier-expansions

Cannonball model

- Uses orbit tracking data
- Averages out the time-variation

$$\widetilde{C_d} = C_{d,nom}(t) + \Delta C_d$$

Fourier what?

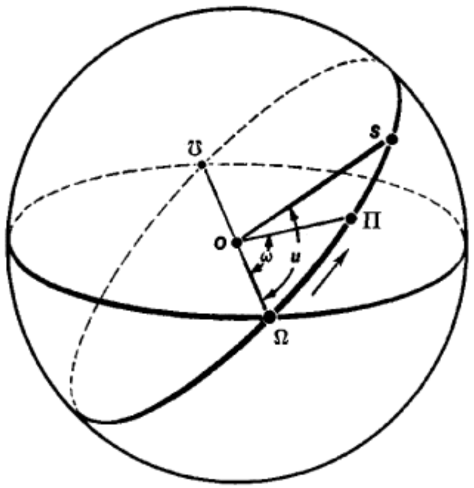


Body-fixed Fourier (BFF) model

- Fourier expansions around orientation of velocity vector in body frame
- Captures variations due to attitude

Orbit-fixed Fourier (OFF) model

- Fourier series expansion around the argument of latitude of the satellite
- Captures variations due to ambient parameters in orbit



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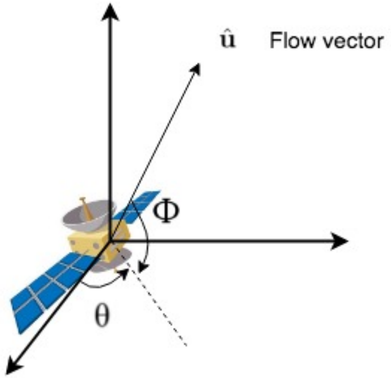
Body-orbit models

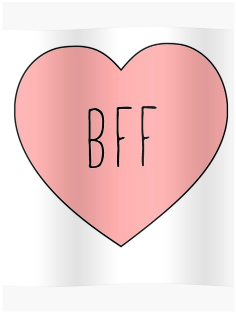
Body-orbit double Fourier (BODF) model

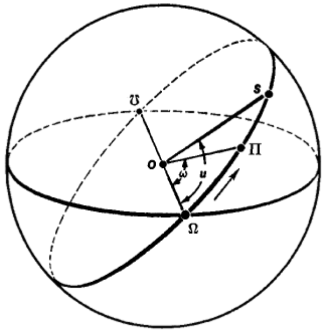
- Capture variations due to both attitude and ambient parameters

