ODE representation of tile drainage on hillslopes of varying topography

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Abstract

We use numerical solutions of the Richard's Equations for 3D porous media to investigate the influence of agricultural subsurface drainage as a hydrologic process and its effect on the hydrologic regime of a watershed. Specifically, we determine the relation between subsurface seepage and subsurface storage in hillslopes with (drained) and without (undrained) subsurface drainage. Simulations are performed in Hydrus3D and the output is analyzed with MATLAB's curve fitting tools, to create simple ordinary differential equations that represent the relationship between subsurface flow and subsurface storage for hillslopes of varying topographical gradients and shapes. We have determined an 'activation point' below which the seepage/storage relationship is roughly linear, and above which the drained and undrained simulations behave according to different nonlinear functional forms. Although the seepage/storage relationship of flat hillslopes have parametric consistencies independent of the hillslope gradient, the addition of curvature increases the complexity. In this work, we describe approximations to account for curved hillslopes. From our formulation, subsurface flow for varying hillslopes can be approximated using only the water storage and the topography of the hillslope. Reducing the system from partial differential equations (Hydrus) to ordinary differential equations improves scalability of the model. Simplified equations are used to study the consequences of large-scale changes in agricultural landscapes due to subsurface drainage.

Limitations on ODE representation of drainage tiles in a hillslope runoff model

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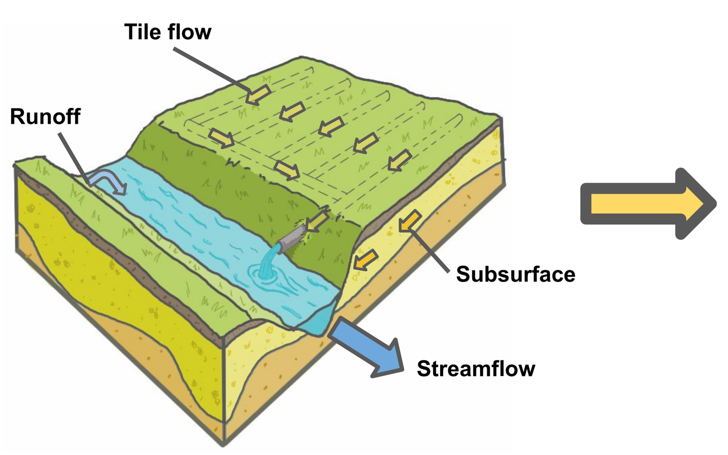
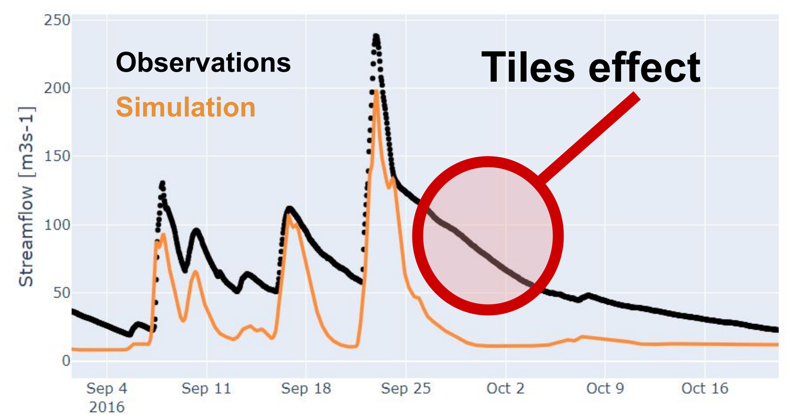
1. Introduction

Tile drainage at the hillslope scale



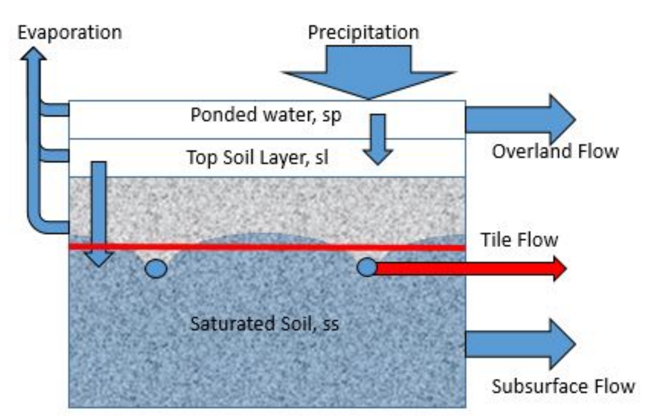
Slow recession at catchment scale



- Tile drainage induces changes in the **recession** curve increasing the **duration time of flooding conditions**.
- We use **Hydrus** simulations to find a relationship between **seepage** and **storage** at hillslope scale to incorporate into an **ODE model**.
- Finally, we couple the new ODEs to our HLM model for regional scale simulations.

