Saturation-ratio Fluctuations from Scalar Transport in Moist Rayleigh-Bénard Convection: One-dimensional-turbulence simulation

Kamal Kant Chandrakar[1], Will Cantrell[1], Steven Krueger[2], Raymond Shaw[1], and Scott Wunsch[3]

[1]Michigan Technological University
[2]Univ of Utah-Meteorology
[3]Johns Hopkins University

November 22, 2022

Abstract

A careful characterization of moisture fluxes and saturation-ratio statistics in atmospheric convection is significant for cloud microphysical processes and dynamics. The saturation-ratio of water vapor is defined as the ratio of actual water vapor pressure and its equilibrium value at a given air temperature. Therefore, it is a function of two scalars (water vapor and temperature) and is coupled through the nonlinear Clausius-Clapeyron equation. Participation of both scalar fields in the convection process and the nonlinear coupling of both scalars in saturation-ratio make this problem more complex, as compared to its dry-convection counterpart. We have explored heat and water vapor fluxes and saturation-ratio statistic in the moist Rayleigh-Bénard convection case, using the one-dimensional-turbulence (ODT) model developed by Wunsch et al. JFM 2005. This idealized small-scale simulation is a step toward understanding the full atmospheric convection problem at a more fundamental level. We have obtained the thermal and moisture fluxes as a function of the non-dimensional buoyancy parameter, also known as moist Rayleigh number, and compared it with the scaling relations. Moreover, we have examined the mean and variance profiles of saturation-ratio, and analyzed the different contributing terms for saturation-ratio fluctuations. Based on the scaling analysis, a simplified relation between saturation-ratio variance and moist Rayleigh number has been derived and compared with the simulation results. Additionally, we found that different values of water vapor and thermal diffusivities make the saturation-ratio pdf broader than the case when they are considered equal.

Supersaturation Fluctuations from Scalar Transport in Moist Rayleigh-Bénard Convection: One-Dimensional-Turbulence Simulation

Kamal Kant Chandrakar [1], Will Cantrell [1], Steven Krueger [2], Raymond A. Shaw [1], and Scott Wunsch [3]

[1]Department of Physics, Michigan Technological University [2]Department of Atmospheric Sciences, University of Utah [3]Applied Physics Laboratory, Johns Hopkins University

Introduction

A careful characterization of moisture fluxes and supersaturation statistics in atmospheric convection is significant for cloud microphysical processes and dynamics. The small-scale supersaturation fluctuation could be an important mechanism for droplet size-distribution broadening [1]. In an idealized sense, atmospheric boundary-layer convection is equivalent to Rayleigh-Bénard convection [2, 3]. Here, continuous plume eruption from boundaries transports heat and moisture and produces the mixed layer [4]. The presence of multiple driving scalars (water vapor and temperature), with slightly different diffusivities, adds to the degree of complexity. Additionally, in the supersaturation statistics, a nonlinear coupling between water-vapor and temperature fields makes the problem intriguing. Important parameters for this problem are [5]:

$$Ra_{moist} \approx \frac{g\Delta T H^3}{T_0 \nu D_t} + \frac{g\epsilon \Delta q_v H^3}{\nu D_t}, \quad Pr \equiv \frac{\nu}{D_t}, \quad \text{and} \quad Sc \equiv \frac{\nu}{D_v}.$$

Numerical Approach

We approach this problem using an idealized, one-dimensional-turbulence (ODT) model that faithfully represents the processes of advection and diffusion in turbulent convection, including fully-resolved boundary layers [6]. The range of Rayleigh number $2.05 \times 10^8 \leq Ra_{moist} \leq 2.75 \times 10^9$ covered in simulations is relevant to the Π-chamber. In order to understand the relative roles of the two diffusivities, we use the following four combinations.

- Actual D_v and D_t: actual diffusivities for the water vapor and thermal fields at 283 K ($D_v/D_t = 1.16$).
- $D_v = D_t$: D_v = the actual thermal diffusivity ($D_v/D_t = 1$).
- $4 \times D_v$: D_v = four times the actual value, and D_t = same as the actual value ($D_v/D_t = 4.63$).
- $4 \times D_t$: D_v = same as the actual value, and D_t = four times the actual value ($D_v/D_t = 0.29$).

