

# A Singularity-Free Crack Model Inferred from Contained Laboratory-Generated Earthquakes

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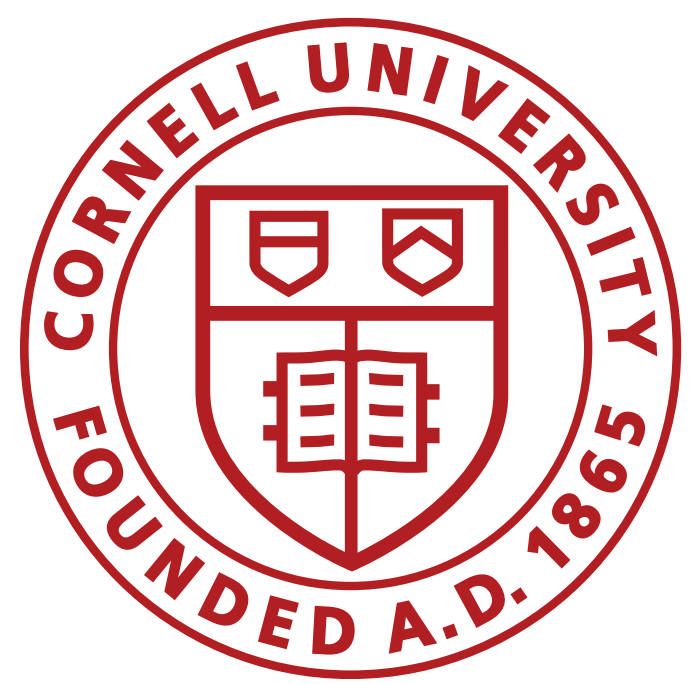
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## Abstract

Earthquakes are commonly modeled as shear cracks, where the slip profile of an earthquake rupture is the spatial distribution of relative displacement between fault surfaces. It is an accumulated result of all the processes during the earthquake: nucleation, propagation, and arrest. Understanding the characteristics of a slip profile gives insight into the associated stress changes, which is generally immeasurable on natural faults, and is useful for understanding the underlying friction law. Most models focus on simplicity for application purposes. For instance, the elastic crack model (Bilby & Eshelby, 1968) established that a perfect crack with uniform shear stress drop leads to an elliptical slip profile; Cowie and Scholz (1992) proposed a crack model with constant-stress cohesive zones at the crack tips, which results in “bell-shaped” slip profiles. However, the elliptical model results in unphysical stress singularities at crack tips, and the plateau in stress drop distribution near crack tips in the “bell-shaped” model is infeasible for friction dominated ruptures because it is commonly believed that slip is always accompanied by shear stress drop, e.g. slip-weakening friction law (Andrews, 1976). We present results from recent large-scale laboratory experiments where all the rupture processes are contained in a 3-meter long saw-cut granite fault (Ke et al., 2018) and slip local fault slip and shear stress changes are measured at 16 locations along the fault. Guided by the laboratory experiments, we derived an analytical model to faithfully represent measured slip profiles  $\delta(x)$ , and shear stress changes  $\Delta\tau(x)$ , resulting from laboratory earthquakes. Field measurements of slip profiles revealed that slip profiles are commonly tapering roughly linearly toward the tips. The proposed model includes this feature, and thus fits slip profiles measured from natural earthquakes on isolated faults better than other idealized analytical models. For more complex natural earthquakes, our model can be used as a basis function. Our results suggest that inelastic earthquake processes can be solely originated from friction, and the shape of an earthquake rupture is likely between the elliptical and bell-shaped idealized models.