2. Physical model

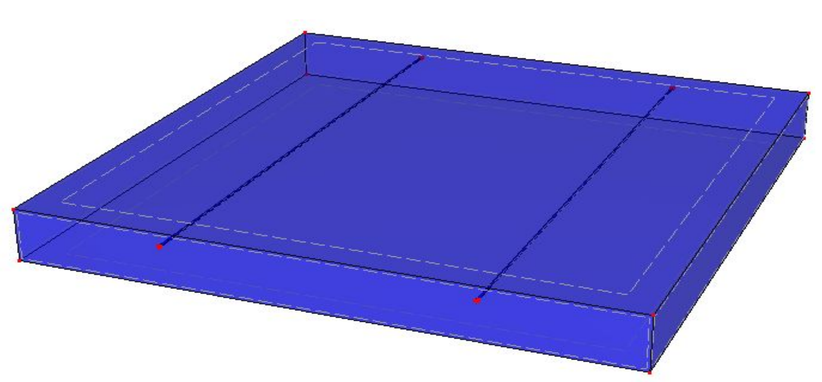
DRAINMOD Model



$$Q_{tile} = \frac{8k_e d_e\left(\frac{4(s-H)}{0.2\pi}\right) + 4k_e\left(\frac{4(s-H)}{0.2\pi}\right)^2}{L^2}$$

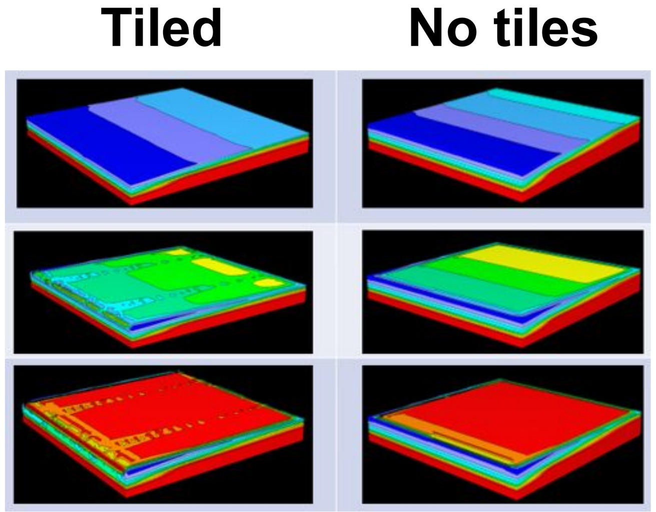
This three-compartment ODE model applies **subsurface flow** and **tile flow** (when valid) as functions of **subsurface storage** (Sloan et al (2016), Sloan et al (2017)). This representation is insufficient when including the effects of hillslope slope.

Hydrus-3D Model



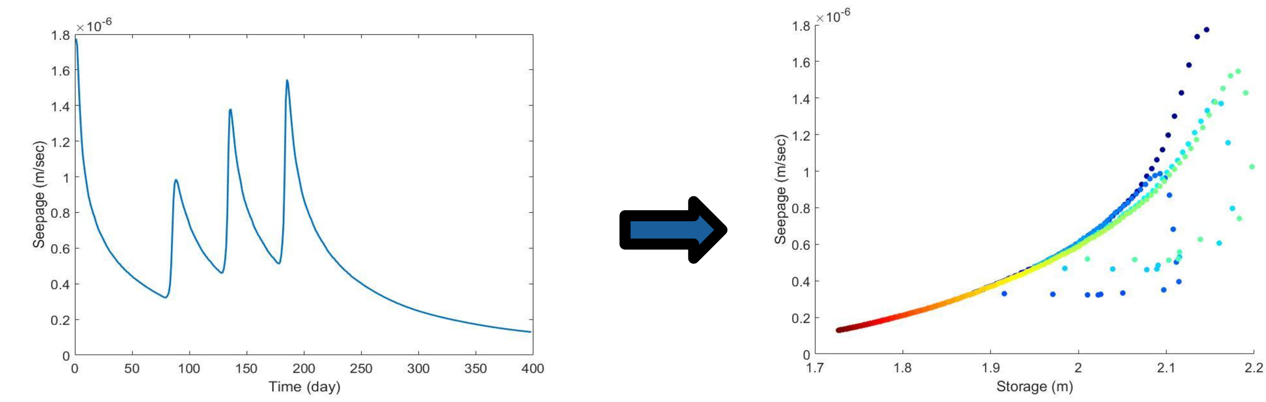
For this case, we setup a **tiled hillslope** in **Hydrus-3D.**