Boundary Fluxes

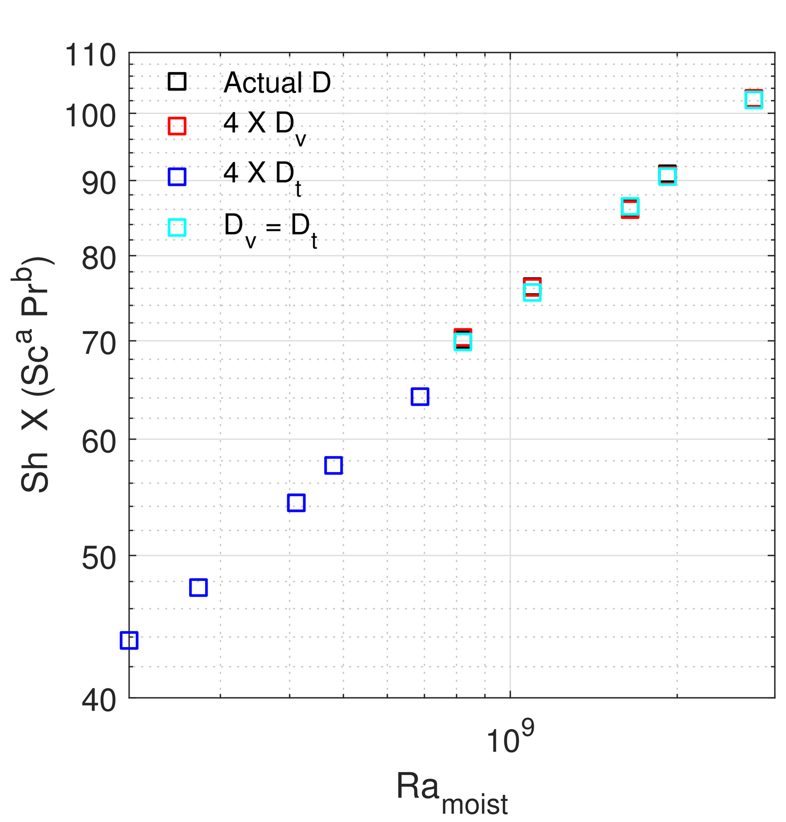

Figure 1: Variation of the scalar fluxes of water vapor (Sh) with moist Rayleigh number (Ra_{moist}). Fitting of the scaled Sh data produce Ra_{moist} exponents around 0.328 ± 0.006.