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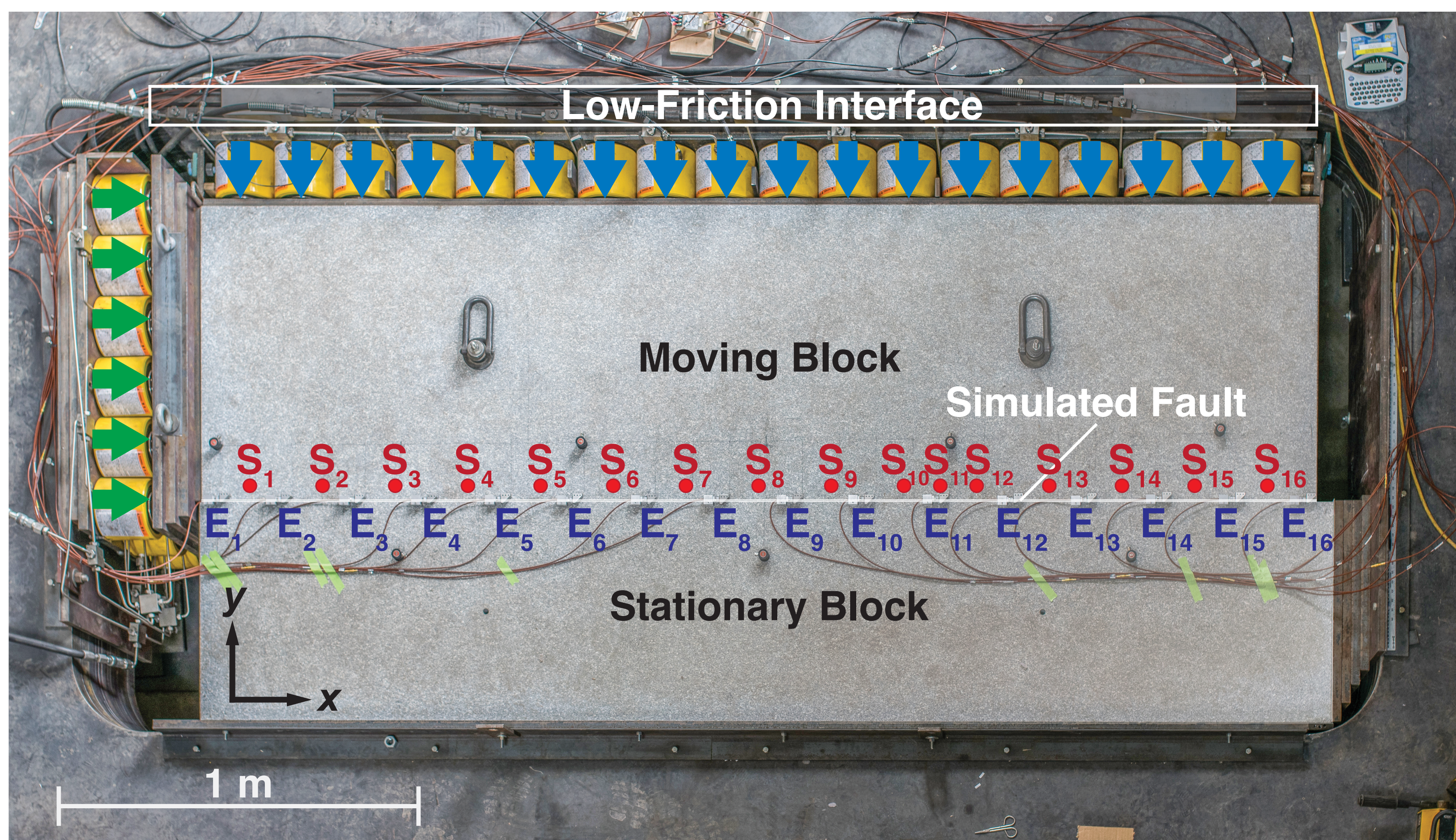
## Key Points

- 3-meter long saw-cut simulated granite fault generates fully and partially contained stick-slip events.
- Shape of slip profiles measured from contained laboratory-generated earthquakes is similar to ones measured from natural faults.
- The proposed slip profile model faithfully represents measured slip profiles and enable further rupture sequence analysis.