Body-orbit summation (BOS) model

- Ignore the cross-coupled terms







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Some results

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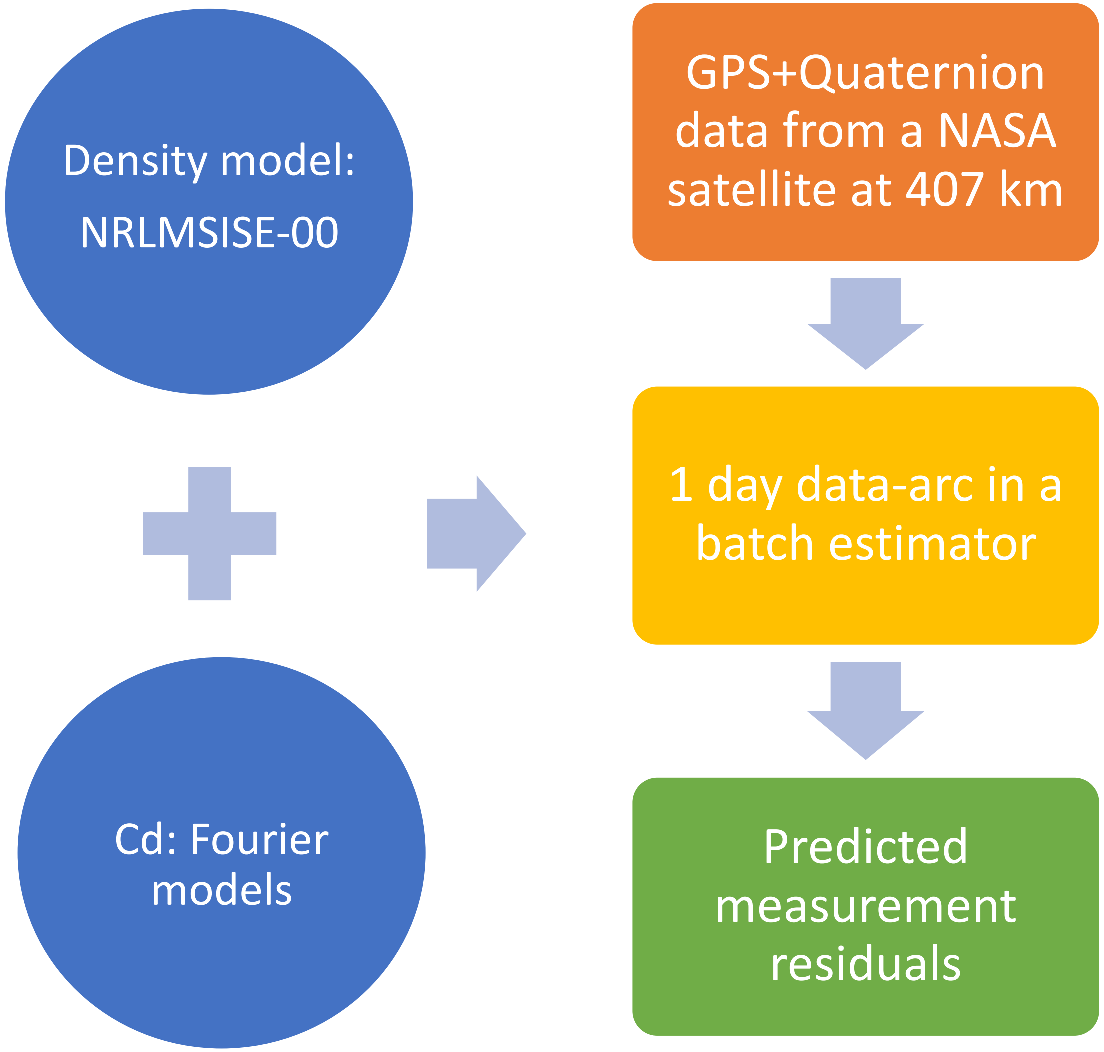
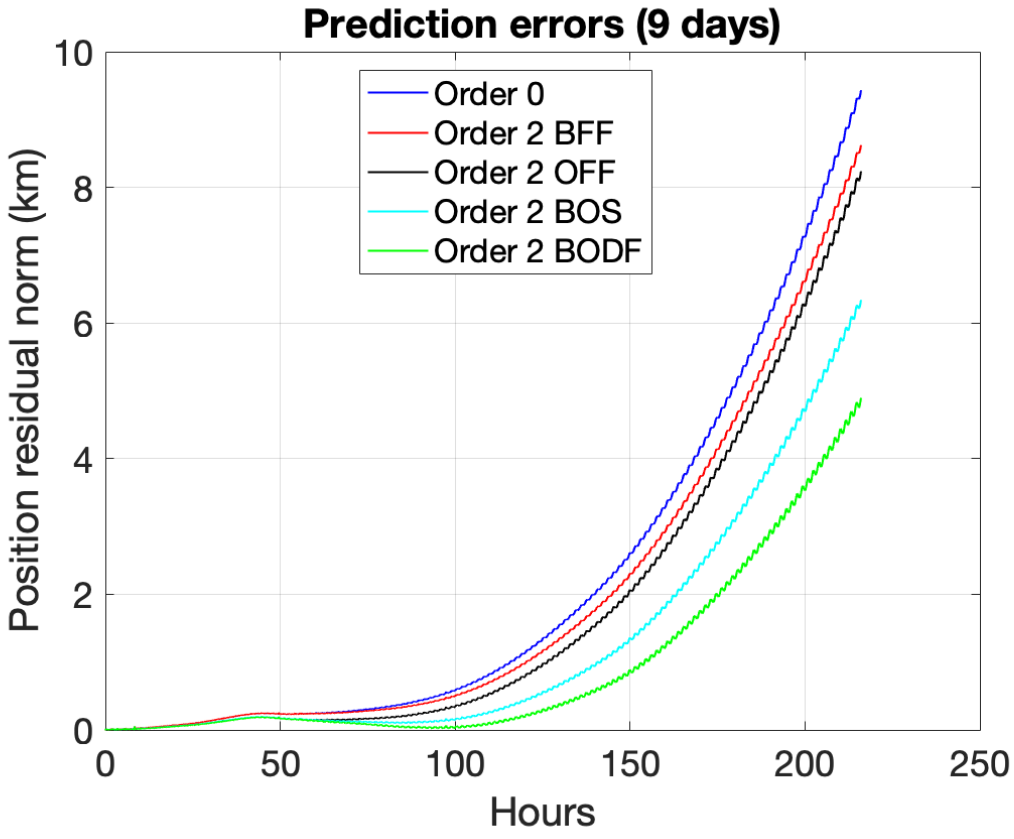
Application to a NASA satellite



Application to a NASA satellite

- Data: Jun 29- Jul 6, 2017
- BODF reduces error by ~ 50 % over cannonball

- Order 0: Cannonball
  - BFF: Body-fixed, OFF: Orbit-fixed, BODF: Body-orbit, BOS: Body-orbit summation