Supersaturation Fluctuations

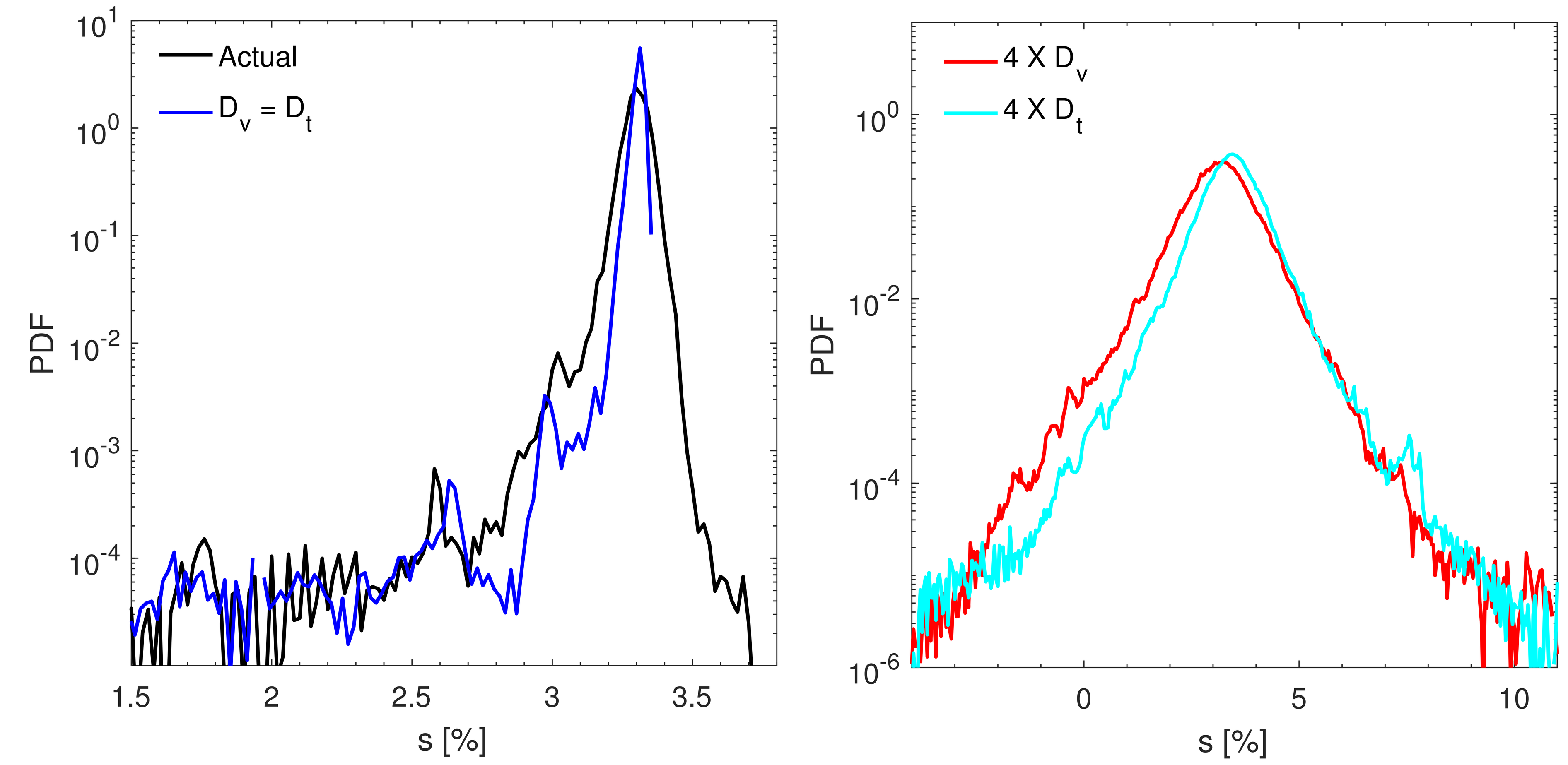

Figure 2: Sample PDFs of supersaturation near the domain center for the different diffusivity cases (8 K applied temperature difference).

Scaling Analysis

Moisture flux:

$$Sh \propto Sc^{1/2} Ra_{moist}^{1/3} \,|Pr \sim 1 \qquad Sh \propto Sc^{1/2} Pr^{-1/3} Ra_{moist}^{1/3} \,|Pr \ll 1$$

Sh : Sherwood Number; Pr : Prandtl Number; Sc : Schmidt Number

Supersaturation Fluctuations:

$$\sigma_s^2 \approx S^2 \left[\left(\frac{\sigma_{e_v}}{\bar{e_v}} \right)^2 + \zeta^2 \left(\frac{\sigma_T}{\bar{T}} \right)^2 - 2\zeta \frac{\overline{T' e_v'}}{\bar{T} \bar{e_v}} \right]$$

$$\left(\frac{\sigma_s}{S} \right)^2 \sim \xi^2 \left[1 - \frac{1}{S} \left(\frac{Pr}{Sc} \right)^{1/2} \right]^2 Pr^{-2} (1 + \sqrt{Pr})^{-1} \, Ra_{moist}^{5/3} \, \left(\frac{z}{H} \right)^{-1}$$

σ_T: temperature STD; σ_{qv} : water-vapor STD; z: vertical position; H : domain height; $\xi \propto H^{-3}$
ΔT: applied temperature difference; Δq_v: applied water vapor mixing-ratio difference
ν: kinematic viscosity; D_t: thermal diffusivity; D_v: water vapor diffusivity

