

Identification of Linear and Nonlinear Causal Relationship Among Low-Frequency Climatic Phenomena in the Last Millennium

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Abstract

Considering that the instrumental climate record covers a period of about a century, it becomes necessary to use paleoclimatic records/models to explore the stability of the climatic variability and investigate the robustness of the teleconnections of the past. It is important to identify if the observed patterns in the current period persist over the last millennium when the changes in the orbital induced climate variations are negligible. In several studies on climatic causality crosscorrelation functions are used and the analysis is based on the relationship between atmospheric structures in pairs, a procedure that has several limitations in the elucidation of the network of possible connections. To mitigate these barriers, this work uses Partial Directed Coherence (PDC) and kernel nonlinear Partial Directed Coherence (knPDC) to allow the inference of the linear or nonlinear couplings between the climatological patterns, respectively. Connections between the two groups of climatic indicators in the last millennium were observed from 850 to 1850. The first group comprises the El Nino-Southern Oscillation (ENSO), Atlantic Multidecadal Oscillation (AMO) and Atlantic Interhemispheric SST Gradient (GTA) and the second, Antarctic Oscillation (AAO), El Nino-Southern Oscillation (ENSO), Pacific-South American (PSA1, EOF2) and QuasiBiennial Oscillation (QBO). The climate indices were computed from a weighted average set of climate model simulations of PMIP3, which represent simulations oriented to the past climate in the climate projection models of CMIP5. For the first group, no significant results were observed on the low-frequency band, observing only linear relationships between the Pacific and Atlantic Oceans. For the second group, the causal analysis point to linear relationships between ENSO-AAO, and nonlinear between ENSO-PSA in the low and high band and QBO-AAO, QBO-ENSO and QBO-PSA in the low-frequency band. In summary, the results indicate a higher nonlinear connection between low-frequency phenomena.

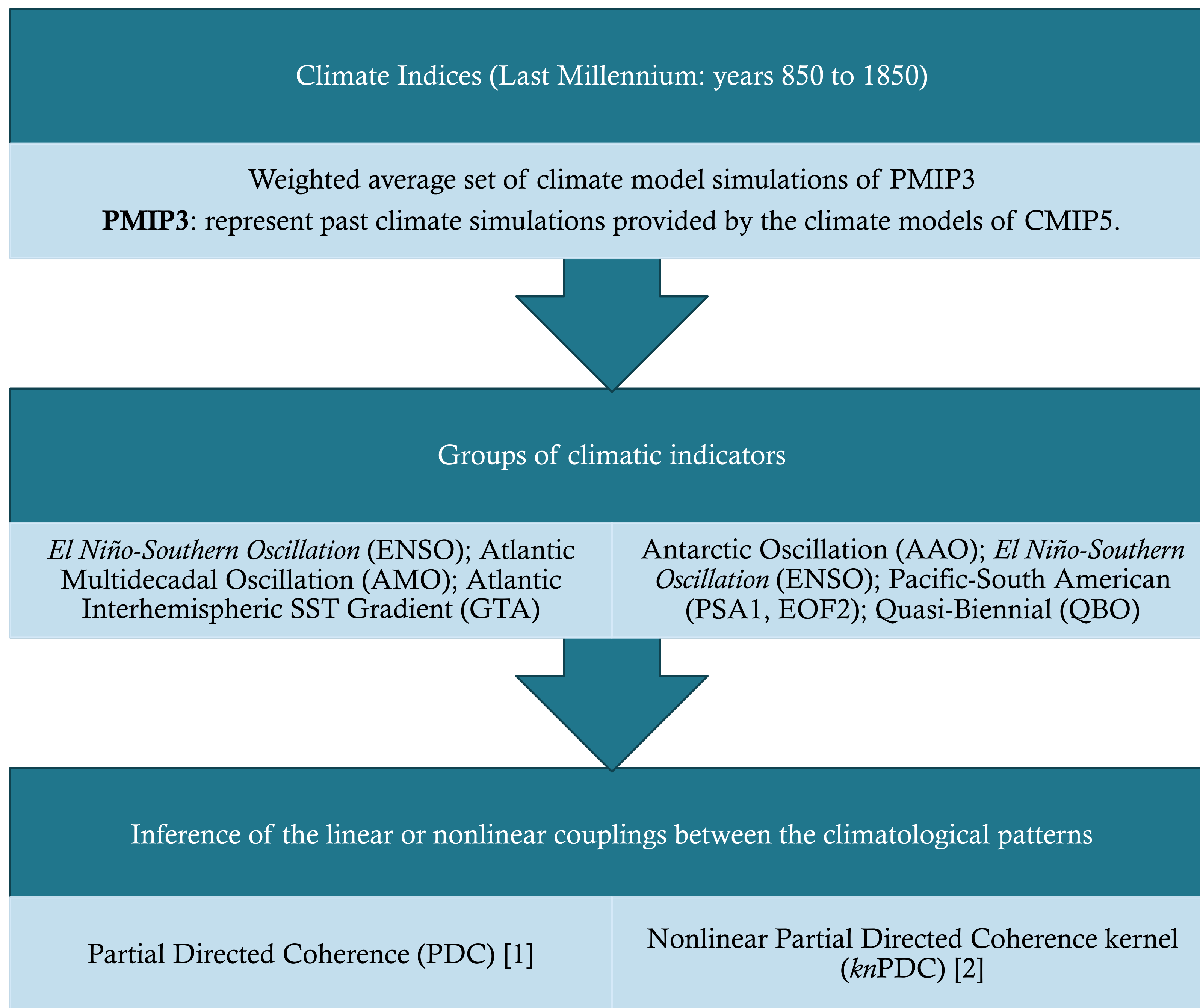
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INTRODUCTION