Jun 29-Jul 6, 2017: Attitude maneuver

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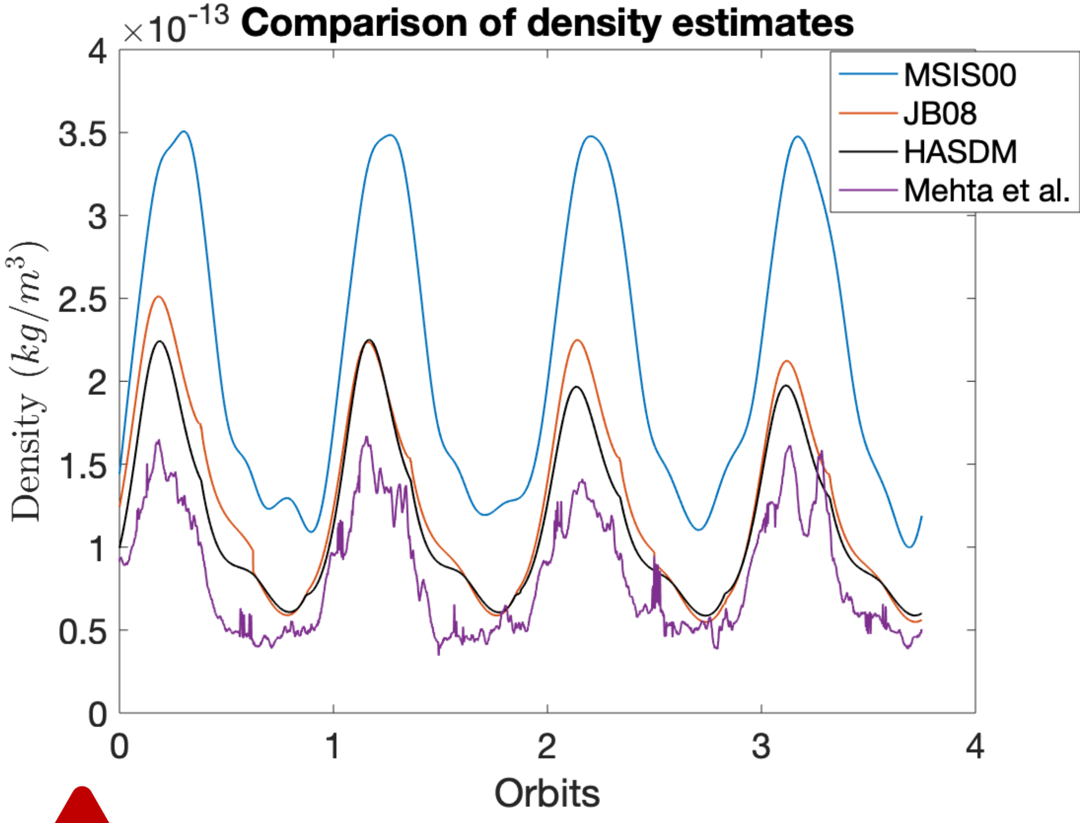
Application to GRACE

- Day 82-87, 2007, quiet geomagnetic conditions
- Densities used in estimator
  - NRLMSISE-00
  - JB08
  - HASDM
  - Estimates from Mehta et al.[1]
- Drag coefficient model:
  - Nominal drag coefficients from Mehta et al.
  - Fourier model to estimate corrections