Supersaturation Fluctuations in Bulk: Contributions from Both Scalars

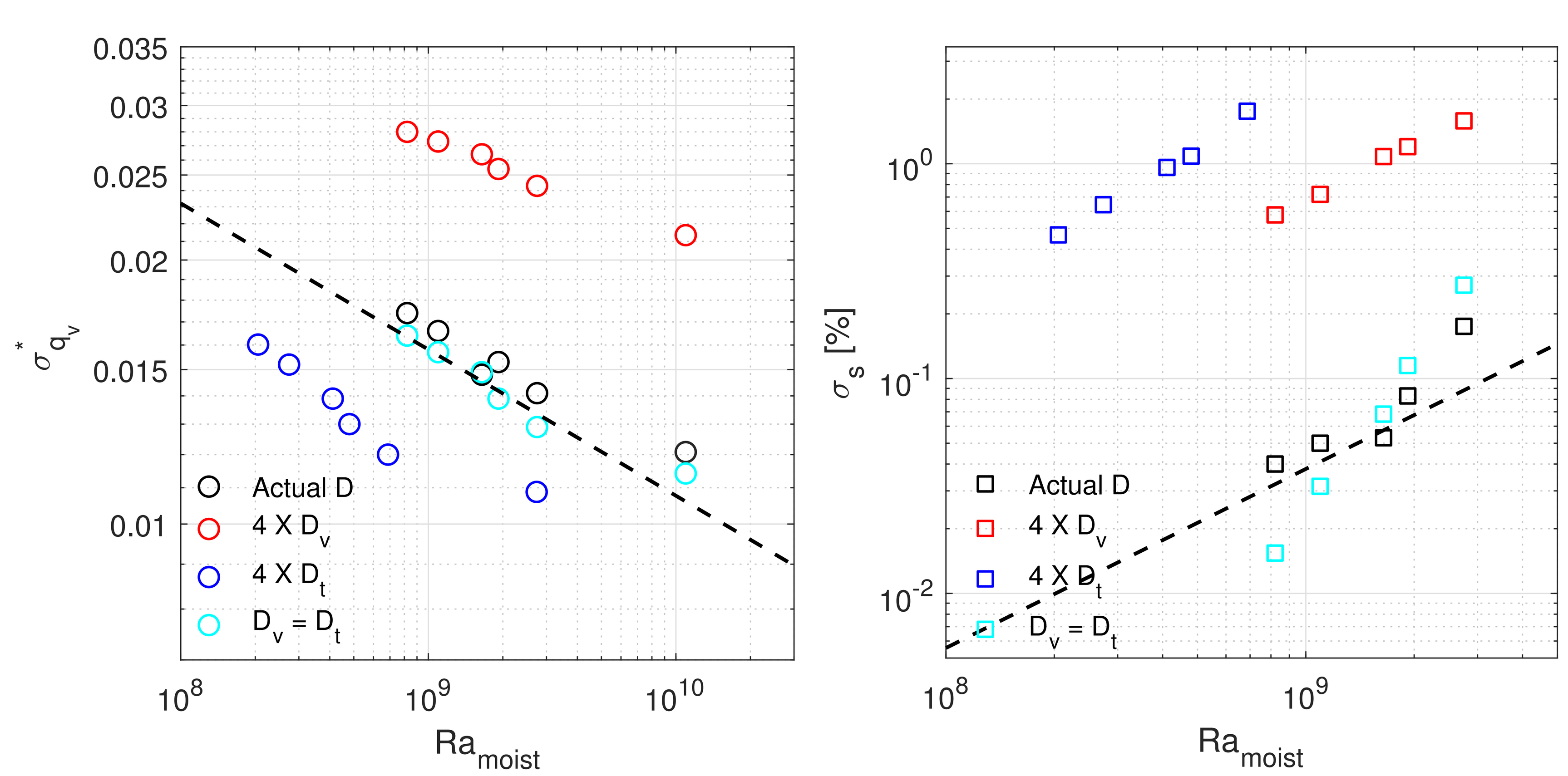

Figure 3: STDs of normalized water vapor mixing-ratio (**left**) and supersaturation fluctuation (**right**) at the domain center versus Ra_{moist}, and their comparison with scaling relations (dash-line) $Ra_{moist}^{-1/6}$ and $Ra_{moist}^{5/6}$.