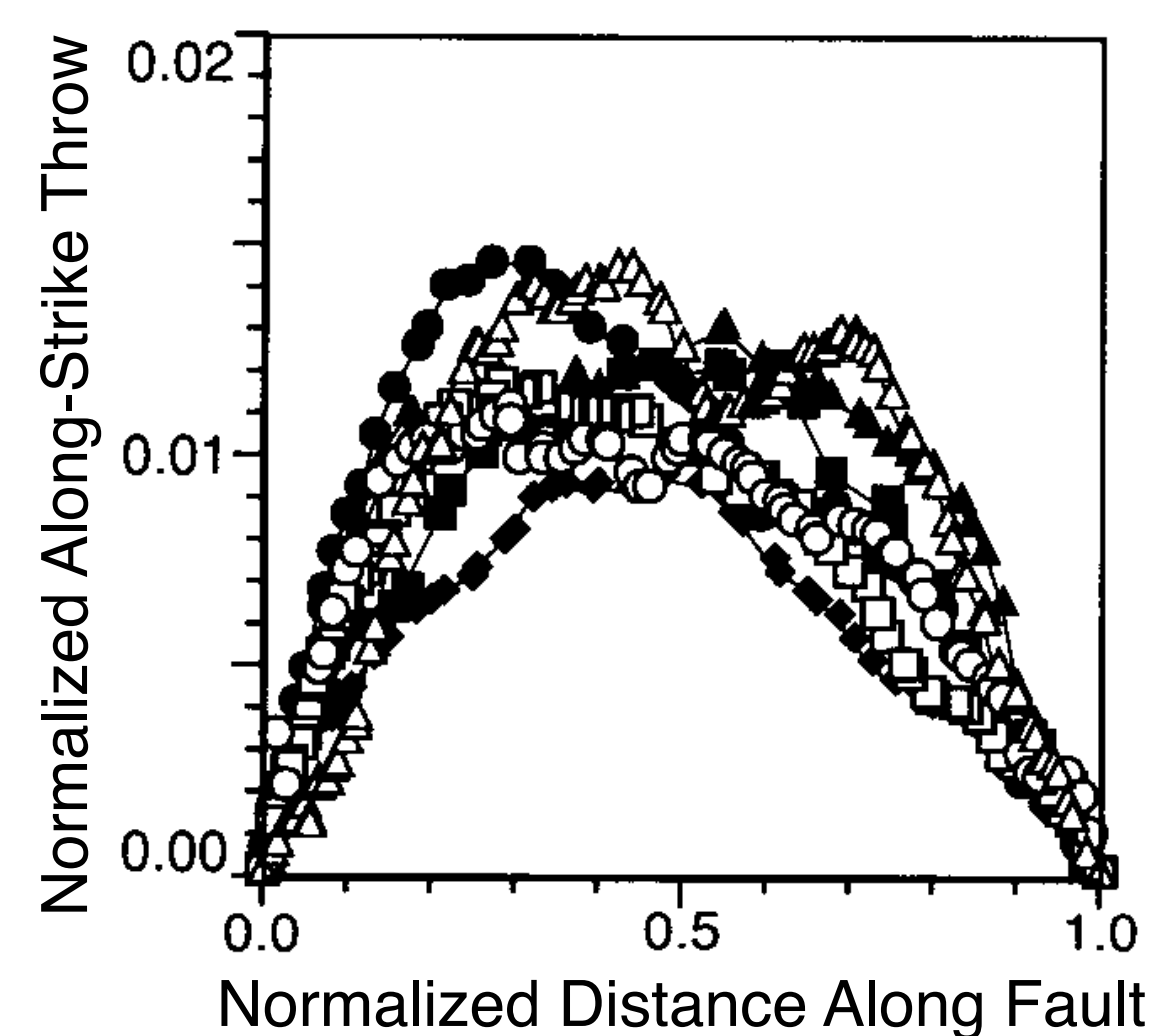
## Introduction

Earthquakes are commonly modeled as shear cracks, where the distribution of slip distance on a fault is an accumulated result from all the processes during an earthquake. The simplest linearly elastic crack model predicts elliptical slip profiles for constant stress drop within the earthquake rupture but it casts stress singularity ahead of the crack tips. Studies have shown that the slip gradients near the tips of earthquake ruptures are approximately linear<sup>1,2,3</sup> instead of ellipse-like. Previous studies assumed the fault's end zones undergo plastic deformation, which results in bell-shaped slip profiles<sup>4,5</sup>. Our experiments suggest that the shape of an earthquake rupture should be something in between those idealized models.