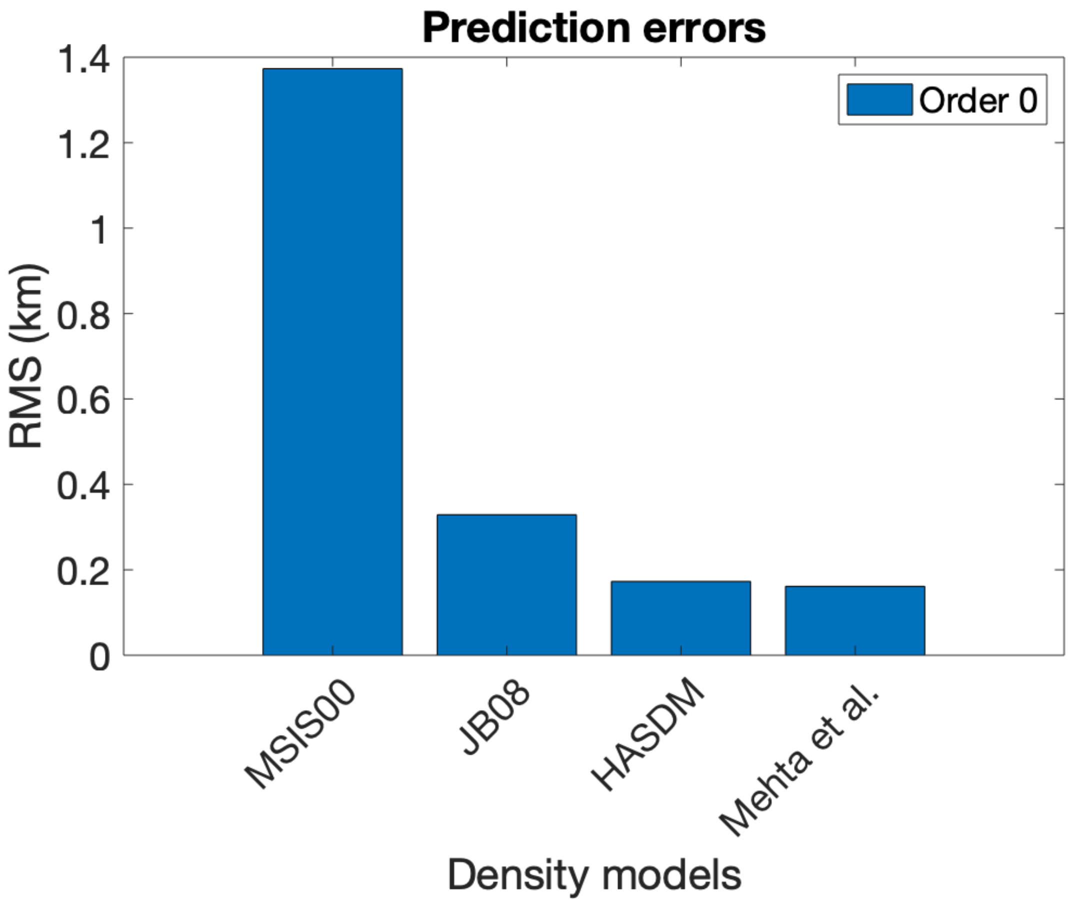
Highly correlated
Order of truncation dependent on density accuracy

[1] Mehta et al., New density estimates derived using accelerometers on board the CHAMP and GRACE satellites, *Space Weather,* DOI:10.1002/2016SW001562

Application to GRACE

Order 0 Cd (Cannonball model)

- MSIS00 > JB08>HASDM> Mehta et al.



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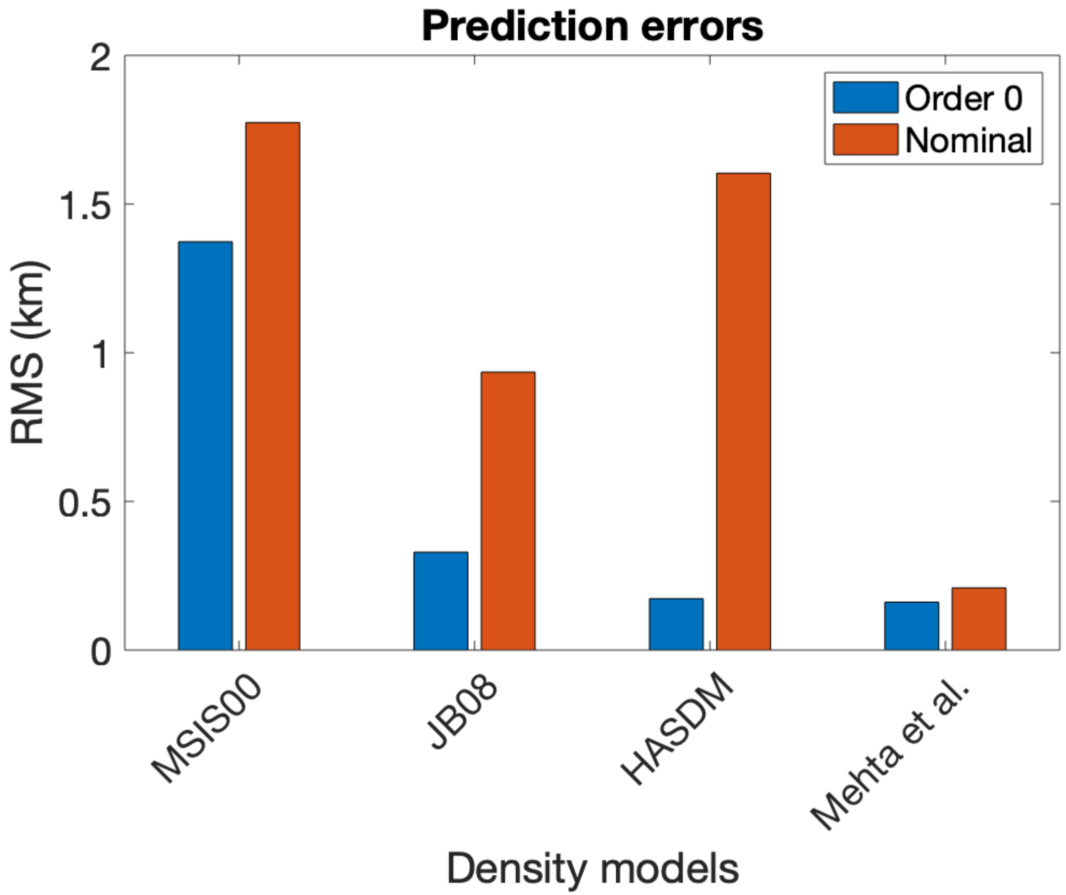
Application to GRACE