- In meteorology, identification of teleconnections between climatic patterns plays an important role in the validation of atmospheric models which are used for weather and climate prediction and for the development of future climate scenarios under global warming forcing.
- In order to evaluate the connectivity between climatic patterns, correlation analysis is often used, but this type of analysis may lead to oversimplified relationships, which do not imply causality between different scales of time (i.e., a nonlinearity).
- In this work, Partial Directed Coherence (PDC) and kernel non-linear Partial Directed Coherence (*kn*PDC) were used to infer the influence between atmospheric compartments (atmosphere and ocean), allowing the detection of linear and non-linear connections, respectively, between variables representative of important climatic variability modes in the PMIP3 simulations for the last millennium.

DATA AND METHODOLOGY



We represent the input series $\{x_i(n)\}_{n=1}^N$ (input space) through a Kernel Vector Autoregressive (kVAR) model, such as (Massarope and Baccalá, 2019)

$$\langle \phi(x(n)) | = \sum_{r=1}^p A_k \langle \phi(x(n-k)) | + \langle \tilde{w}(n) |,$$

where

- $\{\langle \tilde{w}(n) | \}_{n \in \mathbb{Z}} \sim i.i.d. WN(0, \Sigma_{\langle \tilde{w}(n) |})$
- $\phi: \mathbb{X} \rightarrow \mathbb{F}$ represents a nonlinear mapping (Parzen, 1959), such that $\mathbb{E}\{\langle \phi[x_i(n)] | \phi[x_i(n-k)] \rangle\} = \mathbb{E}\{x_i(n), x_i(n-k)\}$;
- $\kappa(\cdot)$: a Mercer kernel;
- $\langle \cdot | \cdot \rangle$: Dirac's 'bracket' notation.

The *kernel-nonlinear*-Partial Directed Coherence defined, in the phase space, as

$$\kappa_{ij} \pi_{ij}(f) = \frac{\bar{A}_{ij}(f) / \sqrt{\sigma_{ii}}}{\sqrt{\bar{a}_{ij}^H(f) \Sigma_{\langle \tilde{w}(n) |}^{-1} \bar{a}_{ij}(f)}}$$

where

- $\bar{A}_{ij}^\phi(f) = \delta_{ij} - \sum_{r=1}^p a_{ij}^\phi(r) e^{-i2\pi f r}$, ($i^2 = -1$);
- $a_{ij}^\phi(r)$ are the coefficients of na adequately fit kVAR model;
- $\bar{a}_{ij}(f)$ represent the columns of the $[\bar{A}_{ij}^\phi(f)]$ matrix.

PDC is similarly defined and can be seen in (Baccalá et al., 2013).