## 3-m Biaxial Earthquake Apparatus



## Slip Profiles



Slip profile is the along-strike distribution of relative displacement (slip distance). Slip profiles measured on natural faults revealed that the slip distance of earthquake ruptures are commonly tapering roughly linearly toward the tips<sup>1,2,7</sup>. While Walsh & Waterson (1987) argue it as a result of cumulative displacement of growing ellipses, Scholz & Lawler (2004) consider it to be an implication of constant fault tip taper rupture criterion (analogy to constant CTOA in Fracture Mechanics). In our experiment, contained ruptures often share this feature.

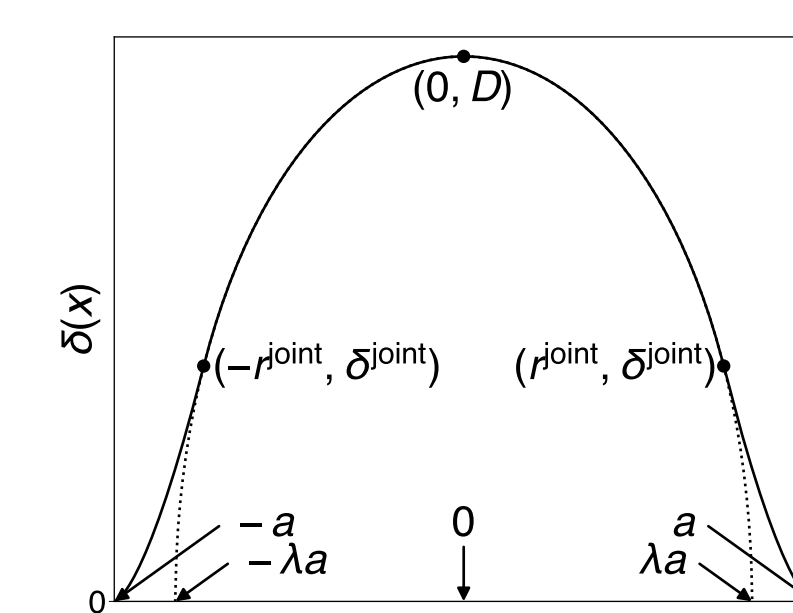
## Crack Models

Bilby and Eshelby (1986) derived the constitutive relationship between the distribution of slip parallel to the fault  $\delta(x)$  and shear stress change distribution  $\Delta\tau(x)$ ,

$$\Delta\tau(x) = -\frac{\mu^*}{2\pi} \int_{a_-}^{a_+} \frac{d\delta(\xi)/d\xi}{x-\xi} d\xi,$$

where  $\mu^* = \mu/(1-\nu)$  for mode II, in which  $\mu$  is shear modulus and  $\nu$  is Poisson's ratio,  $a_{\pm}$  are the location of rupture tips. This formulation assumes the material surrounding the rupture is linearly elastic and takes the first derivative of slip profile  $d\delta(x)/dx$  as input and gives its respective stress change distribution  $\Delta\tau(x)$ .