Nominal Cd (Mehta et al.)

- Increase in prediction errors



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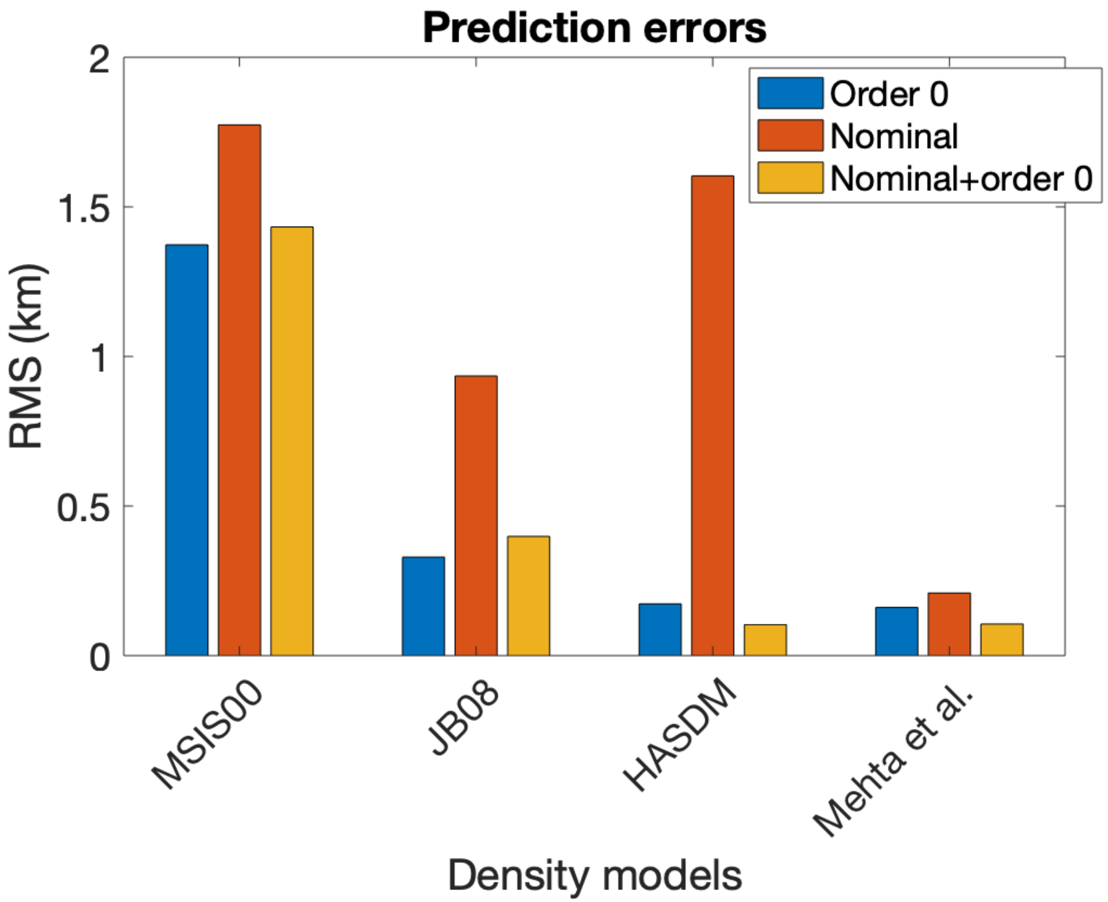
Nominal + Cannonball

- Improved performance for HASDM and Mehta et al.