Tiled | **No tiles**

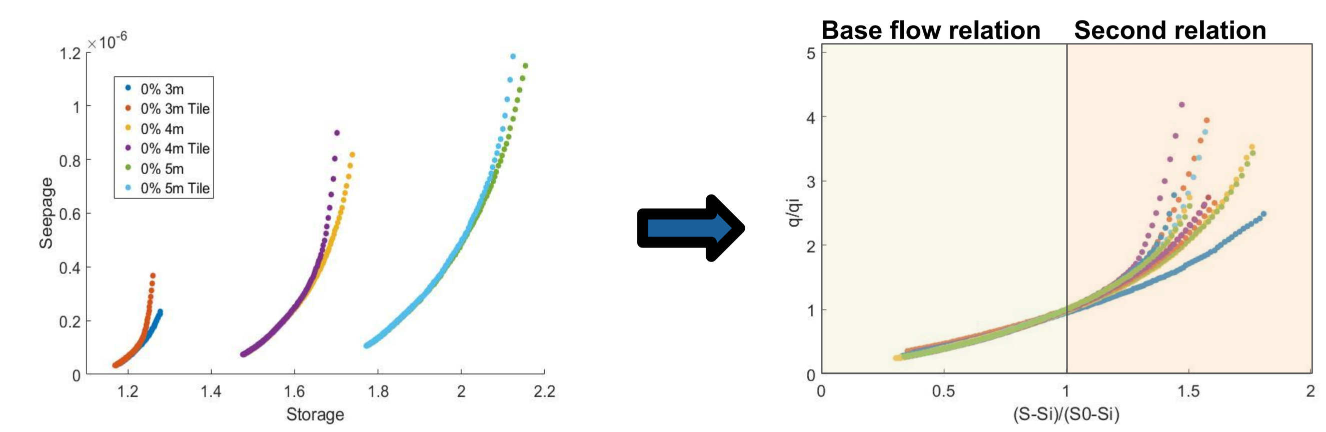


3. ODE representation

3D simulation results .

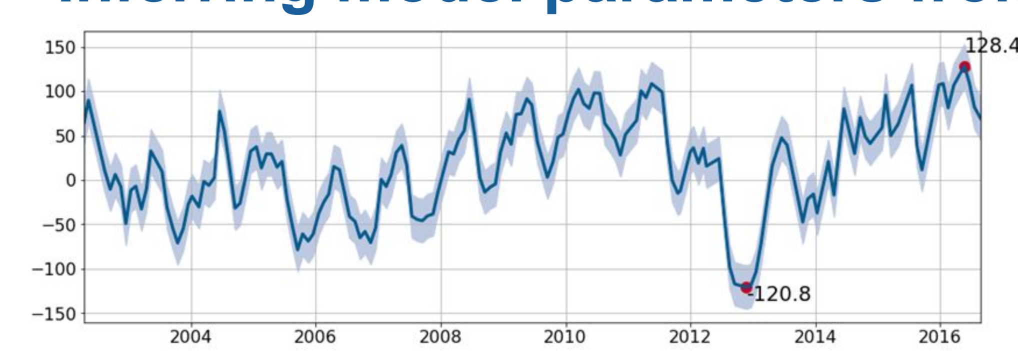


Storage / Seepage relationship obtained from Hydrus-3D model.