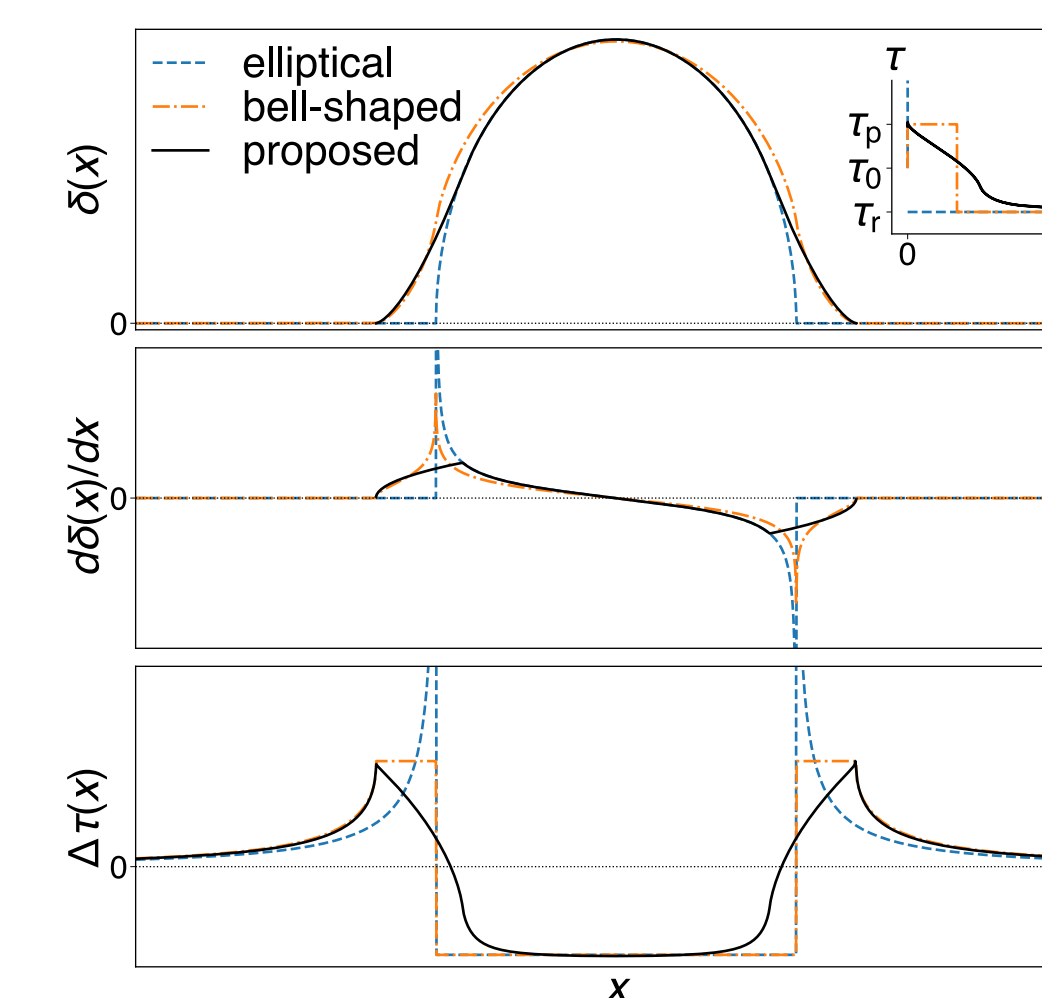
### Proposed Model



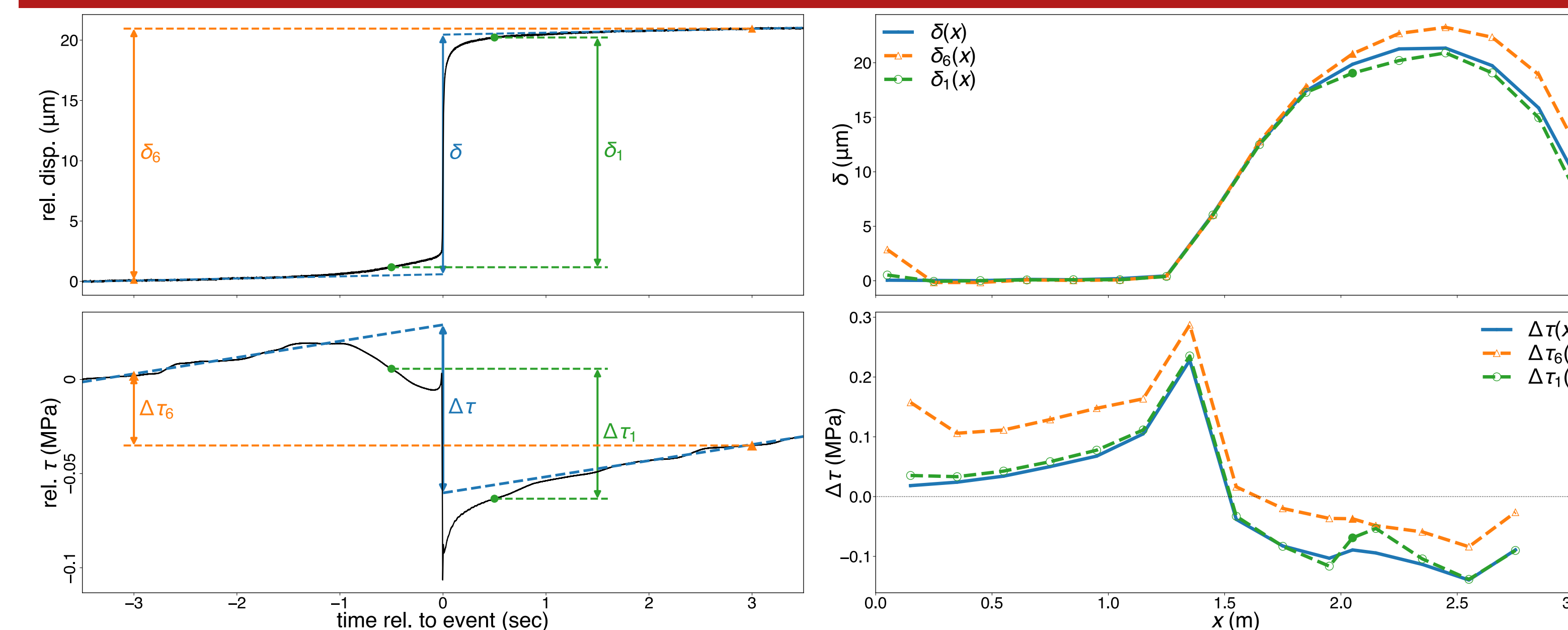
$\lambda$  : controls radius of the ellipse,  $\lambda a$ ,  $0 < \lambda < 1$   
 $x^{\text{joint}}$  : locations where  $\delta(x)$  switches form,  $x^{\text{joint}} = (\sqrt{1+3\lambda^2} - 1)a$   
 $\delta^{\text{joint}}$  : value of  $\delta$  at the joints,  $\delta^{\text{joint}} = \delta(x^{\text{joint}})$

$$\delta(r) = \begin{cases} D \left[ 1 - \left( \frac{r}{\lambda a} \right)^2 \right]^{1/2}, & 0 \leq r \leq x^{\text{joint}} \\ \delta^{\text{joint}} \left( \frac{r - a}{x^{\text{joint}} - a} \right)^{3/2}, & x^{\text{joint}} < r \leq a \\ 0, & a < r \end{cases}$$