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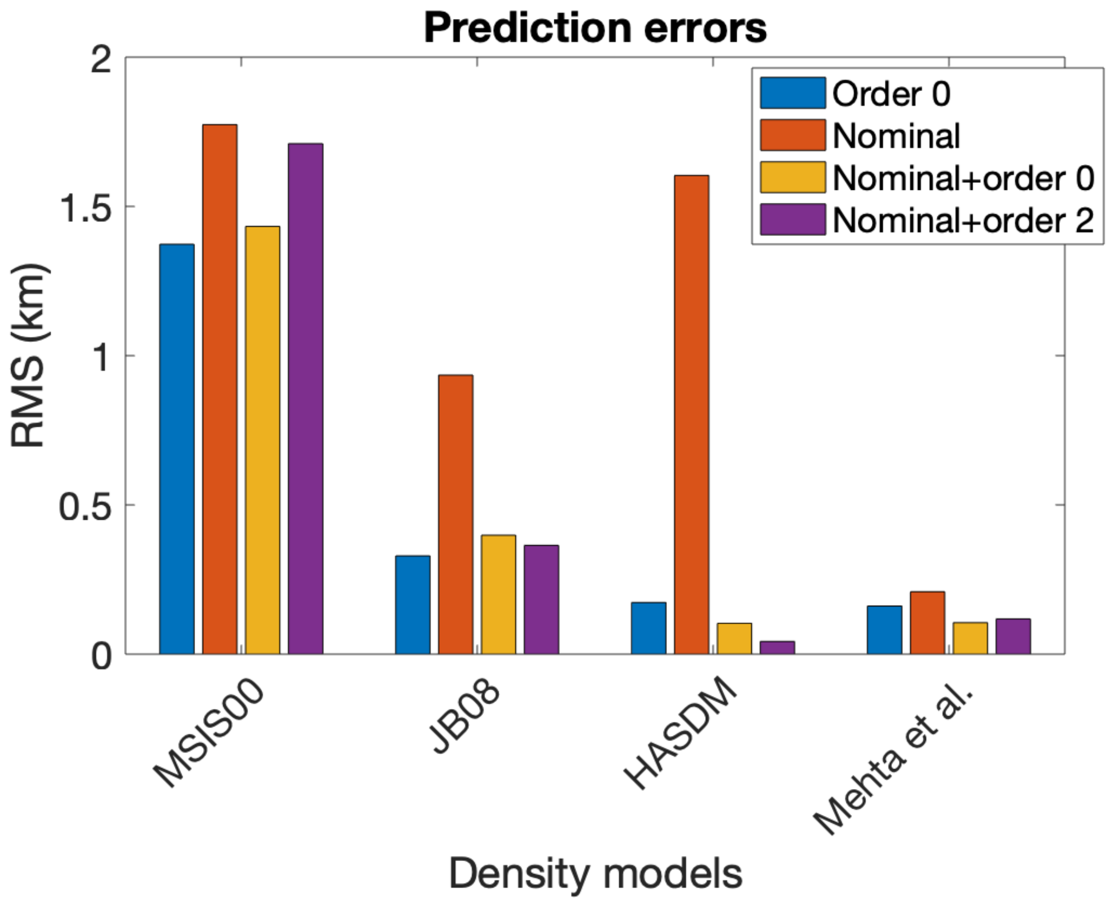
Application to GRACE

Nominal + Order 2 OFF

- Improved performance for HASDM

Possibilities of improving MSIS00 results?