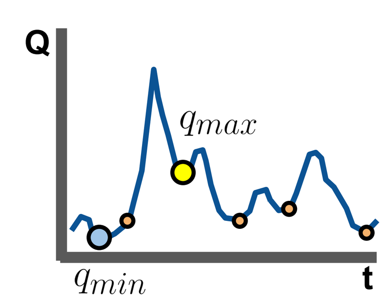
We identify a non-dimensional non-linear storage-seepage relationship that accounts for different hillslope slopes, subsurface depths, and tiling configurations

Inferring model parameters from data



Active water storage derived from GRACE

$$\Delta S = 0.248 \text{ [m]}$$

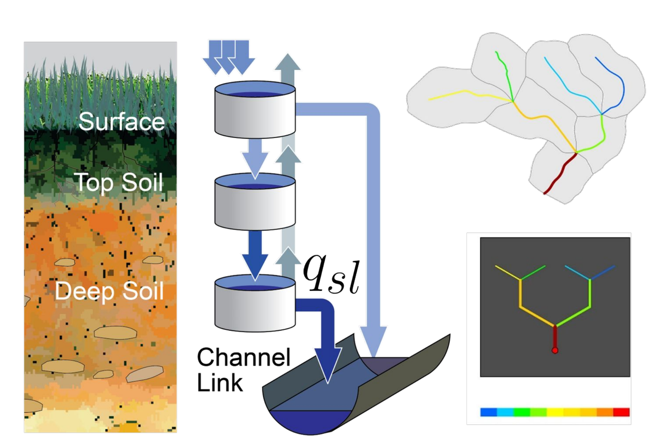


$$\Delta q = \max_t q_{min} - \min_t q_{min}$$

$$K_3 = \frac{\Delta q}{\Delta S \cdot A} \qquad \Delta S = \frac{\Delta q}{K_3 \cdot A}$$

$$\frac{\min_t q_{min}}{\max_t q_{min}}$$

HLM model modification:



HLM - Linear reservoir:

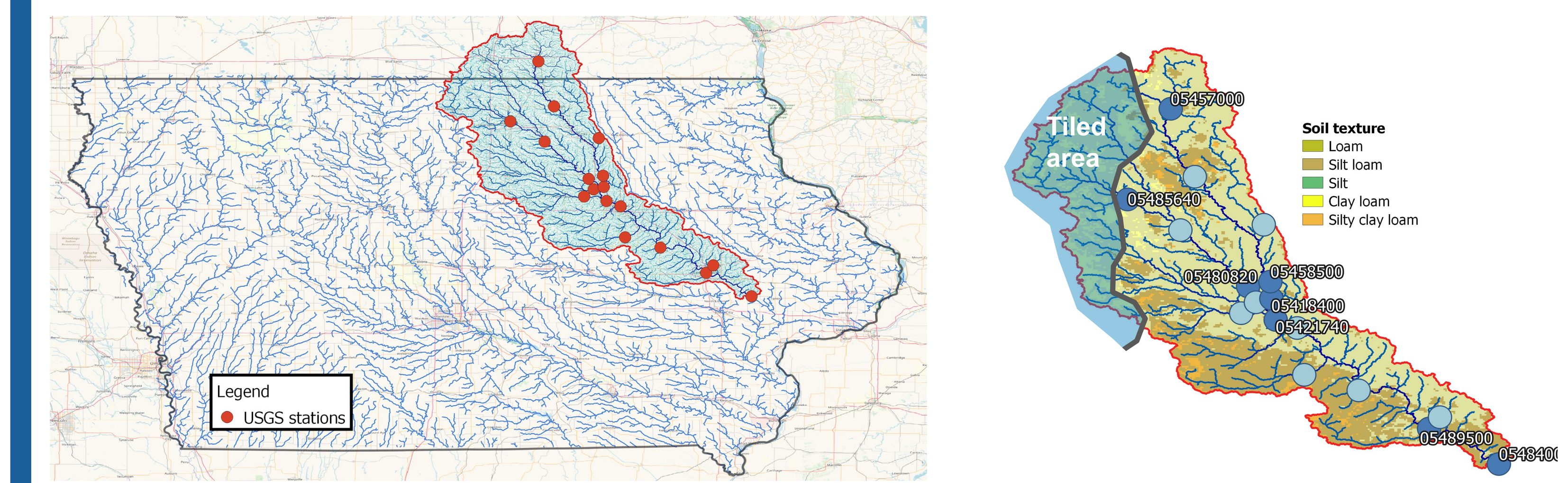
$$q_{sl} = S_s \cdot K_3$$

HLM - Tile:

$$q_{sl} = \begin{cases} S_3 \cdot k_3 & \text{if } S_s < S_i \\ \alpha e^{\beta S_s} & \text{if } S_s \geq S_i \end{cases}$$