Mixing Diagram

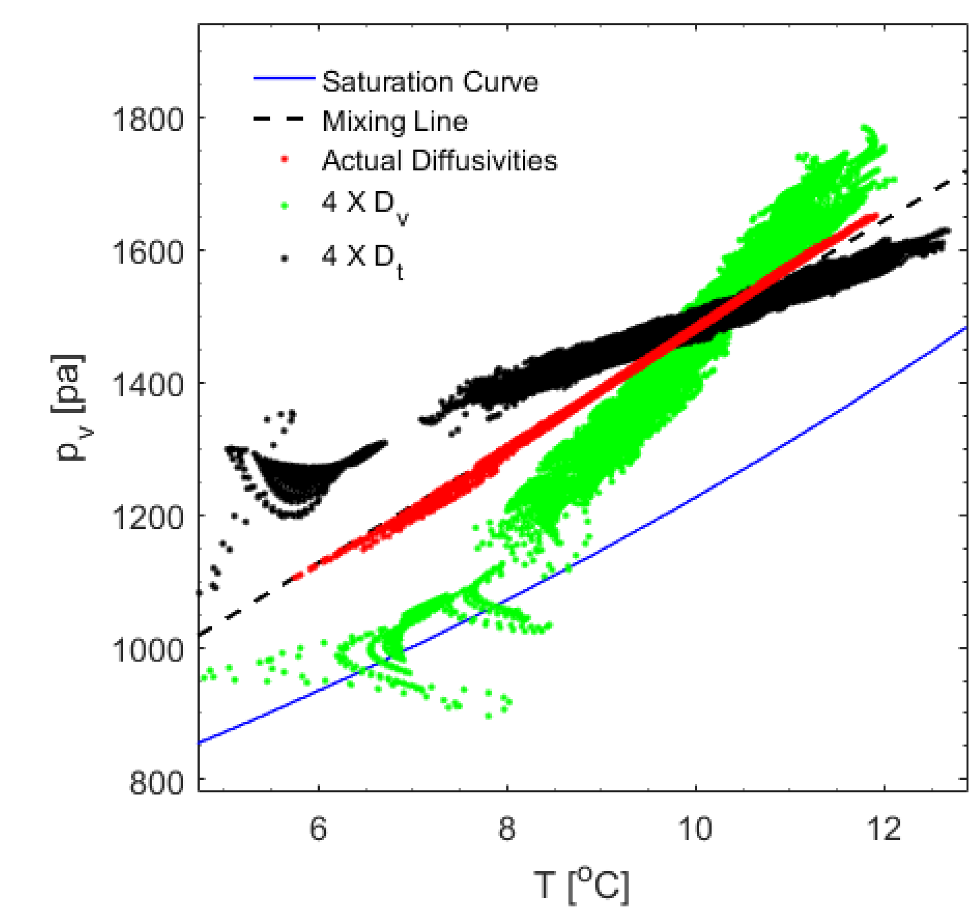

Figure 4: Mixing diagram for different diffusivity cases at 20K applied temperature difference.

Conclusion

- Scaling relations for heat and moisture fluxes are obtained as a function of Ra_{moist}, Pr, and Sc.
- In the bulk fluid, σ_T^* and σ_{qv}^* both follow a $Ra_{moist}^{-1/6}$ scaling relation. Moreover, the magnitude of scalar fluctuations increases with an increase in the respective scalar diffusivity.
- PDFs of supersaturation become broader with an increase in absolute value of diffusivity difference. Also, PDFs are slightly negatively skewed for cases with low diffusivity difference, unlike the T and q_v PDFs.
- Both, scaling and numerical output suggests: $\sigma_s^2 \propto Ra_{moist}^{5/3}/H^6$.
- The analysis of numerical output shows similar order contributions to the supersaturation variance from both scalar variance and covariance.
- Distribution of points in a pressure-temperature mixing diagram deviate from the classical mixing line for isobaric mixing, when $D_v \neq D_t$.

References

[1] Chandrakar *et al.* *Proc. Natl. Acad. Sci. (USA)*, 113(50).
[2] James W Deardorff. *J. Atmos. Sci.*, 27(8):1211–1213, 1970.
[3] Douglas K Lilly. *Quart. J. Roy. Meteor. Soc.*, 94(401):292–309, 1968.
[4] Kerry A Emanuel. Oxford University Press, 1994.
[5] Niedermeier *et al.* *Phys. Rev. Fluids*, 3(8):083501, 2018.
[6] Wunsch *et al.* *J. Fluid Mech.*, 528:173–205, 2005.

Acknowledgements

This work was supported by National Science Foundation Grant AGS-1754244 and by the NASA Earth and Space Science Fellowship Program.