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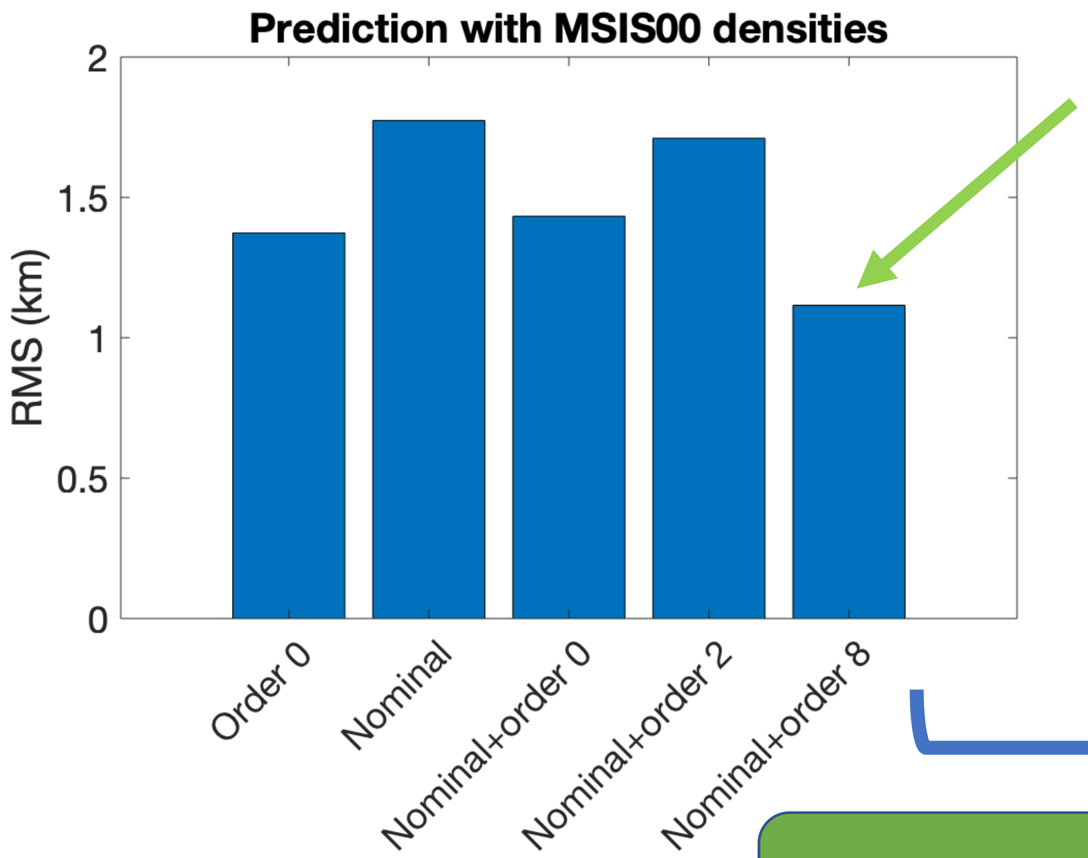
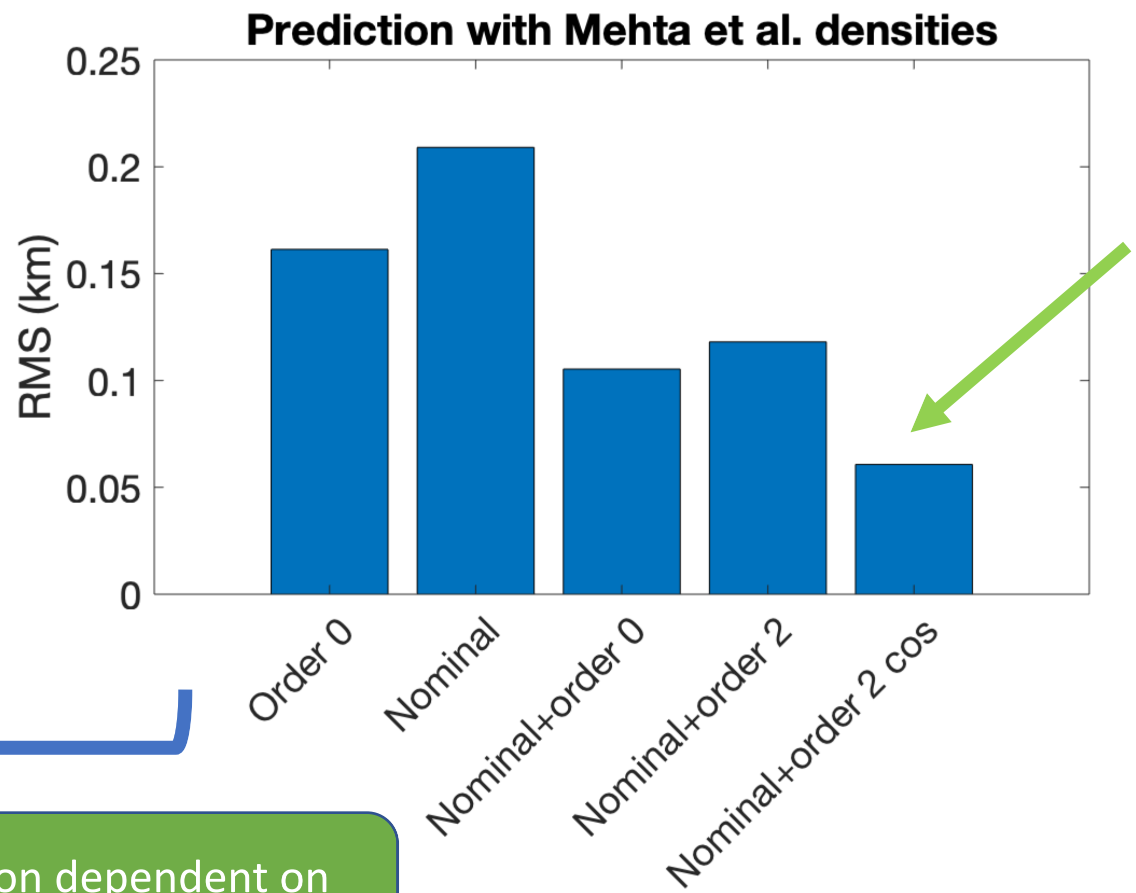
Application to GRACE