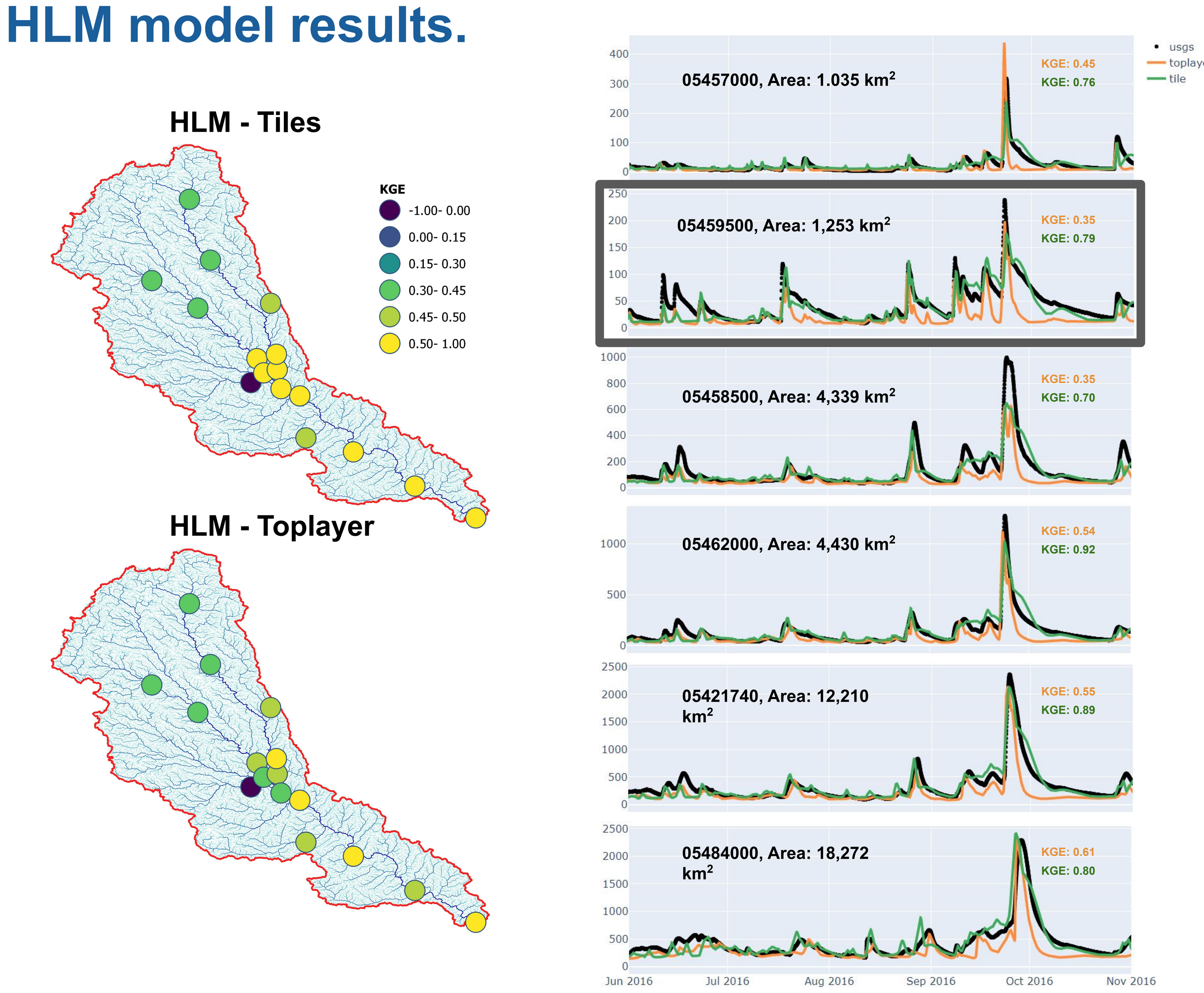
4. Results

Watershed scale simulations



We validate model results at **19 USGS** stations.

HLM model results.



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