RESULTS

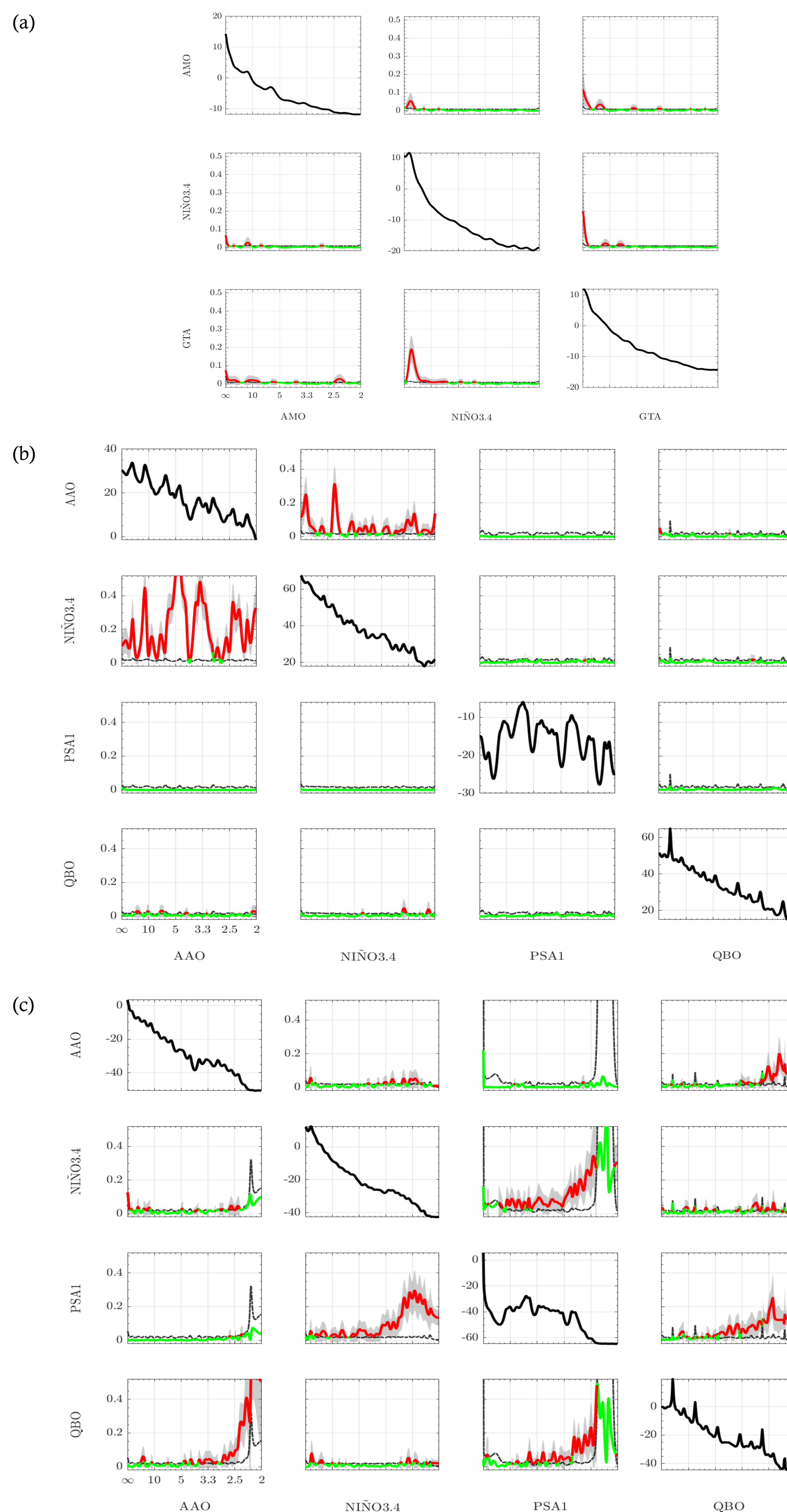


Fig. 1. The black line represents the (pseudo-) spectral density of the series, in dB.; the red line represents the statistically significant PDC / *kn*PDC values; the dashed black line represents Patnaik's threshold approximation (Baccalá et al., 2013); the green line the statistically non-significant PDC / *kn*PDC values. Therefore, using 1% significance level, the figures depict, respectively: (a) PDC for time series set (AMO, Niño 3.4 and GTA) using an autoregressive model of order $p = 48$, (b) PDC for time series set (AAO, Niño 3.4, PSA1 and QBO) using an autoregressive model of order $p = 24$ (c) *kn*PDC for the time series set (AAO, Niño 3.4, PSA1) and using the polynomial kernel $[k(x, y) = (x \cdot y)^2]$ and using a *kernel*-autoregressive model of order $p = 24$.

Fig.1a shows linear causality relationship (PDC) between: AMO \rightarrow GTA; AMO \rightarrow Niño 3.4; Niño 3.4 \rightarrow GTA; Niño 3.4 \rightarrow AMO; GTA \rightarrow Niño 3.4; GTA \rightarrow AMO. For nonlinear causality relationship (*kn*PDC), no results were observed in the low frequency period. The observed results suggests that the Atlantic and Pacific oceans influence each other at different times.

Fig.1b depicts a linear causality relationship (PDC) between: AAO \leftrightarrow Niño 3.4; AAO \leftrightarrow QBO; Niño 3.4 \rightarrow QBO. Some studies corroborate the results obtained with AAO and QBO (Gava et al., 2017), AAO and Niño 3.4 (Yu et al., 2015; Wang et al., 2017) and Niño 3.4 and QBO (Kane, 2005; Li et al., 2016).

*kn*PDC results (Fig.1c) suggests a nonlinear causal relationship between the same patterns observed in PDC, but with higher statistically significant values at high and low frequency. In addition, nonlinear causal relationships between PSA1 \leftrightarrow Niño3.4 and PSA1 \leftrightarrow QBO are observed. These results find support in other studies such as in Yu et al. (2015) for a relationship between PSA and ENSO and in Kane (2005) between PSA and QBO.

DISCUSSION AND CONCLUSIONS

For the first group, no significant results were observed on the low-frequency band, observing only linear relationships between the Pacific and Atlantic Oceans. For the second group, the causal analysis point to linear relationships between ENSO \leftrightarrow AAO, and nonlinear between ENSO \leftrightarrow PSA in the low and high band and QBO \leftrightarrow AAO, QBO \leftrightarrow ENSO and QBO \leftrightarrow PSA in the low-frequency band. In summary, the results indicate a higher nonlinear connection between low-frequency phenomena.

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