Order of truncation dependent on density accuracy

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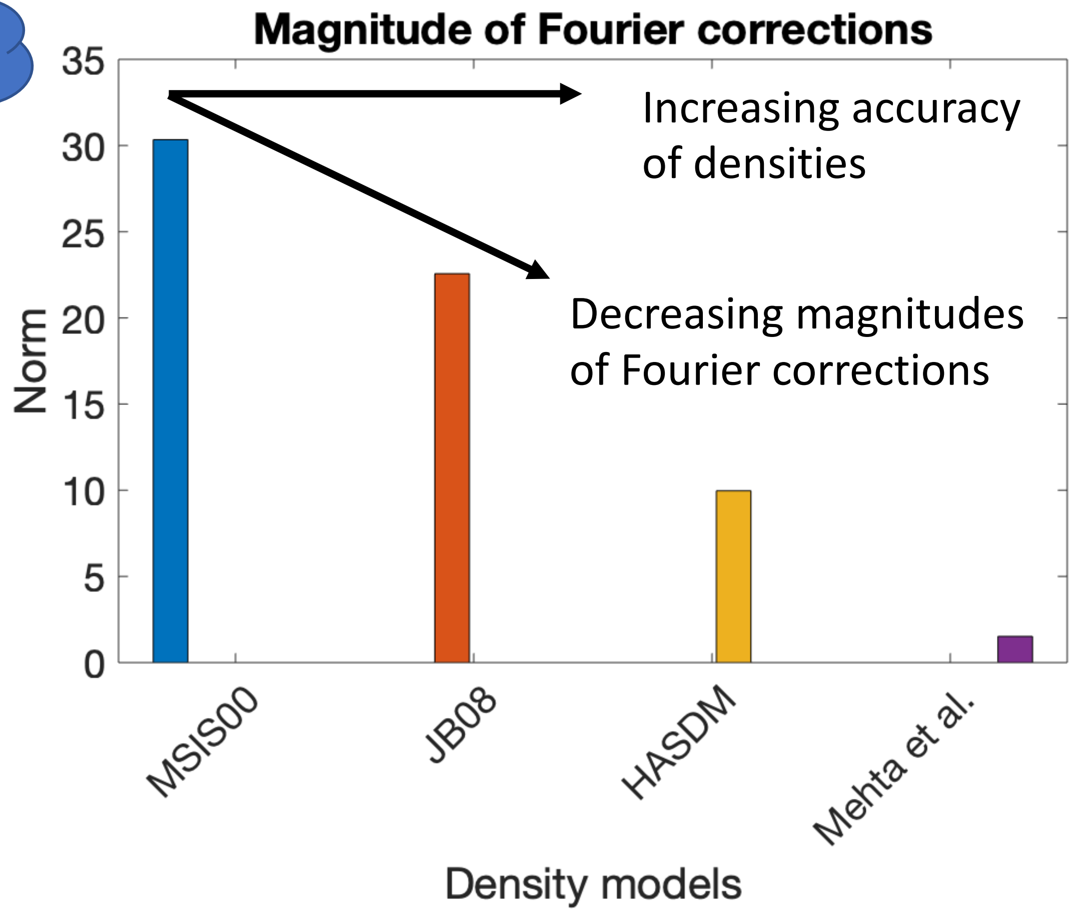
Application to GRACE

Bar charts

Magnitude of Fourier corrections

- Significant implications as a validation tool for models



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Take home points

Fourier expansions can provide periodic corrections to nominal models

Coefficient magnitudes: Validation of Cd and densities

Performance highly dependent on density models

Future work →

Time-varying Fourier coefficients: Markov process

Evaluation for geomagnetically active conditions

Further details: Ray et al., A drag coefficient modeling approach using spatial and temporal Fourier expansions for orbit determination, *Journal of Astronautical Sciences,* Doi: 10.1007/s40295-019-00200-4

Additional slides

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Filter implementation

Initial estimates

- GSIM with a macro-model
- Numerically integrate to find Fourier coefficients
- Truncate at chosen order

Initial covariance

- Monte-Carlo simulation
- Calculate mean and standard deviations

Estimate

- Correlations with mismodeled dynamics: Atmospheric density, truncated geopotential, time-varying SRP

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Body-orbit models

Body-orbit double Fourier (BODF) model

- Capture variations due to both attitude and ambient parameters

Body-orbit summation (BOS) model

- Ignore the cross-coupled terms

$$C_d = \sum_{n=0}^{\infty}(\bar{A}_n \cos n\psi + \bar{B}_n \sin n\psi).$$

$$C_d = \sum_{n=0}^{\infty}(\bar{A}_n(u) \cos n\psi + \bar{B}_n(u) \sin n\psi).$$

$$C_d = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\bar{A}_{mn} \cos mu \cos n\psi + \bar{B}_{mn} \sin mu \cos n\psi + \bar{C}_{mn} \cos mu \sin n\psi + \bar{D}_{mn} \sin mu \sin n\psi)$$