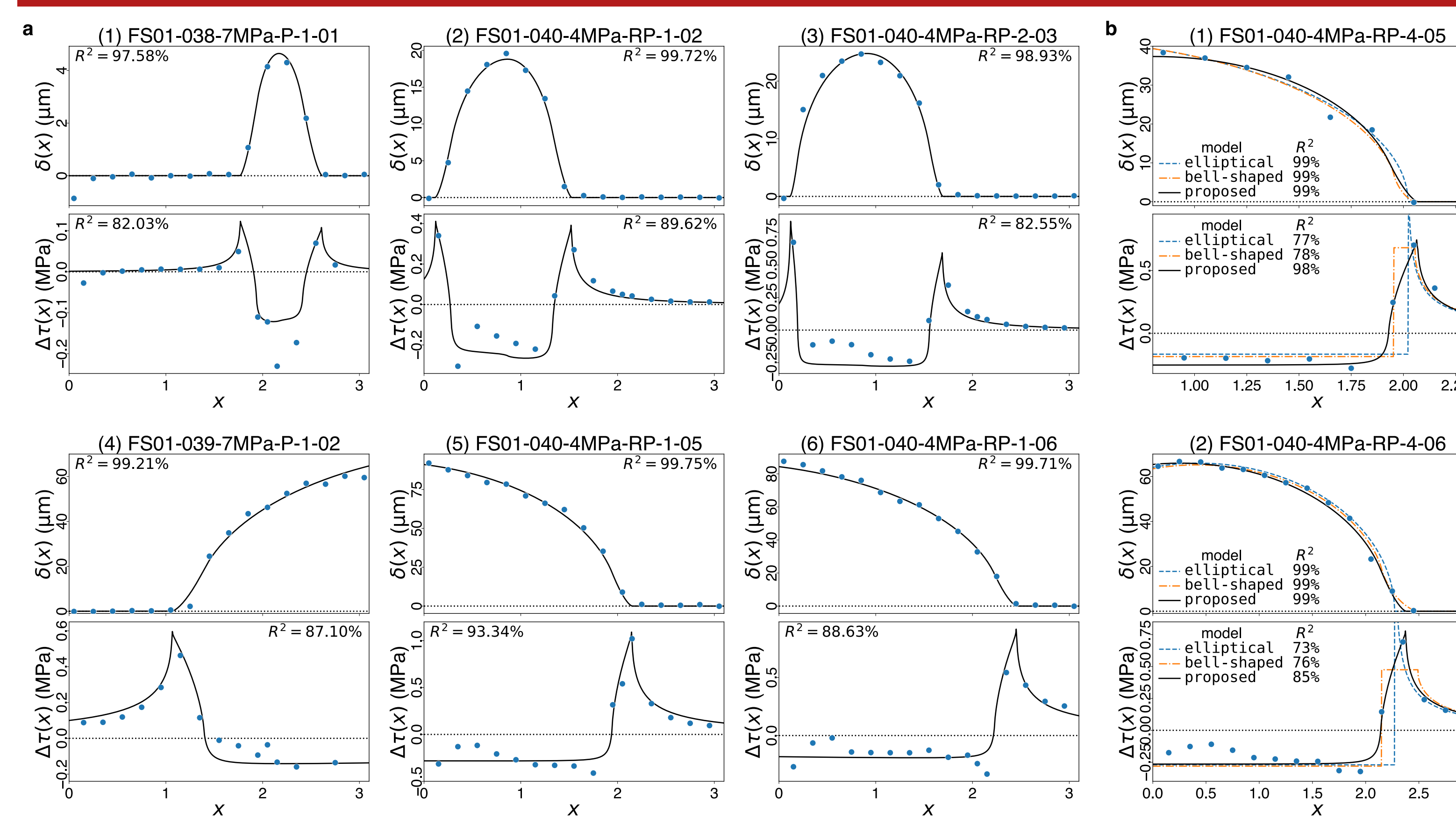
### Comparisons



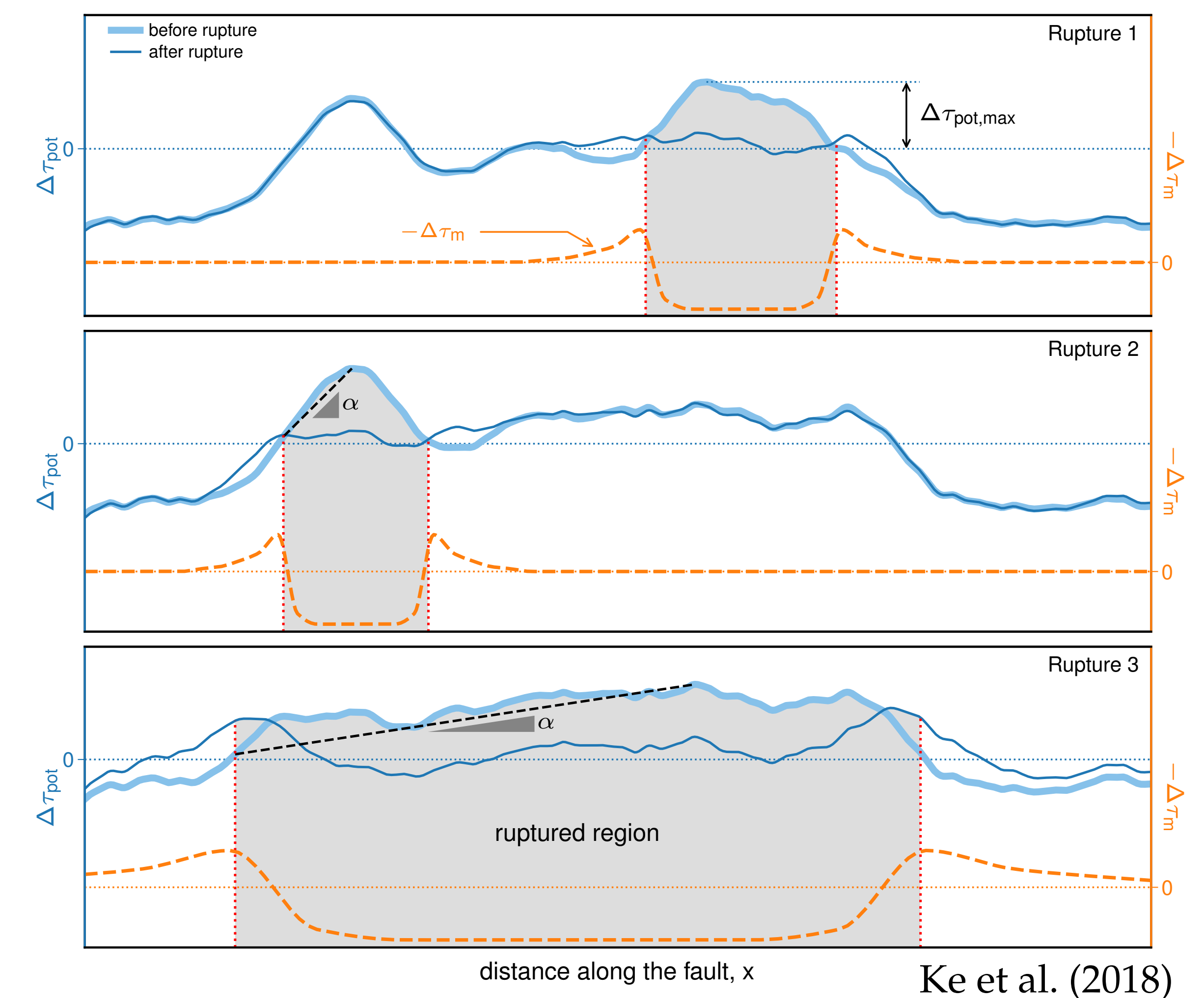
## Definition of Slip Profile & Stress Changes



## Results



## Hypothesized Rupture Sequence



## Conclusions

- Large scale granite samples enable ruptures to nucleate, propagate, and terminate within the simulated fault.
- Slip profiles on natural faults and on contained laboratory earthquakes share a similar shape.
- The proposed slip profile model effectively removes stress singularity ahead of crack tips.
- The proposed slip profile model fits well to both slip and strain measurements from laboratory earthquakes.
- Fitted slip profiles are quantitatively validated through measurements of spatial distribution of shear stress changes.
- A well-posed crack model enables further analysis on the stress evolution throughout a rupture sequence.

## Acknowledgement

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